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Field Pattern of the Vertical Dipole Antenna above a Lossy Half-Space

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Abstract: The field pattern of the vertical dipole antenna (VDA) placed above a homogenous and isotropic medium is determined in this paper. The unknown current distribution (UCD) on the antenna is evaluated by solving the system of integral equations of Hallen's type (SIE-H) using a point-matching method (MoM) and a polynomial current approximation. The influence of the lossy half-space, expressed by the Sommerfeld's integral kernel (SIK), was modelled with a new, very simple and accurate approximate expression. In the surroundings of the antenna, this expression is valid for all positions of the antenna conductors, as well as all possible combinations of the electrical parameters of the ground. A certain number of numerical results were compared to the corresponding ones from the references.

Keywords: Vertical dipole antenna, Field pattern, Lossy half-space, Sommerfeld's integral.

1 Introduction

The problem of characterization of wire antennas placed above a semi-conducting ground has engrossed authors for almost one century since the all in mathematical formulation of this problem (Sommerfeld, 1909), which is notable from a rather rich bibliography of papers on this topic.

This paper presents a sequel of researches performed at the Faculty of Electronic Engineering in Niš, whose aim was to model the SIK in a simple way, but at the same time, to introduce as fewer limitations as possible and give the results of satisfying accuracy [1- 3].

At the last year International PhD seminar (Computation of Electromagnetic Field, Sept. 2004., Budva, S&M), the first author informed the participators about the results of her researches, and presented a new simple model for the SIK, which was in the paper [1] at that time offered, and in the mean time accepted for publishing. Values of the UCD polynomial approximation and the

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input impedance/admittance of the vertical asymmetrical dipole antenna placed above a homogenous and isotropic medium were, in a rather simple manner, very accurately numerically determined using the MoM method and this new model for the SIK [1, 2].

The UCD output results obtained by the proposed method were used, and the VDA field pattern was determined according to the far field model from [5]. The results for the field pattern were graphically illustrated for numerous examples. In the certain number of these examples, the results were compared to the available ones from [3, 4].

2 Theoretical Background

2.1 Description of the VDA model geometry

The VDA placed above a homogenous and isotropic lossy half-space, fed by an ideal voltage δ - generator, of voltage $U = 1\text{V}$ and angular frequency $\omega = 2\pi f$, is considered, Fig. 1. The antenna conductors are of length l_k, $k = 1,2$ and circular cross-section of radius a_k, $a_k << l_k$ and $a_k << \lambda_0$ (λ_0 - wave length in air). The air and ground parameters are: $\sigma_0 = 0$, ε_0, μ_0, σ_1, $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$, $\mu_1 = \mu_0$. The following labels were also introduced: $\underline{\gamma}_i = (j\omega\mu_i\underline{\sigma}_i)^{1/2}$, $i = 0,1$ - complex propagation constant, $\underline{\sigma}_i = \sigma_i + j\omega\varepsilon_i = j\omega\varepsilon_0\underline{\varepsilon}_{ri}$, $i = 0,1$ - complex conductivity, $\underline{n}_{10} = \underline{\gamma}_1 / \underline{\gamma}_0 = \underline{\varepsilon}_{r1}^{1/2}$ – refraction index and $\underline{\varepsilon}_{r1} = \varepsilon_{r1} - j\varepsilon_{i1} = \varepsilon_{r1} - j60\sigma_1\lambda_0$ - complex relative permittivity. The UCD localised along the conductor axis is denoted by $I_k(s'_k)$, $0 \le s'_k \le l_k$, $k = 1,2$. Beginnings of s'_k - axes are at points $z_{A1} = h$ and $z_{A2} = h - l_2$ (i.e. $z'_k = z_{Ak} + s'_k$, $k = 1,2$). The height of the antenna feed point is h, $h \ge l_2$.

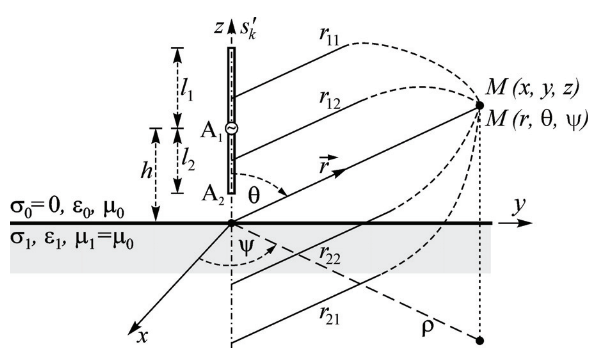

Fig. 1 – *The illustration of the VDA above a lossy half-space with the far field point M.*

2.2 Brief description of the UCD evaluation using the new SIK model

The first step in solving of the proposed problem relates to determination of the UCD along the VDA conductors, as described in [1].

For the purpose of determining the UCD and the input impedance/ admittance of the VDA, the moment method for numerical solving of the SIE-H (Appendix A1 in [3]) with the polynomial approximation for the UCD, was used. In order to form the SIE-H it is primarily necessary to model the Hertz's vector in the vicinity of the VDA, which is described by the general expression:

$$\vec{\Pi}_0 = \Pi_{z0}\,\hat{z} = \frac{1}{4\pi\underline{\sigma}_0}\sum_{k=1}^{2}\int_{s_k'=0}^{l_k} I_k(s_k')[K_0(r_{1k}) + S_{00}^{v}(r_{2k})]\,\mathrm{d}s_k'\;\hat{z}\,, \tag{1}$$

where $I_k(s_k')$ is the UCD, $K_0(r_{1k})$ is the standard potential kernel and $S_{00}^{v}(r_{2k})$ is the SIK of the vertical Hertz's dipole.

According to the used method, the UCD $I_k(s_k')$ is presumed to be in a polynomial form with unknown current coefficients I_{km},

$$I_k(s_k') = \sum_{k=0}^{M_k} I_{km}\left(\frac{s_k'}{l_k}\right)^m,\; k = 1,2\,. \tag{2}$$

The influence of a lossy half-space defined by the SIK was modelled in a simple and very accurate way, as shown in [1] and [2] and partially illustrated in Appendix 1, i.e.

$$S_{00}^{v}(r_{2k}) \cong R_\infty K_0(r_{2k}) + (R_0 - R_\infty)\,\underline{\gamma}_0 \int_{v=z+z_k'}^{\infty} \frac{\exp(-\underline{\gamma}_0\sqrt{\rho^2+v^2})}{\sqrt{\rho^2+v^2}}\,\mathrm{d}v\,, \tag{3}$$

where: R_∞ and R_0 - complex constants, $R_\infty = (\underline{n}_{10}^2 - 1)/(\underline{n}_{10}^2 + 1)$ and $R_0 = (\underline{n}_{10} - 1)/(\underline{n}_{10} + 1)$; $K_0(r_{2k}) = \exp(-\underline{\gamma}_0 r_{2k})/r_{2k}$ - the standard potential kernel from the image; and $z_k' = z_{Ak} + s_k'$, $s_k' \in [0, l_k]$. The new SIK model (3) is used only in the phase of the UCD and the VDA input admittance determination.

2.3 Transversal electric field in the far field zone

After the UCD is obtained, the Hertz's vector in the far field zone is, according to [5], in the following form

$$\vec{\Pi}_0^{zr} = \Pi_{z0}\,\hat{z} = \sum_{k=1}^{2}\int_{s_k'=0}^{l_k} \frac{I_k(s_k')}{4\pi\underline{\sigma}_0}[K_0(r_{1k}) + R_{CR}(\underline{n}_{10},\theta)K_0(r_{2k})]\,\mathrm{d}s_k'\;\hat{z}\,, \tag{4}$$

where $R_{CR}(\underline{n}_{10},\theta)$ is the plane wave reflection coefficient,

$$R_{CR}(\underline{n}_{10},\theta) = \frac{\underline{n}_{10}^2 \cos\theta - \sqrt{\underline{n}_{10}^2 - \sin^2\theta}}{\underline{n}_{10}^2 \cos\theta + \sqrt{\underline{n}_{10}^2 - \sin^2\theta}} . \tag{5}$$

Transversal electric field in the far field zone $E_\theta(\vec{r})$ ($E_\psi(\vec{r})$ - component is omitted) and the field pattern $F_{zr}(\theta)$ are as follows:

$$E_\theta(\vec{r}) \cong -\underline{\gamma}_0^2 \left(\vec{\Pi}_0^{zr} \cdot \hat{\theta} \right) \cong 60 \frac{e^{-\underline{\gamma}_0 r}}{r} |I_1(0)| F_{zr}(\theta) , \tag{6}$$

$$F_{zr}(\theta) = \sum_{k=1}^{2} j \frac{H_k \sin\theta}{2} \sum_{m=0}^{M_k} I_{km}^* \left[e^{j D_k} I_m(C_k) + R_{CR}(\underline{n}_{10},\theta) e^{-j D_k} I_m(-C_k) \right] \tag{7}$$

where: $\underline{\gamma}_0 = j\beta_0 = j 2\pi/\lambda_0$ - complex propagation constant in air; $H_k = \beta_0 l_k$, $Z_{Ak} = \beta_0 z_{Ak}$ - electrical lengths, and $D_k = Z_{Ak}\cos\theta$, $C_k = H_k \cos\theta$ - new values; $I_{km}^* = I_{km} / |I_1(0)|$ - normalized current coefficients obtained in the first step, ref. [1]; and $I_m(\pm C_k)$ - integrals in the following form:

$$I_m(\pm C_k) = \int_{u_k'=0}^{1} u_k'^m \, e^{\pm j C_k u_k'} \, \mathrm{d} u_k' , \; m = 0,1,2,\ldots,M_k . \tag{8}$$

The solution of the (8) is given in the Appendix 2.

3 Numerical Results

Based on the previously described procedure, the VDA field pattern was determined for various parameters describing the model of the VDA placed above a homogenous and isotropic medium.

The field patterns of the half-wave dipole placed at heights $h = 0.25\lambda_0$, $h = 0.30\lambda_0$ and $1.00\lambda_0$, are shown in Fig. 2. The parameters of the lossy half-space (moist earth) are $\varepsilon_{r1} = 10$ and $\sigma_1 = 0.01$ S/m, and the frequency is $f = 100$ MHz. Dash lines represent the field pattern of the VDA placed above a perfectly conducting medium, $\sigma_1 \to \infty$, i.e. $R_{CR}(\underline{n}_{10},\theta) = 1$. For the sake of comparing, using discrete points, the results from [3] and [4] were shown in the same figures. The input admittances and impedances that correspond to these examples are arranged in **Table 1**, and were calculated for the second polynomial degree $M_1 = M_2 = 2$.

Table 1

The input admittance and impedance of the VDA above moist earth ($\sigma_1 = 0.01$ S/m) and perfect ground plane ($\sigma_1 \to \infty$) for different heights h: $l_1 = l_2 = 0.25\lambda_0$, $a_1 = a_2 = 0.007\lambda_0$, $\varepsilon_{r1} = 10$, $f = 100$ MHz, $\underline{n}_{10} = 3.175 - j\,0.283$.

h/λ_0	Y_a in [mS]	Z_a in [Ω]	Y_a in [mS]	Z_a in [Ω]
	$\sigma_1 = 0.01$ S/m		$\sigma_1 \to \infty$	
0.25	6.775 - j 2.017	135.58 + j 40.37	5.812 - j 1.909	155.31 + j 51.01
0.30	8.931 - j 2.333	104.82 + j 27.38	8.436 - j 1.874	112.96 + j 25.10
1.00	9.187 - j 3.632	94.14 + j 37.22	9.226 - j 3.671	93.57 + j 37.23

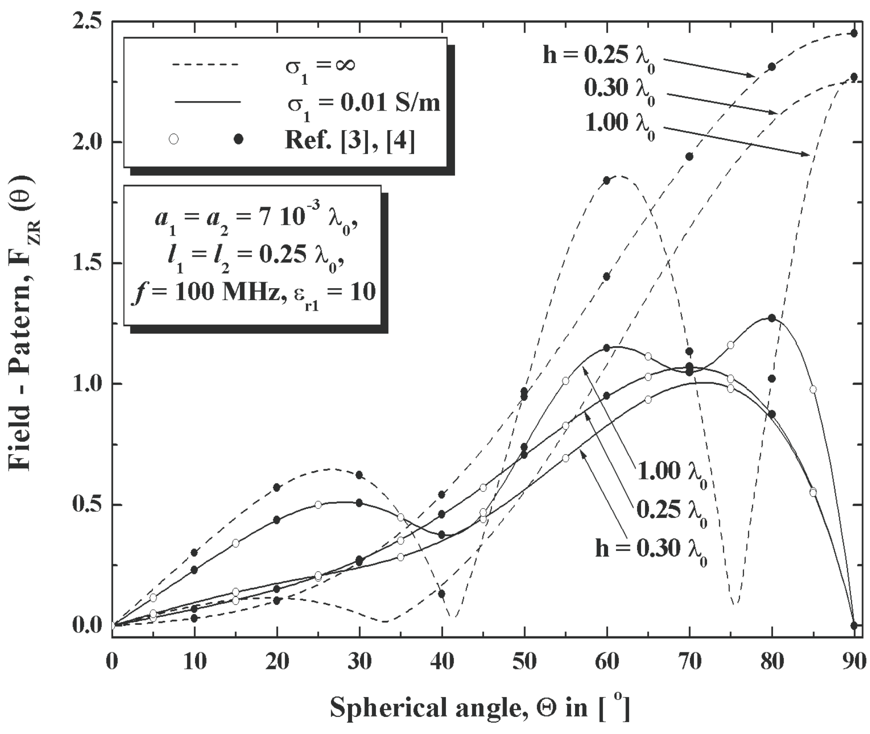

Fig. 2 – *The field pattern of the vertical half-wave dipole versus the spherical angle θ, with heights $h = 0.25\lambda_0$, $h = 0.30\lambda_0$ and $1.00\lambda_0$ as parameters. Relative permittivity is $\varepsilon_{r1} = 10$, conductivity $\sigma_1 = 0.01$S/m and frequency $f = 100$ MHz.*

The field patterns and corresponding polar diagrams of the VDA, whose geometry and parameters are taken from [6], are presented in Figs. 3, 4 and 5. Namely, a half-wave dipole placed at heights $h = 5.0\,\mathrm{m}$, $8.0\,\mathrm{m}$ and $20.0\,\mathrm{m}$ is considered. The length of antenna conductors is $l_1 = l_2 = 5.0\,\mathrm{m}$, their cross-section radii $a_1 = a_2 = 0.05\,\mathrm{m}$ and the feeding frequency $f = 15\,\mathrm{MHz}$. Relative permittivity and conductivity of the lossy half-space are parameters, i.e.: $\varepsilon_{r1} = 40$ and $\sigma_1 = 1.0\,\mathrm{S/m}$; $\varepsilon_{r1} = 10$ and $\sigma_1 = 0.01\,\mathrm{S/m}$; $\varepsilon_{r1} = 5$ and $\sigma_1 = 0.001\,\mathrm{S/m}$ and $\varepsilon_{r1} = 40$, $\sigma_1 \to \infty$, i.e. $R_{CR}(\underline{n}_{10}, \theta) = 1$. Corresponding input admittances and impedances are given in **Table 2**.

Table 2

The input admittance and impedance of the VDA above a lossy half-space, for three different heights h and different electrical parameters of the medium: $l_1 = l_2 = 5\,\mathrm{m}$, $a_1 = a_2 = 0.05\,\mathrm{m}$, $f = 15\,\mathrm{MHz}$, $\lambda_0 = 20\,\mathrm{m}$.

A: $\varepsilon_{r1} = 5$, $\sigma_1 = 0.001\,\mathrm{S/m}$, $\underline{n}_{10} = 2.252 - j0.266$

h/λ_0	Y_a in [mS]	Z_a in [Ω]	Y_a in [mS]	Z_a in [Ω]
	$\sigma_1 = 0.001$ S/m		$\sigma_1 \to \infty$	
0.25	7.432 - j 3.107	114.54 + j 47.88	6.053 - j 2.857	135.10 + j 63.76
0.40	9.697 - j 4.412	85.44 + j 38.87	10.100 - j 4.030	85.41 + j 34.08
1.00	9.282 - j 4.524	87.05 + j 42.43	9.332 - j 4.580	86.36 + j 42.38

B: $\varepsilon_{r1} = 10$, $\sigma_1 = 0.01\,\mathrm{S/m}$, $\underline{n}_{10} = 3.579 - j1.675$

h/λ_0	Y_a in [mS]	Z_a in [Ω]	Y_a in [mS]	Z_a in [Ω]
	$\sigma_1 = 0.01$ S/m		$\sigma_1 \to \infty$	
0.25	6.776 - j 2.630	128.26 + j 49.78	6.053 - j 2.857	135.10 + j 63.76
0.40	9.947 - j 4.369	84.27 + j 37.01	10.100 - j 4.030	85.41 + j 34.08
1.00	9.290 - j 4.555	86.79 + j 42.56	9.332 - j 4.580	86.36 + j 42.38

C: $\varepsilon_{r1} = 40$, $\sigma_1 = 1.0$ S/m, $\underline{n}_{10} = 24.948 - j23.909$

h/λ_0	Y_a in [mS]	Z_a in [Ω]	Y_a in [mS]	Z_a in [Ω]
	$\sigma_1 = 1.0$ S/m		$\sigma_1 \to \infty$	
0.25	6.106 - j 2.805	135.24 + j 62.11	6.053 - j 2.857	135.10 + j 63.76
0.40	10.104 - j 4.077	85.11 + j 34.34	10.100 - j 4.030	85.41 + j 34.08
1.00	9.325 - j 4.579	86.40 + j 42.43	9.332 - j 4.580	86.36 + j 42.38

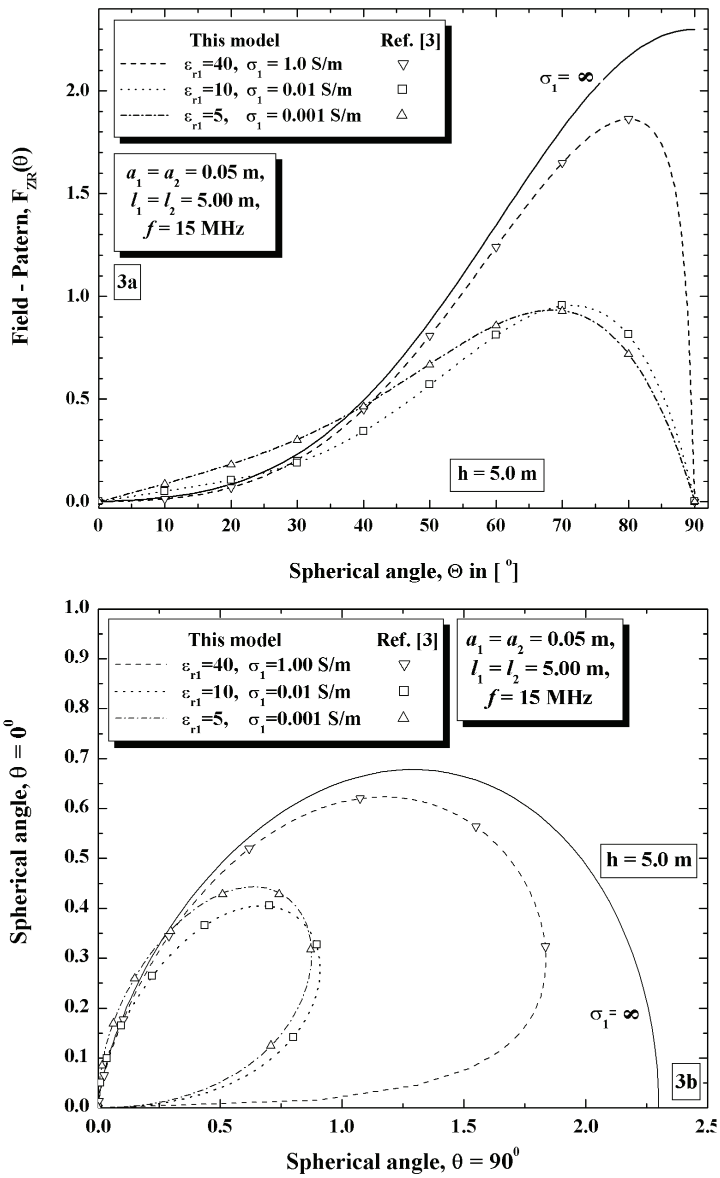

Fig. 3 – *The field pattern and corresponding polar diagram of the half-wave dipole, placed at height* $h = 5.0\,\mathrm{m}$ *above a homogenous and isotropic lossy half-space, versus the spherical angle* θ*.Conductivity* σ_1 *and relative permittivity* ε_{r1} *are parameters.*

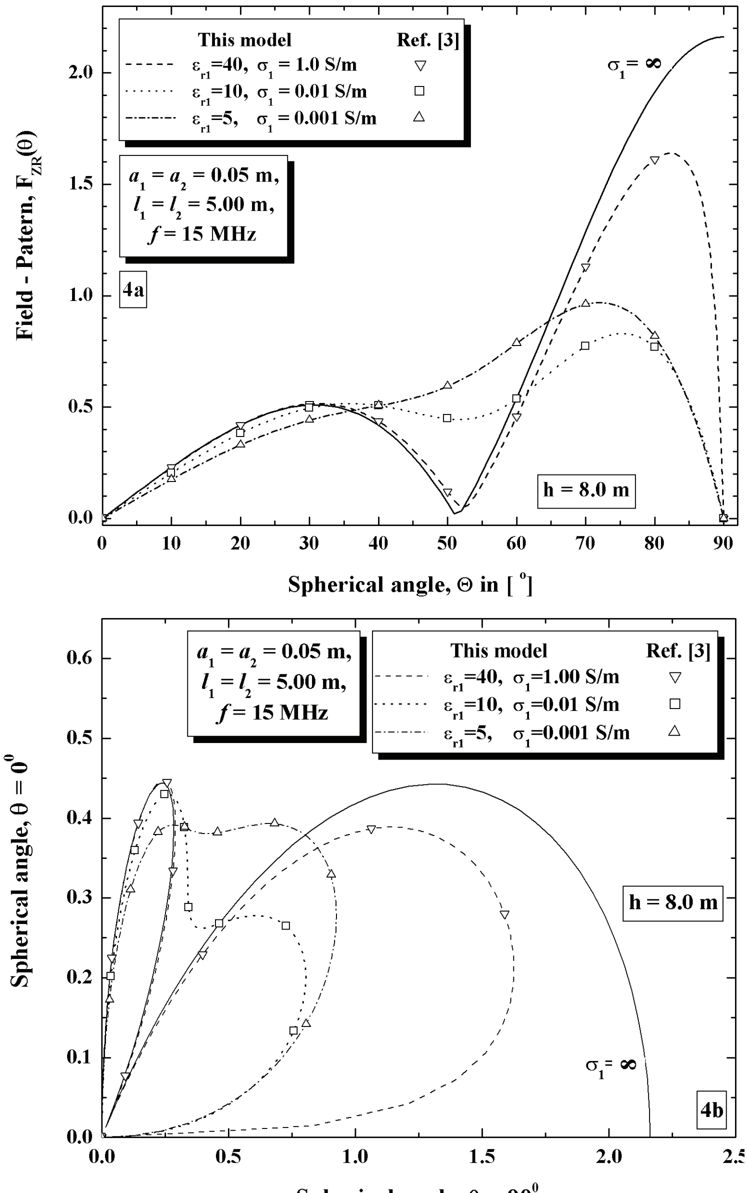

Fig. 4 – *The field pattern and corresponding polar diagram of the half-wave dipole, placed at height* $h = 8.0\,\mathrm{m}$ *above a homogenous and isotropic lossy half-space, versus the spherical angle* θ. *Conductivity* σ_1 *and relative permittivity* ε_{r1} *are parameters.*

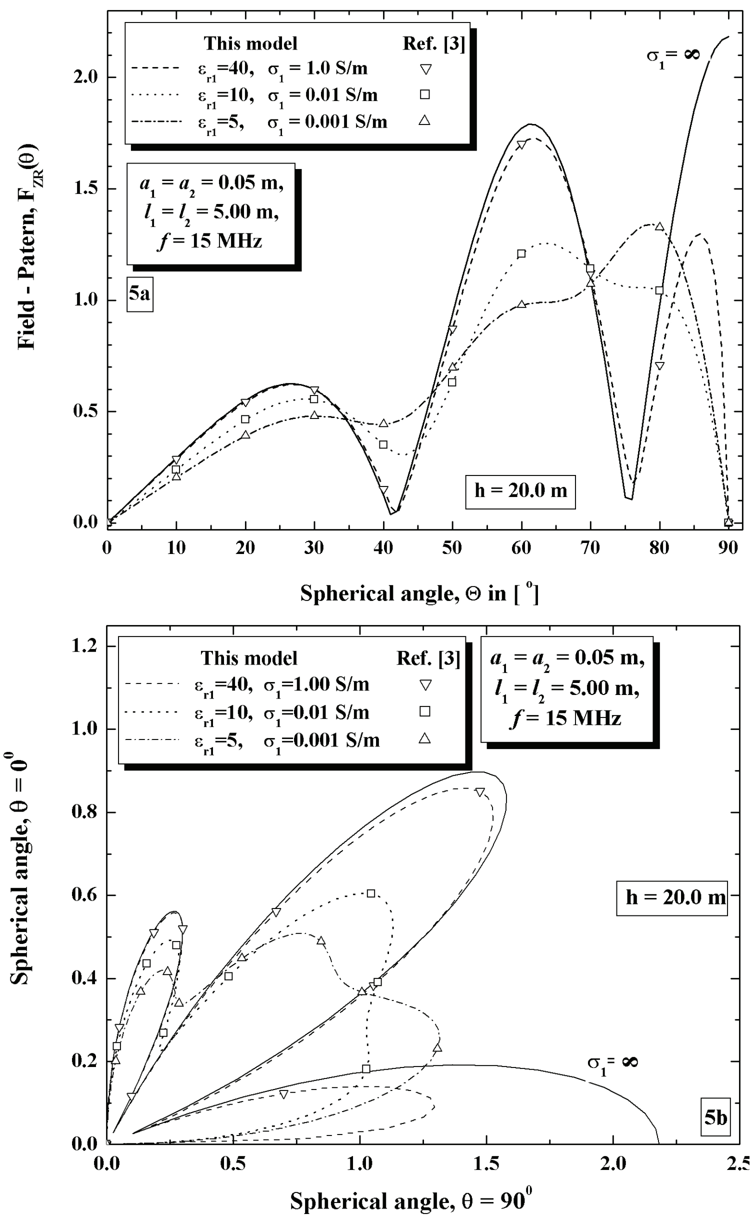

Fig. 5 – *The field pattern and corresponding polar diagram of the half-wave dipole, placed at height* $h = 20.0\,\text{m}$ *above a homogenous and isotropic lossy half-space, versus the spherical angle* θ*. Conductivity* σ_1 *and relative permittivity* ε_{r1} *are parameters.*

4 Conclusion

The field pattern of the VDA placed above a homogenous and isotropic lossy half-space was determined in this paper. The new SIK model from [1] was used for numerical evaluation of the UCD, and for the field pattern determination, the expression for the far field from [5].

From the presented results one can conclude:

- The field patterns have the expected shape;
- All three SIK models used for the near EM field determination, i.e. the UCD evaluation, give practically the same results when the field pattern is determined according to [5], i.e. (7) (see Fig. 2);
- Differences that occur when different SIK models are used will be notable only in the value of E_θ – component magnitude because of factor $|I_1(0)|$ in (6);
- When the surface wave is considered in the far field evaluation, for angles $\theta \in [80^0, 90^0]$, greater differences can be expected. For the remaining values of θ, i.e. $\theta \in [0^0, 80^0]$, the field pattern will detain the form given in Figs. 2-5.

5 Acknowledgement

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The authors, as a sign of gratitude, dedicate this paper to late Prof. dr Dragutin M. Veličković, who has passed away on December 20, 2004. About thirty years ago, the second author became the first student that professor Veličković promoted into a dipl. ing. of electrotechnics, and during the next 15 years was his first associate.

6 Appendix 1

Appendix 1 relates to illustration of accuracy of the SIK numerical evaluation using the new model (3) proposed in [1]. For the sake of comparing, corresponding values of exact SIK evaluation according to [7] and [8] are also presented in Figs. A1a and A1b. Discrete points (solid and open triangles and squares) denote these values. Satisfying agreement of the results, in a wide range of relative permittivity and conductivity ($\varepsilon_{r1} \in [1, 81]$ and $\sigma_1 \in [0, \infty)$), is evident.

The accuracy of the current distribution evaluation along the VDA is, comparing to [3], increased in this manner.

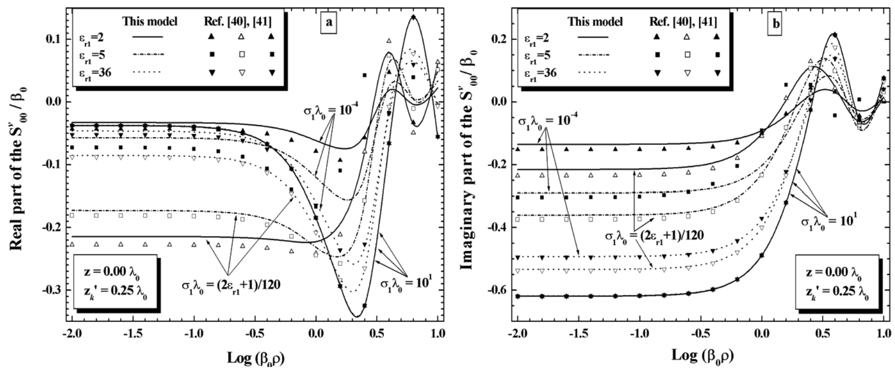

Fig. A1 – *Real and imaginary parts of the SIK* $S_{00}^{v}(r_{2k})/\beta_0$ *versus the normalized radial distance* $\text{Log}\,(\beta_0\rho)$ *, with complex permittivity as a parameter. The position of the VHD is* $z_k' = 0.25\lambda_0$ *and the field point is in plane* $z = 0.00\lambda_0$.

7 Appendix 2

The integral (8) can be solved using the recurrent relations that are obtained by applying the method of partial integration. The solution is in the form:

$$I_m(\pm C_k, C_k \geq 10^{-2}) = \begin{cases} \dfrac{e^{\pm jC_k} - 1}{\pm jC_k}, & m = 0, \\[2ex] \dfrac{e^{\pm jC_k} - mI_{m-1}(\pm C_k)}{\pm jC_k}, & m \geq 0, \end{cases}$$

$$I_m(\pm C_k, C_k < 10^{-2}) = \left(\frac{1}{m+1} - \frac{C_k^2}{2(m+3)}\right) \pm jC_k\left(\frac{1}{m+2} - \frac{C_k^2}{6(m+4)}\right).$$

8 References

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