SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 19, No. 1, February 2022, 15-32

UDC: 621.315.1: 621.315.55 DOI: https://doi.org/10.2298/SJEE2201015L

Multi-conductor Transmission Line with Non-linear Impedances: Application to 50Hz Studies

Giovanni Lucca[1]

Abstract: This paper presents an algorithm for the evaluation of currents and voltages along a multi-conductor transmission line having non-linear impedances due to the presence of ferromagnetic conductors; we assume that these impedances are function of the current carried by the relevant conductor so that the equivalent circuit modelling the line is non-linear and has to be solved by means of an iterative procedure. The present paper can be considered as a generalization of our previous works successfully applied to single-conductor lines with earth return. Some examples of application to typical 50-60 Hz interference studies between power/railway lines on telecommunication cables and pipelines are shown.

Keywords: Multi-conductor transmission lines, Ferromagnetic conductors, Electromagnetic interference, Electromagnetic compatibility, Non-linearity.

1 Introduction

The multi-conductor transmission line model has been applying since several years in many Electromagnetic Compatibility (EMC) problems both in frequency domain and in time domain; here we deal with 50-60 Hz electromagnetic interference problems involving power and railway lines with telecommunication cables and pipelines under the condition that all these structures may be considered as composed by parallel conductors.

In this case, the well-known propagation equations, applied to a multi-conductor system, rule the voltages and currents distribution along the N conductors forming the line [1 – 4].

In the frequency domain and by using phasors, they are given, in matrix form, as:

$$\begin{cases} -\dfrac{\mathrm{d}\left[V(x)\right]}{\mathrm{d}x} = [z][I(x)] - [f(x)], \\ -\dfrac{\mathrm{d}\left[I(x)\right]}{\mathrm{d}x} = [y][V(x)] - [j(x)], \end{cases} \tag{1}$$

[1]Retired; formerly with MERMEC STE, Milano, Italy; E-mail: vanni.luccapc@gmail.com

where x is the abscissa along the line, the vectors of N order $[V(x)]$ and $[I(x)]$ are the voltage and current along the line, the matrices of $N\times N$ order $[z]$ and $[y]$ are respectively the per unit length (pul) impedance and coefficients of admittance among the N coupled circuits and the vectors of N order $[f(x)]$ and $[j(x)]$ are respectively the pul electromotive force (emf) and the pul current generators modelling the coupling of any external electromagnetic field (if existing) with the multi-conductor line.

Let us focus our attention on the matrix $[z]$; in many cases it is a constant quantity that does not depend on the value of the unknowns $[V(x)]$ and $[I(x)]$ so that the system (1) is linear. Nevertheless, in certain cases, when some among the conductors forming the line are composed by ferromagnetic materials, the linearity assumption may fail because some elements of matrix $[z]$ depend on the value of the currents flowing in those conductors having non-linear behaviour.

In the frame of the 50-60 Hz electromagnetic interference, that phenomenon occurs when one has to deal with telecommunication cables provided with armouring composed by steel tapes wrapped around the cable sheath and with steel pipelines that, for sufficiently high value of the induced current, present a non-negligible non-linear behaviour.

In two previous papers we have faced these problems [5–6], but with the limitation that we considered only one conductor with non-linear behaviour so that we studied an induced line composed by a single conductor with earth return while the electromagnetic influence from the inducing system (power/railway line) was modelled by means of suitable pul emf generators and pul current generators to be applied to the induced single-conductor line.

Thus, we adopted a two-step procedure:

– Solve the inducing system and determine the pul emf and current generators to be applied to the induced line

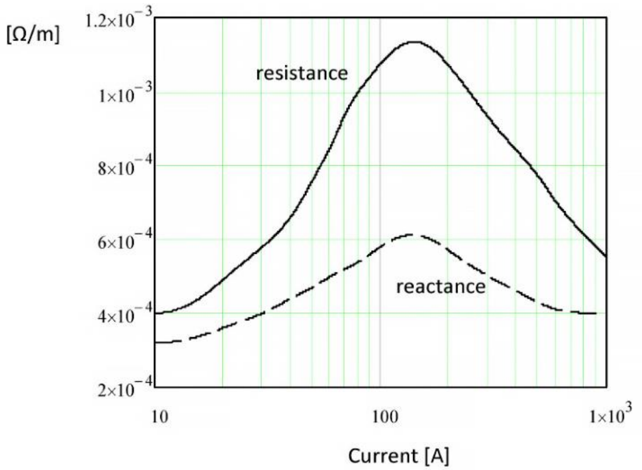
– Solve the induced single-conductor line, characterized by a non-linear impedance, under the action of the above-mentioned generators.

The main assumption at the basis of such an approach is that the influence of the induced system on the inducing one can be neglected; nevertheless, this hypothesis is not always valid (the first example of application, that follows in par.4, is a typical case where the above assumption fails); therefore, a more general approach consisting in the study of an only multi-conductor line including, at the same time both inducing and induced systems, must be adopted. The novelty here presented is that one or more conductors, which form the multi-conductor line, can have a non-linear self-impedance that depends on the current carried by the conductor itself.

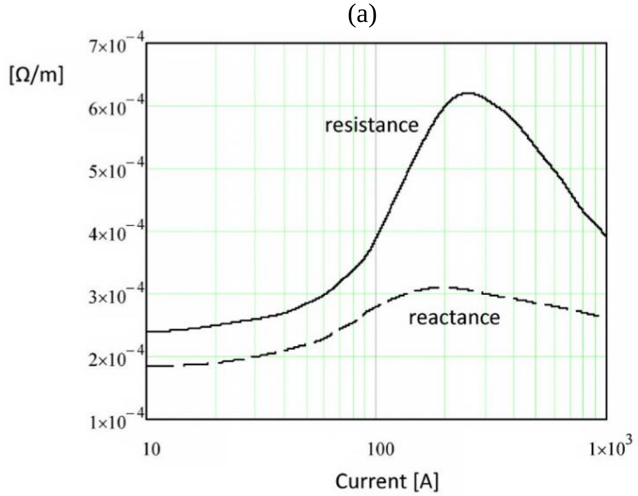
2 Impedance of the Conductor in Function of the Current

The algorithm we are going to describe in the next paragraph, is based on the knowledge, due to measurements, of the pul internal impedance of each conductor composed by magnetic material, in function of the current flowing inside it.

As an example, in Fig. 1, from the data available in [7], we drew the curves of the of the pul resistance and reactance at 50 Hz for two different types of pipelines versus the current.



(a)



(b)

Fig. 1 – (a) *Measured pul internal impedance of a steel pipe versus the current;* f = 50Hz, *diameter* 80mm*;* (b) *Measured pul internal impedance of a steel pipe versus the current;* f = 50Hz*, diameter* 160mm*.*

The figures manifestly put into evidence the significative dependence of resistance and reactance on the current flowing in the pipe for values greater than 10 A. For values lesser than 10 A, it can be practically assumed, a linear behaviour of the steel pipe so that the value of the pul internal impedance is not function of the current.

In other cases, one has at disposal the experimental curves of the complex permeability (real and imaginary part), at the frequency of interest, of the ferromagnetic material composing the conductor, versus the value of the applied magnetic field ***H*** (see for example [8−9]). From those curves it possible to construct other curves, analogous to the ones of Fig. 1, representing the pul internal impedance in function of the current.

Therefore, in the following we shall assume that for each conductor forming the multi-conductor line and having non-linear behaviour, due to its ferromagnetic characteristics, the curve of its pul internal impedance in function of the current is known i.e.:

$$z_{\mathrm{int},m} = z_{\mathrm{int},m}(I_m), \quad m = 1, 2, \ldots, M \ , \tag{2}$$

where m is the index relevant to the m-th non-linear conductor and M is the number of conductors having non-linear behaviour; clearly, M may range from 1 to N meaning respectively that only one conductor or all the conductors have ferromagnetic characteristics.

3 Description of the Algorithm

3.1 General

The algorithm we present here is largely based on the one described in [1] which consists essentially in the discretization of the multi-conductor transmission line equations (1); we generalize them by assuming that the pul square matrix [z] is not constant but, on the contrary, one or more, among its diagonal elements, have non-linear characteristics and depend on the current. The next paragraph is devoted to the description of the multi-conductor algorithm core; for reasons of brevity, this description shall be concise while, for detailed information, one should refer to [1] or more shortly in [10].

A further paragraph is devoted to the description of the iterative procedure that must be combined and applied to the multi-conductor algorithm core in order to apply it to the non-linear case.

3.2 Core of the algorithm

Firstly, let us define by N the number of parallel conductors forming the multi-conductor line and by n the number of points chosen to discretize the line itself.

Therefore, the whole line is discretized by a chain of n–1 cells of π type, with concentrated parameters, like the one shown in Fig. 2.

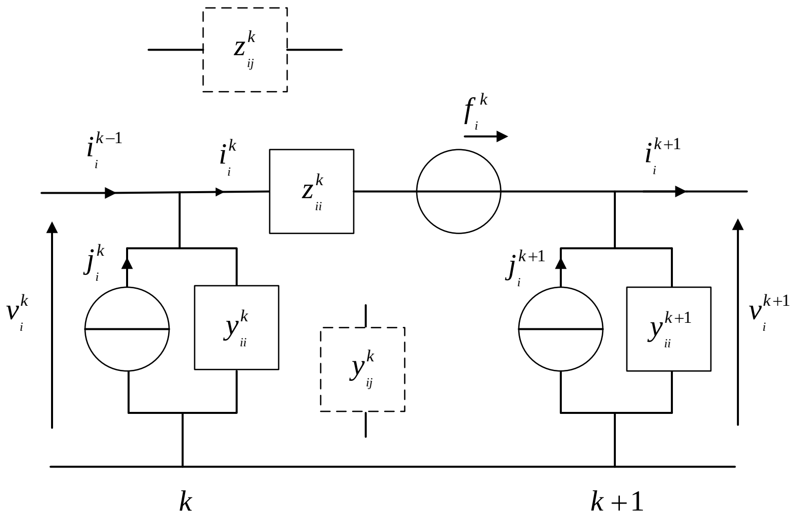


Fig. 2 – *k-th cell of the discretized multi-conductor line.*

In the following, we shall use the upper index to indicate a quantity relevant to the cell i.e.: impedances, emf generators and currents or to the point i.e.: coefficients of admittance, current generators and voltages.

Hence, we have that the generic k-th cell is characterized by the following parameters:

– The matrix of self and mutual impedances $[z]^k$ (NxN order);

– The emf (electromotive force) generators vector $[f]^k$ (N order),

while the generic k-th point is characterized by the following parameters:

– The matrix of self and mutual coefficients of admittances $[y]^k$ ($N\times N$ order);

– The current generators vector $[j]^k$ (N order).

More explicitly, these parameters are defined as follows:

$$[z]^k = [z]_l^k + [z]_p^k + [z]_a^k, \tag{3}$$

$$[y]^k = [y]_l^k + [y]_p^k + [y]_a^k, \tag{4}$$

$$[f]^k = [f]_e^k + [f]_a^k, \tag{5}$$

$$[j]^k = [j]_e^k + [j]_a^k, \tag{6}$$

In order to better explain relationships from (3) to (6) we add the following information:

– The matrices with lower index "l" are related to the pul parameters of the line and depend on the electrical and geometric characteristics of the plant and on the length of the *k*-th cell;

– The matrices with lower index "*p*" characterize passive loads inserted along the line (e.g.: groundings, bondings between different conductors, insulating joints, equivalent impedances that model devices or apparatuses);

– The matrices and vectors with lower index "*a*" are related to active loads inserted along the line (e.g.: generators that model power substations, autotransformers, generators internal impedance);

– The vectors with lower index "*e*" characterize any external electromagnetic field that influences the line;

By means of the application of Kirchhoff laws to the chain of *n*-1 cells describing the whole line and after some algebraic steps, one can obtain, in compact form, the following linear system [1]:

$$[Q][V]=[T], \tag{7}$$

The role of the unknown is here played by the block vector [*V*] of *n* order whose elements are the vectors $[v]^k$ of *N* order representing the voltages to earth of the conductors in the *k*-th point along the line. [*Q*] is a tri-diagonal block matrix of $n\times n$ order; each element forming it is a matrix of $N\times N$ order; in particular, the main diagonal elements $[M]^k$ can be obtained by means of the relation:

$$[M]^k=[y]^k+\left([z]^{k-1}\right)^{-1}+\left([z]^k\right)^{-1}, \tag{8}$$

The lower and upper sub-diagonal elements $[D]^k$ and $[H]^k$ can be respectively obtained by means of the following relations:

$$[D]^k=-\left([z]^{k-1}\right)^{-1}, \tag{9}$$

$$[H]^k=-\left([z]^k\right)^{-1}, \tag{10}$$

Finally [*T*] is a block vector of *n* order whose elements are the vectors $[t]^k$ that can be obtained through:

$$[t]^k=[j]^k+\left([z]^{k-1}\right)^{-1}[f]^{k-1}-\left([z]^k\right)^{-1}[f]^k, \tag{11}$$

It is useful to better visualize (7), by means of a block structure, as follows [1]:

$$\begin{bmatrix} [M]^1 & [H]^1 & [0] & [0] & \cdots & [0] & [0] \\ [D]^2 & [M]^2 & [H]^2 & [0] & \cdots & [0] & [0] \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ [0] & [0] & \cdots & \cdots & [D]^{n-1} & [M]^{n-1} & [H]^{n-1} \\ [0] & [0] & \cdots & \cdots & [0] & [D]^n & [M]^n \end{bmatrix} \begin{bmatrix} [v]^1 \\ [v]^2 \\ \cdot \\ \vdots \\ \cdot \\ [v]^{n-1} \\ [v]^n \end{bmatrix} = \begin{bmatrix} [t]^1 \\ [t]^2 \\ \cdot \\ \vdots \\ \cdot \\ [t]^{n-1} \\ [t]^n \end{bmatrix}. \quad (12)$$

After the solution of linear system (6), from the knowledge of the vectors $[v]^k$, the currents $[i]^k$ can be obtained by means of relationship:

$$[i]^k = \left([z]^k\right)^{-1}\left([v]^k - [v]^{k+1} + [f]^k\right), \quad (13)$$

Note that, in our case, we suppose that the multi-conductor line is not subjected to the influence of any external electromagnetic so both vectors $[f]_e^k$ and $[j]_e^k$ are assumed to be zero.

3.3 Iterative procedure

The algorithm described in the previous paragraph is strictly valid when the impedance $[z]^k$ is constant so that the system to be solved is linear; nevertheless, by means of an iterative procedure, it can be applied also to the non-linear case provided that the curves, expressed by (2), for each conductor with ferromagnetic characteristics are known.

Let us start with arbitrary values of the current carried by the conductors in the linear region of the curves (for example, in Fig. 1, for values very close to 10A); in such a way, one obtains an initial set of matrices $[z]^{k,0}$ (k = 1,2,…,n–1) where the second superscript 0 means “starting value”.

By applying the algorithm described in par.3.2 and by posing $[z]^k = [z]^{k,0}$, one determines the voltages and current at the first step, i.e.: $[v]^{k,1}$ (k = 1, 2,…,n) and $[i]^{k,1}$ (k = 1,2,…,n–1). From the knowledge of $[i]^{k,1}$ and from (2), one determines $[z]^{k,1}$ (k = 1,2,…n–1) by means of the following equation:

$$[z]^{k,1} = \begin{bmatrix} z_{11}^{k,0}\left(i_1^{k,1}\right) & z_{12}^{k,0} & \ldots & z_{1N}^{k,0} \\ z_{21}^{k,0} & z_{22}^{k,0}\left(i_2^{k,1}\right) & \ldots & z_{2N}^{k,0} \\ \vdots & \vdots & \ddots & \vdots \\ z_{N1}^{k,0} & z_{N2}^{k,0} & \ldots & z_{NN}^{k,0}\left(i_N^{k,1}\right) \end{bmatrix}, \quad k = 1,2,\ldots,n-1. \quad (14)$$

When writing (14), we have assumed that $M = N$ that is all the conductors have non-linear behaviour; nevertheless, such an equation can be readily adapted when $M < N$.

In turn, by solving the circuit, one gets $[v]^{k,2}$ ($k = 1,2,\ldots,n$) and $[i]^{k,2}$ ($k = 1,2,\ldots,n-1$).

At the j-th step of the procedure one has:

$$[z]^{k,j+1} = \begin{bmatrix} z_{11}^{k,j}\left(i_1^{k,j+1}\right) & z_{12}^{k,0} & \cdots & z_{1N}^{k,0} \\ z_{21}^{k,0} & z_{22}^{k,j}\left(i_2^{k,j+1}\right) & \cdots & z_{2N}^{k,0} \\ \vdots & \vdots & \vdots & \vdots \\ z_{N1}^{k,0} & z_{N2}^{k,0} & \cdots & z_{NN}^{k,j}\left(i_N^{k,j+1}\right) \end{bmatrix}, \quad k = 1,2,\ldots,n-1. \tag{15}$$

The iterative procedure continues for p steps as long as one has $[i]^{k,p} \cong [i]^{k,p+1}$. Generally, few iterations are enough to obtain satisfactory results so that the relative difference between the currents for two consecutive steps is less than few percent.

It is useful to notice, by looking at (14) and (15), that only the diagonal elements of the matrices $[z]^{k,j}$ depend on the values of the currents evaluated at the previous step while all the other elements (i.e., the mutual ones) are constant.

An important remark has to be done in order to use the multi-conductor transmission line model above described; due to the presence of non-linear material, one loses the sinusoidal dependence of all currents and voltages, thus the use of phasors is not strictly correct. By using them all the same, means to consider only the basic frequency (in our case 50 Hz) and not to look at all the harmonics whose energy is lost. Nevertheless, we have to remember that the major part of the energy is still in the basic frequency and thus we assume to neglect the harmonics contribution.

Therefore, by starting from the experimental curves of the non-linear impedance at the basic frequency (like the ones of Fig. 1) or from the experimental curves of the complex magnetic permeability at the basic frequency for each non-linear conductor belonging to the multi-conductor line and by applying the iterative procedure just described, one can get approximated solutions that are adequate for engineering applications.

4 Examples of Application

4.1 General

In this paragraph we deal with two real examples of application in the frame of 50 Hz electromagnetic interference produced by railway lines/power lines on telecommunication cables and pipelines.

We present two specific case studies:

1. Interference by a double track High Speed Railway Line (HSRL) fed at 2×25 kV-50 Hz and equipped with autotransformers, on a parallel telecommunication/signalling cable provided with steel armouring;
2. Interference produced by a single circuit 400 kV-50 Hz power line on two steel pipelines and parallel to the power line conductors.

The main purpose of this paragraph is to compare the final results of the interference on the induced plants (i.e.: induced voltages and currents) when considering or not the non-linear behaviour associated to the internal impedance of the interfered conductors.

4.2 Interference between HSRL and armored telecommunication cable

In this case, the multi-conductor line consists of 15 conductors; 14 conductors belong to the double track HSRL and the last conductor represents the aluminium sheath, provided with steel tapes armouring, of the telecommunication cable; such a cable, lies parallel to the railway line at a lateral distance of 5.8 m from the traction line axis for the whole its length, i.e., 38.866 km.

The model of the railway line 2×25 kV-50 Hz equipped with auto-transformers has been built according to [11] while the model of cable sheath with steel tapes armouring is based on the information contained in [5, 8 – 9], [12 – 13] and on known experimental curves of the 50 Hz complex magnetic permeability in function of the applied magnetic field [8]; in relation to this last point, we remind that the presence of the armouring is modelled by a pul additional impedance to be added to the pul cable sheath with earth return impedance.

We also remind that the real part of the pul armouring additional impedance is associated to the hysteretic losses inside the steel tapes while the imaginary part is related to their inductance.

A section view of the 15 conductors forming the HSRL together the telecommunication cable is shown in Fig. 3.

It is important to remark that the sheath of the telecommunication cable is metallically bonded to the earth electrode, placed on its same side, in many points along the line; thus, the return of the traction current is significantly modified with respect to the case when the above-mentioned connections do not exist. This implies that a multi-conductor description involving all the 15 conductors at one time is needed; for such a reason, a two-step analysis by first studying the traction line alone and then evaluating its electromagnetic influence on the telecommunication cable alone would not be correct.

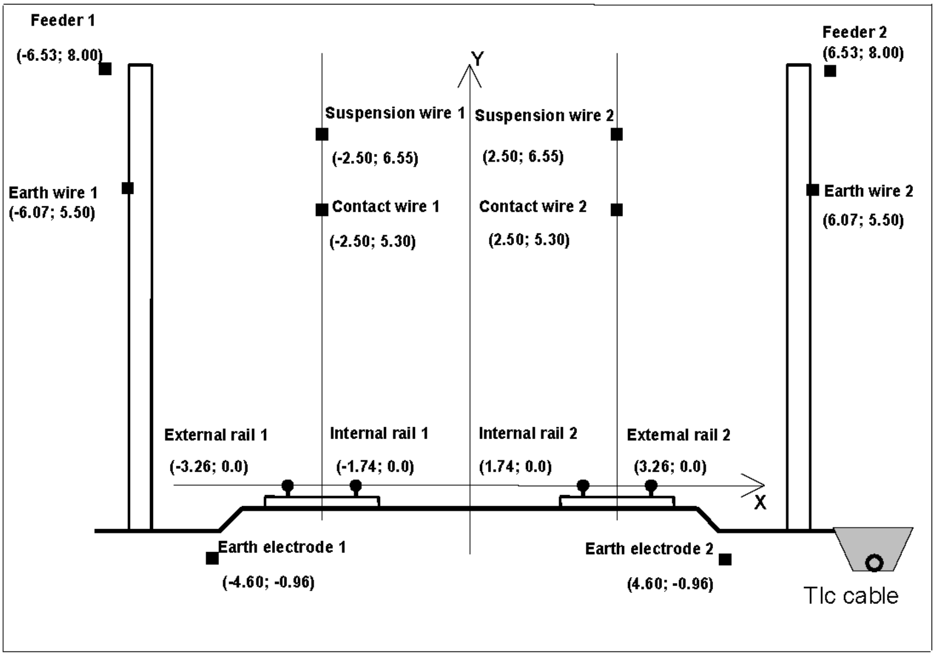


Fig. 3 – *Section view of the railway line and telecommunication cable.*

The level of electromagnetic interference on the telecommunication cable is strongly influenced by the number and position of locomotives along the line as well as the current absorbed by them. Among the very large number of possible configurations, we chose only one among them that is quite meaningful and it is enough for our purposes i.e.:

– Four locomotives are present along the line; two on track 1 and two on track 2

– The locomotives on tracks 1 and 2 are placed in the same position, i.e., two locomotives at km 10.026 and two locomotives at km 24.927. In such a way, the constraint of the minimum distance (related to safety reasons) of about 25 km between two consecutive trains on the same track is respected and, at the same time the level of electromagnetic interference on the cable is enhanced just because the two couples of locomotives on tracks 1 and 2 are located at the same abscissa of the railway line so that the induction effect on the cable is doubled.

– The current absorbed by each locomotive is about 390 A.

The next figures show some results obtained by considering or not the non-linearity due the presence of the steel tape armouring wrapped along the aluminium cable sheath.

Fig. 4 shows the modulus of the pul additional impedance due to the armouring versus the abscissa along the railway line; as expected, in the linear case, it is a constant quantity, while in the non-linear case, it depends on the currents flowing inside the sheath which in turn depends on the above-mentioned abscissa.

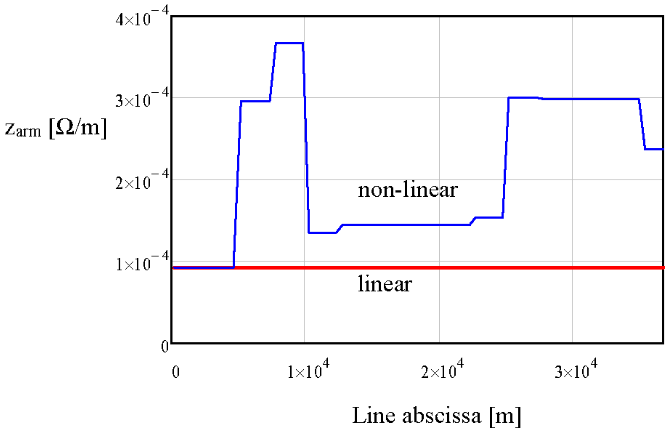


Fig. 4 – *Pul armouring impedance versus railway line abscissa.*

Fig. 5 is relevant to the modulus of the current flowing on the cable sheath versus the railway line abscissa; it is useful to notice the similar trend of the curves in Figs. 4 and 5 which reflects the strict connection between the pul impedance of the armoured sheath and the sheath current.

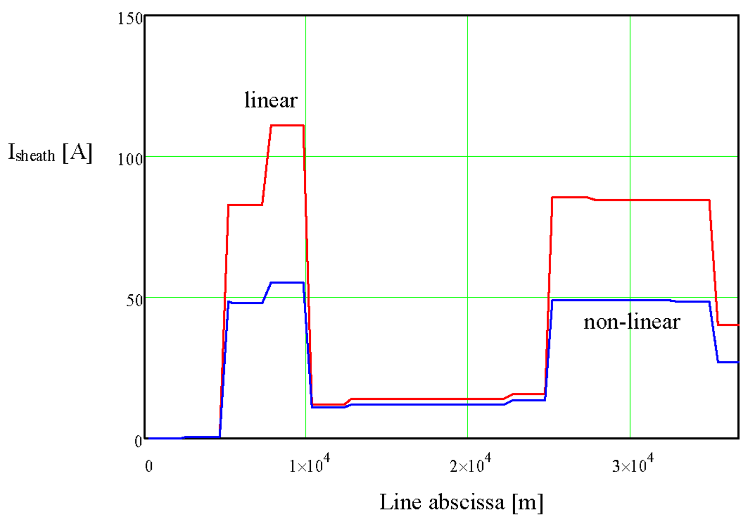


Fig. 5 – *Current circulating on the telecommunication cable sheath.*

Fig. 6 shows to the modulus of the pul emf $f_c(x)$ induced on the cable core that is on the bunch of the symmetrical copper telecommunication pairs that are disposed inside the sheath.

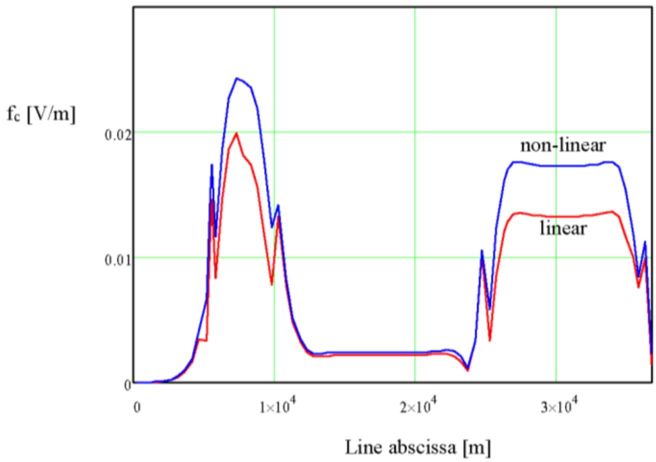


Fig. 6 – *Pul emf induced on the telecommunication cable core.*

These figures put into evidence the significant difference between the results; the higher pul sheath-earth impedance in the non-linear case compared to the linear case, explains the large difference relevant to the current circulating on the sheath (see Fig. 5) where such a quantity, in the linear case is higher; in turn we have also that, in the linear case, the pul induced emf on the cable core is smaller due to the better shielding action which, as known, is related to the intensity of the current flowing on the sheath. (See Fig. 6).

Let us now assume that the cable core has initial point at $s1$ and final point at $s2$; from the knowledge of the pul induced emf $f_c = f_c(s)$ together the hypotheses that the core is not connected to the sheath and has constant pul admittance to earth, it is possible to calculate its voltage to earth $V_c = V_c(s)$ by means of the following formula [14]:

$$V_c(s) = \int_{s1}^{s} f_c(\alpha)\,\mathrm{d}\alpha - V_{cm}\,. \tag{16}$$

Being V_{cm} given by:

$$V_{cm} = \frac{1}{s2 - s1}\int_{s1}^{s2} f_c(\alpha)\,\mathrm{d}\alpha\,. \tag{17}$$

In formula (16), the integral has the meaning of cumulated emf on the cable core between the initial point $s1$ and the evaluation point s while V_{cm} in formula (17) has the meaning of mean cumulated emf on the cable core between abscissas $s1$ and $s2$.

We have considered, in our example, that $s1 = 0$ m and $s2 = 17927$ m; Fig. 7 shows the modulus of the voltage to earth induced on the cable core $V_c = V_c(s)$ versus the cable core abscissa s.

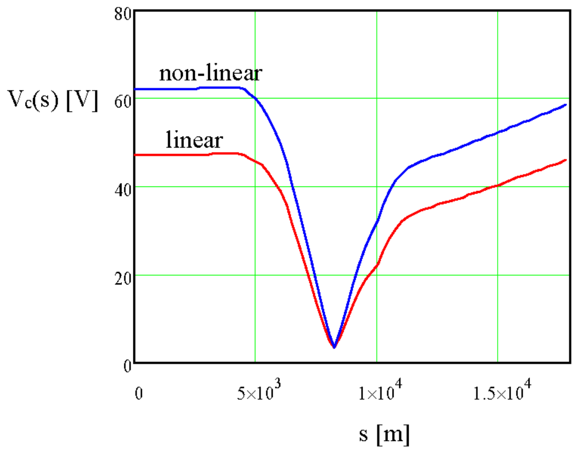


Fig. 7 – *Voltage to earth of the cable core.*

Thus, by looking at Figs. 6 and 7, we can conclude that, when taking into account of the non-linearity of the armouring, one obtains higher values for the pul induced emf and for the voltage to earth of the cable core. Therefore, the results that take into account of the steel tapes non-linearity are more cautionary when evaluating the electromagnetic interference on the cable core.

In conclusion, if the electromagnetic interference calculations are aimed to guarantee electrical safety, an approach that takes into account of the sheath armouring non-linearity should be preferred.

4.3 Interference between HV power line and pipelines

In this case, the study is devoted to a 400 kV-50 Hz HV (High Voltage) power line 60 km long that shares the same corridor with two pipelines having pul internal impedance given by the curves shown in Fig. 1.

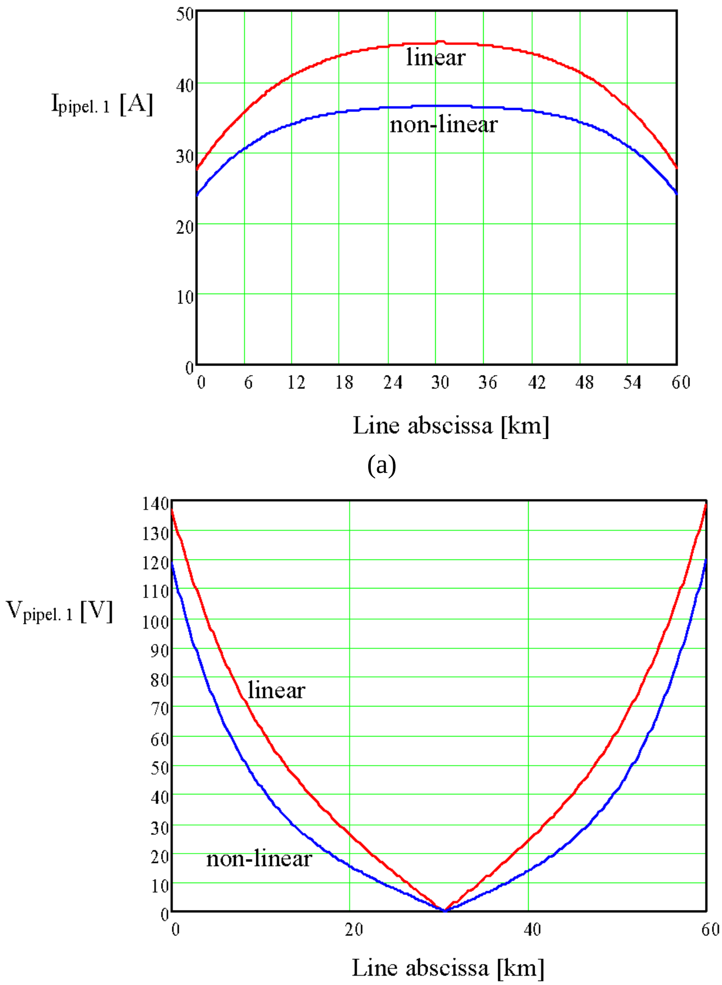
The power line, that has the phase conductors with horizontal disposition, is equipped with one shield wire and carries a balanced current of 2000 A.

The two pipelines are buried on opposite sides with respect to the power line axis at lateral distances of 15 m and 10 m respectively; at the right of the power line, we have pipeline 1 of diameter 80 mm and pul internal impedance according to Fig. 1a while, at the left, we have pipeline 2 of diameter 160 mm and pul internal impedance according to Fig. 1b; both the pipelines are earthed at their ends by earthing impedances of 5 Ω.

In the examples that follow, the currents flowing on the phase conductors are supposed to be balanced, but this is not absolutely a requisite in order to apply the calculation method that can be used also in case of imbalanced currents as well. (See the previous example dealing with the railway line where the inducing currents are strongly imbalanced).

The whole multi-conductor has been discretised by means of 300 cells.

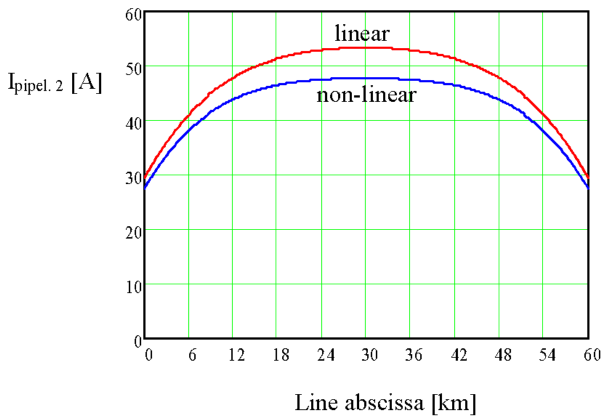
Figs. 8 and 9 and show the modulus of current and voltage induced on pipelines 1 and 2 respectively.



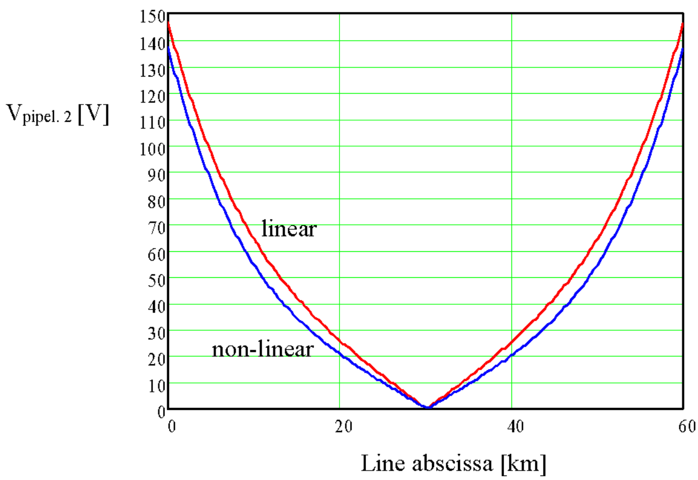
(a)

(b)

Fig. 8 – (a) *Current induced on pipeline 1*; (b) *Voltage induced on pipeline 1*.



(a)



(b)

Fig. 9 – (a) *Current induced on pipeline 2*; (b) *Voltage induced on pipeline 2*.

Also in this example, by looking at Figs. 8 and 9, we can clearly notice the differences in the results between the two approaches: linear and non-linear.
In particular we have lower values for induced current and voltages on both the pipelines when the steel non-linearity is taken into account.

One could expect such results; in fact, by looking at Fig. 1, we can notice that when considering the non-linear behaviour of the pul pipe internal impedance, such a parameter assumes larger values with respect to the linear behaviour (values of the curves corresponding to 10 A). Therefore, the value of the longitudinal current flowing in the pipe-earth circuit is higher in the linear model (lower impedance) than in the non-linear one (higher impedance); hence (being the pul admittance the same in the two models) also the transversal leakage current to soil is higher in the linear model and accordingly the same holds for the values of the voltage to earth.

It may be interesting to examine the influence of the phase current on the induced voltage and current in pipelines 1 and 2; to this aim, we consider the differences between linear and non-linear models $\Delta V_h(x) = V_{h,\mathrm{lin}}(x) - V_{h,\mathrm{non\text{-}lin}}(x)$ and $\Delta I_h(x) = I_{h,\mathrm{lin}}(x) - I_{h,\mathrm{non\text{-}lin}}(x)$ with $h = 1,2$.

In **Table 1** we show the maxima of ΔV_h and ΔI_h for different values of the phase current.

Table 1

Maxima of ΔV_h and ΔI_h for different values of the phase conductors current.

	Pipeline 1		Pipeline 2	
Phase current [A]	Max ΔV_1 [V]	Max ΔI_1 [A]	Max ΔV_2 [V]	Max ΔI_2 [A]
1000	5.726	2.017	2.152	1.084
1500	12.935	5.09	5.612	2.943
2000	21.911	8.968	10.573	5.541
2500	33.416	13.725	17.117	8.775

As we could expect, to the increase of the inducing phase current, it corresponds the increase in the gap between the linear and non-linear model.

Finally, in **Table 2**, by considering the same quantities as in **Table 1**, we show the influence of the number of cells used to discretise the multi-conductor line.

Table 2

Maxima of ΔV_h and ΔI_h for different number of cells;
phase conductors current 2000A.

	Pipeline 1		Pipeline 2	
Number of cells	Max ΔV_1 [V]	Max ΔI_1 [A]	Max ΔV_2 [V]	Max ΔI_2 [A]
75	21.904	8.966	10.567	5.54
150	21.907	8.967	10.572	5.541
300	21.911	8.968	10.573	5.541
600	21.911	8.968	10.573	5.541
1200	21.912	8.968	10.573	5.541

From **Table 2**, we can remark that, in this case, thanks to the uniformity of the multi-conductor line, the influence of the number of cells, on the final results, is very weak.

5 Conclusions

In this paper, we have presented an algorithm that allows to evaluate voltage and current circulating on a multi-conductor transmission line when at least one, among the parallel conductors forming it, is ferromagnetic and its pul internal impedance presents a non-linear behaviour. This paper generalises our previous works that were focused on the same problem but with the limitation of considering only a single-conductor line instead of a multi-conductor line.

We have applied the proposed algorithm to two typical cases of 50-60 Hz interference by comparing the results between two models:

– First model: all the pul parameters of the multi-conductor line have linear behaviour and any non-linearity due to the presence of ferromagnetic conductor has been neglected

– Second model: the non-linearity of the ferromagnetic conductors composing the line has been taken into account starting from suitable experimental curves describing or the complex magnetic permeability of the conductors (i.e.: the steel tapes forming the cable armouring) or their pul internal impedance.

By focusing our attention on those conductors presenting non-linear behaviour, the results based on the two cases studied, show that:

– A lower current is expected in the non-linear case compared to the linear model; this is clearly explained by the higher pul internal impedance of the conductor.

– As far as the voltage to earth is concerned it is not possible to “a priori” determine which, among the two models, gives the more cautionary results; in fact, in example 1 we got more conservative results by means of the non-linear model while, in example 2, the opposite is true.

We can conclude that, in principle, the non-linear model should be preferred to the linear one especially if the value of the current flowing on the conductors presenting non-linear behaviour are sufficiently high.

However, in practice there is a real and actual difficulty which consists in having at disposal the data, that characterize the ferromagnetic conductors, i.e., the measured pul length impedance or the complex magnetic permeability in function of the magnetic field.

Hence, if results of field/laboratory measurements are not available, every effort collecting the necessary data, through literature and/or handbooks or technical reports, should be done.

6 References

[1] ITU-T: Directives Concerning the Protection of Telecommunication Lines Against Harmful Effects from Electric Power and Electrified Railway Lines – Capacitive, Inductive and Conductive Coupling: Physical Theory and Calculation Methods, Vol. III, 1989.

[2] R. P. Clayton: Analysis of Multiconductor Transmission Lines, 1st Edition, John Wiley & Sons, New York, Chichester, Brisbane, 1994.

[3] F. M. Tesche, M. V. Ianoz, T. Karlsson: EMC Analysis Methods and Computational Models, 1st Edition, John Wiley & Sons, New York, Chichester, Brisbane, 1996.

[4] J. A. Brandao Faria: Multiconductor Transmission-Line Structures: Modal Analysis Techniques, 1st Edition, John Wiley & Sons, New York, Chichester, Brisbane, 1993.

[5] G. Lucca, G. L. Solbiati: Transmission Line Circuit with Nonlinear Impedances: Application to EMC Problems, Proceeding of the 11th International Zurich Symposium & Technical Exhibition on EMC, Zurich, Switzerland, March 1995.

[6] G. Lucca: Influence of Steel Non-Linearity in Assessing 50-60 Hz Interference on Pipelines, Progress in Electromagnetics Research M, Vol. 74, October 2018, pp. 1 – 10.

[7] Working Group 36.02: Guide on the Influence of High Voltage AC Power Systems on Metallic Pipeline – Electromagnetic Compatibility with telecommunication circuits, low voltage networks and metallic structures, CIGRE, 1995.

[8] G. Lucca: Impedances Internes de Conducteurs Ferromagnetiques dans les Problemes d'Interference a Frequence Industrielle, 8ème Colloque International sur la compatibilité électromagnétique, Lille, France, September 1996.

[9] J. M. Nahman, V. B. Djordjevic, D. Salamon: Grounding Effects of HV and MV Underground Cables Associated with Urban Distribution Substations, IEEE Transactions on Power Delivery, Vol. 17, No. 1, January 2002, pp. 111 – 116.

[10] G. Lucca: Influence of Railway Line Characteristics in Inductive Interference on Railway Track Circuits, IET Science Measurements & Technology 2019, Vol.13, No.1, pp. 9 – 16.

[11] ITU-T: Directives Concerning the Protection of Telecommunication Lines Against Harmful Effects from Electric Power and Electrified Railway Lines – Inducing Currents and Voltages in Electrified Railway Systems, Vol. IV, 1989.

[12] H. Otoy, M. Yajima, M. Yokota: A Corrugated Wire-Filled Aluminium Sheath to Screen Telephone Cable from Inductive Interference, Proceedings of the 21st International Wire & Cable Symposium, Atlantic City, USA, December 1972, pp. 224 – 234.

[13] K. Kupfmüller: Electricité Théorique et Appliquée, Dunod, Paris, 1959.

[14] R. Pomponi, G. L. Solbiati: On the Calculation of the Effects Caused by the Electric Power Lines on the Telecommunication Cables, CSELT - Rapporti Tecnici, Vol. II, No. 3, December 1974, pp. 49 – 75.