SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 3, No. 1, June 2006, 45 - 54

Reduction the effects of opamp finite gain and offset voltage in LDI termination with a minus one half delay of SC ladder filters

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Abstract: In this paper a combined approach for reducing the effects of op amp imperfections (finite gain A and offset voltage V_{OS}) in first-order SC cell, realizing LDI (lossless discrete integrator) termination with a minus one half delay is presented. First, the conventional integrator is replaced with gain- and offset-compensated (GOC) integrator. Next, the gain errors $m(\omega)$ and the phase errors $\theta(\omega)$ are further reduced by using the precise op amp gain approach in the GOC structure. The variation of the dc gain A from its nominal value A_0 is taken into account.

Keywords: Ladder filters, Gain- and offset- compensation, Operational amplifiers, Switched-capacitor integrators.

1 Introduction

Standard design of switched-capacitor (SC) networks assumes operation with infinite dc gain A and infinite bandwidth *GB* operational amplifiers (op amps). However in the op amp design a tradeoff exists between the speed and the gain. As a consequence when a high sampling frequency f_s is used, the needed large bandwidth limits the op amp gain to low values, therefore limiting the achievable accuracy. For these cases alternative design approaches are needed that address the op amp design tradeoff between speed and gain, allowing the frequency range of SC networks to be extended.

The two known general approaches for reducing the op amps finite gain effects in SC circuits are:

a) The finite-gain-insensitive (FGI) approach;

b) The precise op amp gain (POG) approach.

The FGI approach consists in the replacement of the conventional SC circuits with gain- and offset- compensated (GOC) structures. Several GOC SC

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building blocks (integrators, gain stage, sample-and–hold circuits) have been reported in the literature [1, 2].

The POG approach [3-7] is based on the use of simple and fast amplifiers with low but precisely known and stable gain. The precise nominal dc gain value A_0 is then taken into account in the capacitor sizing of classical SC structures. The result is a classical SC structure with modified capacitor values which implements exactly the required transfer function. The standard op amp design tradeoff between speed and gain is changed into the POG design tradeoff between speed and gain precisions which is more affordable in high-frequency op amps. Using a gain A_0 with maximum relative gain deviation ε in POG design results in the same response accuracy as with a gain of A_0/ε in standard design.

The two types of termination in LDI-transformed SC ladder filters are the termination with a minus one half delay and the termination with an extra half delay [8, 9]. In [4, 5] the POG approach is applied to one-input first-order cell, realizing LDI load termination with a minus one half delay. Analytical expressions for the gain errors $m(\omega)$ and the phase errors $\theta(\omega)$ in the standard design and with the POG approach are obtained and compared. The influence of the input-referred op amp dc offset voltage V_{OS} is not taken into account.

In this paper a combined approach for reduction the effects of op amp imperfections (finite gain A and offset voltage V_{OS}) is the first-order cell from [4, 5] is proposed. It is based on the use of the POG design approach in GOC structure. Analytical expressions for the gain and phase errors, due to a variation of the op amp dc gain A from its nominal value A_0 are derived and compared to the POG design case in the conventional structure.

2 Theoretical Results

In the following the op amp is assumed to have finite dc gain A and infinite bandwidth. This supposition is adequate for the analysis of SC circuits containing fast and relatively low-gain amplifiers.

Fig.1a shows the circuit schema of the conventional first-order cell [4, 5]. This damped integrator has two feed-in branches. When the circuit is used as load termination the inverting integrator branch through the switched capacitor C_q is connected to the output of one another op amp in the ladder filter. The inverting integrator branch through the switched capacitor C_1 is connected to the input of the ladder filter and in conjunction with the branch C_q realizes the source termination. The nonzero input-referred dc offset voltage of the op amp is modelled as a voltage source at the noninverting input terminal.

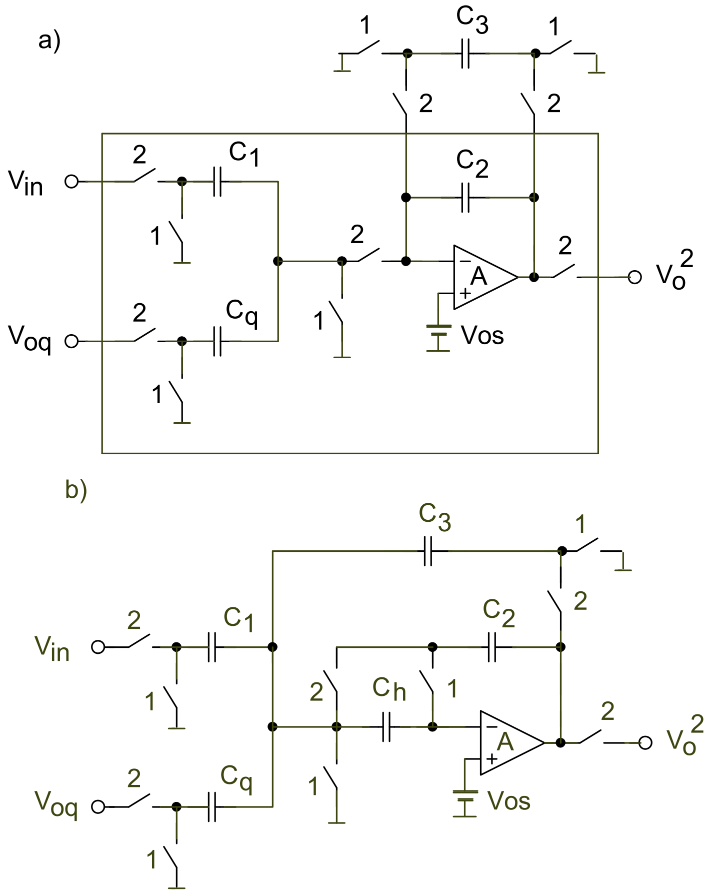

Fig. 1 – *First-order cell*: a) *Conventional structure*. b) *GOC structure.*

At first, the POG approach is applied to the conventional structure (Fig. 1a). In the *z*-domain the actual output voltage is given by

$$\mathrm{V}_o^{\,2}(z) = -\frac{C_{1p}\ \mathrm{V}_{in}^2(z) + C_{qp}\ \mathrm{V}_{oq}^{\,2}(z)}{C_{2p} + C_{3P} + (1/\mathrm{A})(C_{1p} + C_{2p} + C_{3p} + C_{qp}) - C_{2p}(1 + 1/\mathrm{A})\ z^{-1}} = \tag{1}$$
$$= H_{Cp1}^{22}(z)\ \mathrm{V}_{in}^2(z) + H_{Cp\,q}^{22}(z)\ \mathrm{V}_{oq}^{\,2}(z),$$

while the standard case ideal output voltage (for $\mathrm{A} \to \infty$) is

$$\mathrm{V}_o^{\,2}{}_{id}(z) = -\frac{C_1\ \mathrm{V}_{in}^2(z) + C_q\ \mathrm{V}_{oq}^{\,2}(z)}{C_2 + C_3 - C_3\ z^{-1}}\ . \tag{2}$$

To obtain the same transfer function the POG capacitance values C_{ip} are related to the standard values C_{i} as given in the following:

$$C_{1p} = C_1, \; C_{qp} = C_q, \; C_{2p} = C_2\left(1+\frac{1}{A}\right)^{-1},$$
$$C_{3p} = \left[C_3 - \frac{1}{A}(C_1 + C_q)\right]\left(1+\frac{1}{A}\right)^{-1}. \tag{3}$$

The gain error $m(\omega)$ and the phase error $\theta(\omega)$ of an actual z-transfer function $H_a(z)$ with respect to the ideal expression $H_{id}(z)$ are defined as [10],

$$H_a(z) = H_{id}(z)\,[1 + m(\omega)]\,e[j\theta(\omega)] \approx \frac{H_{id}(z)}{1 - m(\omega) - j\theta(\omega)}. \tag{4}$$

In the standard design, for the standard capacitance values C_i , the gain and phase errors of the conventional structure (Fig. 1a), due to a finite dc gain A, are

$$m_C(\omega) \approx -\frac{C_3\left(C_1 + C_3 + C_q\right) + C_2\left(2C_2 + 2C_3 + C_1 + C_q\right)\left(1 - \cos\omega T_s\right)}{A\,[C_3^2 + 2C_2(C_2 + C_3)(1 - \cos\omega T_s)]} \tag{5a}$$

$$\theta_C(\omega) \approx \frac{C_2\left(C_1 + C_q\right)\sin\omega T_s}{A\,[C_3^2 + 2C_2(C_2 + C_3)(1 - \cos\omega T_s)]}. \tag{5b}$$

On the other hand, with the POG approach the gain error $m_{Cp}(\omega)$ and the phase error $\theta_{Cp}(\omega)$, due to an actual gain equal to $A = A_0 + \Delta A_0$, are given by the expressions

$$m_{Cp}(\omega) \approx \frac{\Delta A_0(1 - \Delta A_0/A_0)}{A_0^2[C_3^2 + 2C_2(C_2 + C_3)(1 - \cos\omega T_s)]}\Big\{C_3\left(C_{3p} + C_1 + C_q\right) + \left[C_{2p}C_3 + C_2\left(2C_{2p} + C_{3p} + C_1 + C_q\right)\right]\left(1 - \cos\omega T_s\right)\Big\}, \tag{6a}$$

$$\theta_{Cp}(\omega) \approx -\frac{\Delta A_0(1 - \Delta A_0/A_0)\sin\omega T_s}{A_0^2[C_3^2 + 2C_2(C_2 + C_3)(1 - \cos\omega T_s)]} \times \left[C_2\left(C_{3p} + C_1 + C_q\right) - C_{2p}C_3\right]. \tag{6b}$$

The expressions (5) and (6) for the gain and phase errors are similar to the corresponding expressions given in [4, 5].

Comparing (5) and (6), since the capacitance values C_{ip} and C_i are almost equal (3), it can be seen that the POG effective gain is $A_0(1+\varepsilon)/\varepsilon$ for $\varepsilon = \Delta A_0/A_0$.

The combined approach proposed consists in the following two consecutive steps:

First: For reducing the effect of op amp imperfections (dc gain A and offset voltage V_{OS}) the conventional integrator of Fig. 1a (the block enclosed in broken line) is replaced with Ki-89 GOC SC integrator [11]. The resulting GOC first-order cell is shown in Fig. 1b. The holding capacitance C_h is equal to the integrating capacitance C_2.

Next: The POG design approach is applied to the GOC structure from Fig.1b. In the z-domain the actual output voltage is given by

$$\mathrm{V}_o^2(z) = -\frac{C_{1p}\,\mathrm{V}_{in}^2(z) + C_{qp}\,\mathrm{V}_{oq}^2(z)}{a - b\,z^{-1}}, \tag{7}$$

where

$$a = C_{2p} + C_{3p} + \frac{1}{\mathrm{A}}\,(2\,C_{2p} + C_{3p} + C_{1p} + C_{qp}),$$

$$b = \left[C_{2p} + \frac{1}{\mathrm{A}}\,(2\,C_{2p} + C_{3p} + C_{1p} + C_{qp})\right]\left(1+\frac{1}{\mathrm{A}}\right)\left(1+\frac{2}{\mathrm{A}}\right)^{-1}.$$

Comparing (2) and (7) one gets the following expressions for the POG capacitance values:

$$C_{1p} = C_1,\;\; C_{qp} = C_q,\;\; C_{2p} = C_2(1+\frac{1}{\mathrm{A}})^{-1} - \left(\frac{C_1 + C_3 + C_q}{\mathrm{A}}\right)\left(1+\frac{1}{\mathrm{A}}\right)^{-2},$$
$$C_{3p} = \left[C_3\left(1+\frac{2}{\mathrm{A}}\right) - \frac{C_1 + C_q}{\mathrm{A}^2}\right](1+\frac{1}{\mathrm{A}})^{-2} \approx C_3. \tag{8}$$

In the standard design, for the standard capacitance values C_i, the gain and phase errors of the GOC structure (Fig. 1b), due to a finite dc gain A, are

$$m_K(\omega) \approx -\frac{\left(C_1 + C_2 + C_3 + C_q\right)\left(2C_2 + C_3\right)\left(1-\cos\omega T_s\right)}{\mathrm{A}\,[C_3^2 + 2C_2(C_2 + C_3)\left(1-\cos\omega T_s\right)]} - $$
$$-\frac{[C_3\cos\omega T_s - C_2(1-\cos\omega T_s)]\,(C_3 + C_1 + C_q)}{\mathrm{A}^2(1+2/\mathrm{A})\,[C_3^2 + 2C_2(C_2 + C_3)\left(1-\cos\omega T_s\right)]}, \tag{9a}$$

$$\theta_K(\omega) \approx -\frac{C_3\left(C_1 + C_2 + C_3 + C_q\right)\sin\omega T_s}{\mathrm{A}\,[C_3^2 + 2C_2(C_2 + C_3)\left(1-\cos\omega T_s\right)]} + $$
$$+\frac{(C_2 + C_3)(C_3 + C_1 + C_q)\sin\omega T_s}{\mathrm{A}^2(1+2/\mathrm{A})\,[C_3^2 + 2C_2(C_2 + C_3)\left(1-\cos\omega T_s\right)]}. \tag{9b}$$

With the POG approach the gain error $m_{Kp}(\omega)$ and the phase error $\theta_{Kp}(\omega)$ due to an actual gain equal to $A = A_0 + \Delta A_0$ are given by the expressions:

$$\begin{aligned} m_{Kp}(\omega) \approx & \frac{\Delta A_0 (1 - \Delta A_0 / A_0)}{A_0^2 [C_3^2 + 2C_2 (C_2 + C_3)(1 - \cos\omega T_s)]} \times \\ & \times \Big\{ \big(C_{2p} + C_{3p} + C_1 + C_q\big)\big(C_3 + 2C_2\big)\big(1 - \cos\omega T_s\big) + \\ & + \frac{2}{A_0}\big(C_{3p} + C_1 + C_q\big)\big[C_3 \cos\omega T_s - C_2\big(1 - \cos\omega T_s\big)\big]\Big\}, \end{aligned} \tag{10a}$$

$$\begin{aligned} \theta_{Kp}(\omega) \approx & \frac{\Delta A_0 (1 - \Delta A_0 / A_0) \sin\omega T_s}{A_0^2 [C_3^2 + 2C_2 (C_2 + C_3)(1 - \cos\omega T_s)]} \times \\ & \times \left[C_3\big(C_{2p} + C_{3p} + C_1 + C_q\big) - \frac{2}{A_0}\big(C_{3p} + C_1 + C_q\big)\big(C_2 + C_3\big) \right]. \end{aligned} \tag{10b}$$

Comparing (9) and (10), since the capacitance values C_{ip} and C_i are almost equal (8), it can be again seen that the POG effective gain is $A_0(1+\varepsilon)/\varepsilon$ for $\varepsilon = \Delta A_0 / A_0$.

To verify the effectiveness of the proposed combined approach the source termination of the 3rd-order SC lowpass ladder filter from [12] is considered. The standard capacitance values of the conventional structure used (Fig. 1a) are: C_1=3.7037038, C_2=5.8046478, C_3 = C_q= 1.8518519. The sampling frequency is f_s = 54 MHz. The errors responses of the two terminations from Fig. 1 with standard design are compared in Fig. 2.

The errors are computed from (5) and (9) for $A = A_0 = 100$. It is seen that the gain errors $m_K(\omega)$ of the GOC structure are smaller than those of the conventional structure in the frequency range $0 \le f < f_g$. At signal frequency

$$f_g = \frac{f_s}{2\pi} \arccos\frac{C_2}{C_2 + C_3} = 6.105\,\text{MHz}. \tag{11}$$

The gain errors $m_C(f_g)$ and $m_K(f_g)$ are equal.

The phase errors $\theta_K(\omega)$ of the GOC structure are smaller than those of the conventional structure. The two errors $\theta_C(\omega)$ and $\theta_K(\omega)$ are maximal at signal frequency:

$$f_p = \frac{f_s}{2\pi} \arccos \frac{2C_2(C_2+C_3)}{C_3^2+2C_2(C_2+C_3)} = 2.35\,\mathrm{MHz}\,. \tag{12}$$

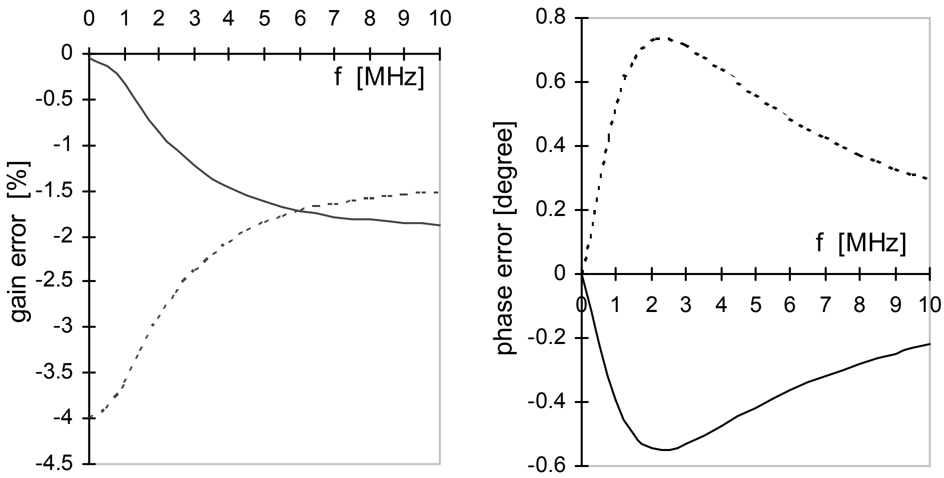

Fig. 2 – *Gain error responses and phase error responses of the source terminations with standard design for* $\mathrm{A} = \mathrm{A}_0 = 100$.

--------------- Conventional structure

———— GOC structure

The POG capacitance values for the two source terminations (Fig. 1) are calculated from (3) and (8) for $\mathrm{A} = \mathrm{A}_0 = 100$. On obtains

a) C_{1p}=3.7037038, C_{2p}=5.747176, C_{3p} =1.7785112, C_{qp}= 1.8518519 for the conventional structure in Fig. 1a;

b) C_{1p}=3.7037038, C_{2p}= C_h = 5.6745615, C_{3p} =1.8511258, C_{qp}= 1.8518519 for the GOC structure in Fig. 1b.

The error responses of the two terminations from Fig. 1 with POG design are depicted in Fig. 3. The corresponding gain and phase errors, due to deviation $\pm 8\,\%$ of the op amp gain A from its nominal values $\mathrm{A}_0 = 100$ [3] are computed from (6) and (10).

As for the standard design the gain errors $m_{Kp}(\omega)$ of the GOC structure for smaller than those of the conventional structure for $f < f_g$ (11). The difference between the absolute values of the gain errors $m_{Kp}(\omega)$ and $m_{Cp}(\omega)$ for $f = f_s/2$ is given by the expression

$$\left|m_{Kp}(f_s/2)\right|-\left|m_{Cp}(f_s/2)\right|=\frac{\left|\Delta A_0\right|\left(1-\Delta A_0/A_0\right)\left(C_3+2C_2\right)\left(C''_{3p}+C_1+C_q\right)}{A_0^2\left[C_3^2+4C_2\left(C_2+C_3\right)\right]}, \tag{13}$$

where $C_{3p}^{''}$ is the POG capacitance value for the GOC structure. One obtains

$$\left|m_{Kp}(f_s/2)\right|-\left|m_{Cp}(f_s/2)\right|=0.0401\% \quad \text{for} \quad \Delta A=8 \;\; (A=108)$$
$$\left|m_{Kp}(f_s/2)\right|-\left|m_{Cp}(f_s/2)\right|=0.0471\% \quad \text{for} \quad \Delta A=8 \;\; (A=108).$$

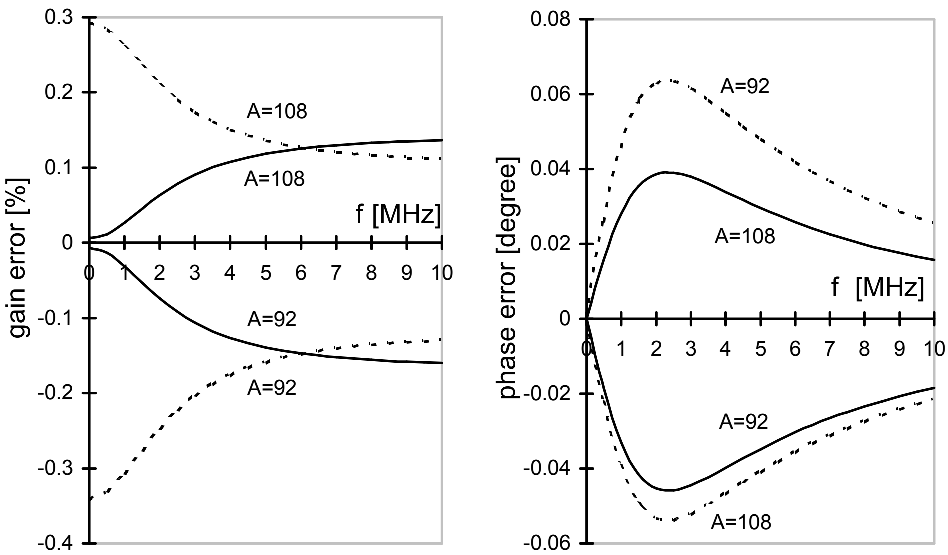

Fig. 3 – *Gain error responses and phase error responses of the source terminations with* POG *design and gain variation* $A=100\pm 8$.

---------------- Conventional structure
———————— GOC structure

Obviously, for $f>f_g$ the gain errors m_{Kp} of the GOC structure are slightly larger than those of the conventional structure. At dc, $f=0$, the following relation holds

$$\left|\frac{m_{Cp}(0)}{m_{kp}(0)}\right|\approx\frac{A_0}{2}. \tag{14}$$

The phase errors $\theta_{Kp}(\omega)$ of the GOC structure are smaller than those of the conventional structure. The two errors $\theta_{Cp}(\omega)$ and $\theta_{Kp}(\omega)$ are maximal at signal frequency f_p (12).

The influence of the input-referred DC offset voltage V_{OS} is evaluated by the corresponding output voltage V_{OSS} in steady state, for $V_{in}=0$, as follows:

a) conventional structure (Fig. 1a)

$$\mathrm{V\,oss}_C = \frac{C_3 + C_1 + C_q}{C_3 + \frac{1}{\mathrm{A}}\left(C_3 + C_1 + C_q\right)} \mathrm{V_{OS}} . \tag{15}$$

b) GOC structure (Fig. 1b)

$$\mathrm{V\,oss}_K = \frac{C_3 + C_1 + C_q}{\mathrm{A}\left[C_3\left(1 + \frac{2}{\mathrm{A}}\right) + \frac{1}{\mathrm{A}^2}(C_3 + C_1 + C_q\right]} \mathrm{V_{OS}} . \tag{16}$$

For the POG capacitance values and $\mathrm{A} = \mathrm{A}_0 = 100$ one obtains

$$\mathrm{V\,oss}_C = 3.960\ \mathrm{V_{OS}},\quad \mathrm{V\,oss}_K = 0.0392\ \mathrm{V_{OS}} .$$

Therefore, the GOC termination of Fig. 1b is less sensitive to the op amp offset voltage.

3 Conclusion

The effects of op amp imperfections in conventional LDI termination with a minus one half delay are reduced by applying the precise op amp gain (POG) approach in the corresponding gain- and offset compensated (GOC) structure. Analytical expressions for the resulting gain, phase and offset errors with gain variation are derived and compared with the corresponding errors, obtained by the use of the POG approach in the classical structure.

The gain errors of the GOC structure are smaller than those of the conventional structure in a given frequency interval. The phase errors of the compensated structure are smaller than those of the conventional structure. The GOC termination is also less sensitive to the op amp offset voltage.

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