SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 2, No. 1, May 2005, 21 - 28

Possible Solution of Parallel FIR Filter Structure

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Abstract: In this paper, a parallel form **FIR** adaptive filter structure with **RLS** (**R**ecursive **L**east **S**quares) type adaptive algorithm is proposed. The proposed parallel form **FIR** structure consists of a recursive orthogonal transform stage and sparse **FIR** subfilters operating in parallel. The adaptive algorithm used to update coefficient vector of the sparse filters is implemented by using modified Hopfield networks. This structure implements the **RLS**-type adaptive algorithm, without an explicit matrix inversion avoiding numerical instability problems. Simulation results which show the desirable features of proposed structure are given.

Keywords: FIR, RLS adaptive algorithm, Hopfield networks, Computer simulations.

1 Introduction

The **RLS** adaptive algorithm presents several advantages over the well known **LMS** (**L**east **M**ean **S**quares) adaptive algorithm, with respect to the convergence speed and insensibility to additive noise. However, its higher computational cost makes difficulty to use it in many practical applications, which require relatively large filters order. Especially analogue implementation of conventional **RLS** algorithm is difficult because it requires matrix operation. This is an obstacle for the implementation of **RLS** type adaptive algorithm in using analogue systems, making necessary to develop adaptive **RLS** algorithms without explicit matrix operation.

Continuous time Hopfield network can be implemented using analogue circuits [1], solving linear simultaneous equations required by the **RLS** algorithm without explicit matrix inversion. This does not require any additional computational efforts and numerical instability, as compared with the conventional one [2- 4]. Using these features, the direct-form **FIR** adaptive filter structure using a modified Hopfield network was proposed [2, 3]. This system shows fairly good performance compared with the conventional **RLS** algorithm. However, when order of the filter increases, the number of required connections of a

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modified Hopfield network increases with the square degree, making the implementation of high order adaptive filters difficult.

On the other hand, the parallel form **FIR** adaptive filter has, potentially, better convergence performance than the direct form **FIR** adaptive structure, because the input signal is orthogonalized. This orthogonalization is carried out by using an orthogonal transformation such as the **DFT**, **DCT**, etc. [5, 6]. In the parallel form **FIR** filter, each subfilter has smaller order than the direct form one.

In this paper is proposed a parallel form **FIR** adaptive filter structure, in which the discrete cosine transform (**DCT)** is used to orthogonalize the input signal, while the adjustment of filter coefficients vector is realized by using continuous time modified Hopfield network.

Computer simulation results are given to show the actual performance of proposed adaptive filter structure.

2 Proposed Structure

Consider an N^{th} order of the transfer function $H(z)$, the direct form of **FIR** filter is given by:

$$H(z) = \sum_{i=0}^{N-1} h_i z^{-i} , \tag{1}$$

which using a subband decomposition approach [5], can be decomposed into M parallel subfilter structures, such that $H(z)$ can be rewritten as:

$$H(z) = \left[1, z^{-1}, \ldots, z^{(M-1)}\right] C^T \begin{bmatrix} G_0(z^L) \\ G_1(z^L) \\ \cdot \\ \cdot \\ \cdot \\ G_{M-1}(z^L) \end{bmatrix} , \tag{2}$$

where C is an $M \times M$ matrix orthogonal transformation, and $G_r(z^L)$ is sparse subfilter is given by:

$$G_r(z^L) = \sum_{l=0}^{K-1} g_{r,l} z^{-lL} , \tag{3}$$

where L is sparsity factor and K is the number of coefficients in each sparse subfilter [5].

A parallel form FIR structure using subband decomposition is shown by Fig. 1, in which as orthogonal transformation C, the discrete cosine transform (DCT) is used to its better orthogonalizing properties [6, 7].

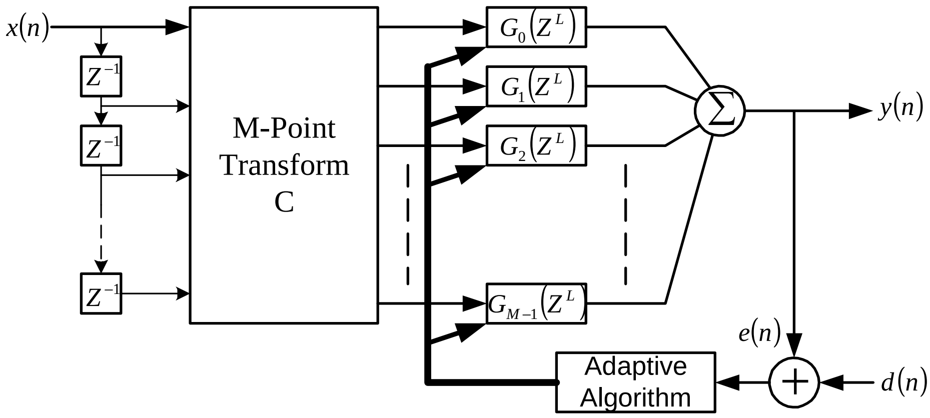

Fig. 1 - *Proposed parallel form **FIR** adaptive filter structure.*

The *k*-th discrete cosine transform coefficient of input signal at time m, $m-1$ and $m-2$ point are given by:

$$C(m,k)=\alpha(k)\sum_{n=0}^{N-1}x(m-[N-1]+n)\cos(\pi(2n+1)k/2N) \tag{4}$$

$$C(m-1,k)=\alpha(k)\sum_{n=0}^{N-1}x(m-1-[N-1]+n)\cos(\pi(2n+1)k/2N) \tag{5}$$

$$C(m-2,k)=\alpha(k)\sum_{n=0}^{N-1}x(m-2-[N-1]+n)\cos(\pi(2n+1)k/2N). \tag{6}$$

Using the fact that:

$$2\cos(a)\cos(b)=\cos(a-b)+\cos(a+b) \tag{7}$$

after some mathematics manipulations, we get:

$$H(z)=\sum_{r=0}^{M-1}C_r(z)G_r(z), \tag{8}$$

where:

$$C(z)=\frac{\cos\frac{\pi k}{2N}\left((-1)^k-(-1)^k z^{-1}-z^{-N}+z^{-N-1}\right)x(z)}{1-2\cos\frac{k\pi}{N}z^{-1}+z^{-2}}. \tag{9}$$

The using the discrete cosine transform $C_r(z)$, from (9), using the sparse subfilters $G_r(z^L)$ given by (3), we get the proposed parallel form **FIR** adaptive structure shown in Fig. 2 and Fig. 3.

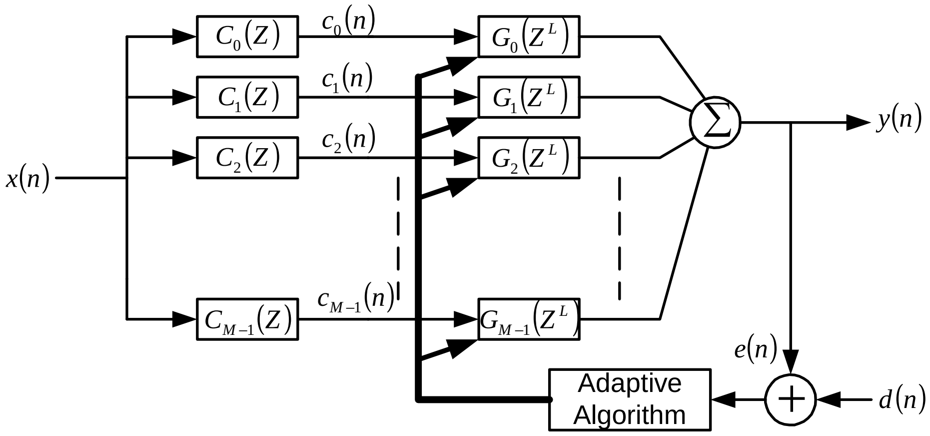

Fig. 2 – *Proposed **FIR** adaptive filter structure using the discrete cosine transform.*

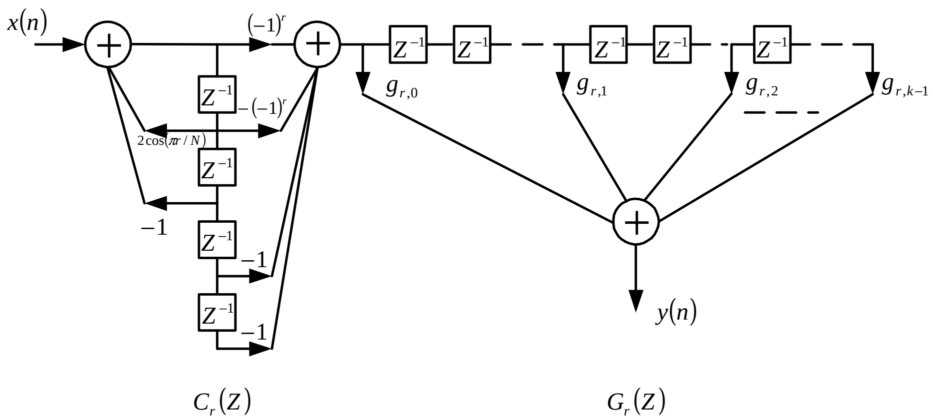

Fig. 3 – *r-th stage of proposed **FIR** adaptive filter structure.*

Thus from Fig. 2 and Fig. 3 it follows that in the proposed structure, the input signal is filtered by the discrete cosine transform to produce a set of M orthogonal signal components. Subsequently these orthogonal signal components are filtered by M sparse subfilters. The output signal of the proposed structure is the sum of the output of each subfilter.

3 Proposed Adaptive Algorithm

The adaptation algorithm used to update the proposed structure coefficient vector is an **RLS**-type adaptive algorithm. Assuming that $g_r(n)$ is the adaptive filter coefficient vector of the *r*-th subfilter at time n. The update equation is given by:

$$g_r(n) = g_r(n-1) + \mu R_r^{-1}(n) e(n) U_r(n), \tag{10}$$

where $0 < \mu < 1$, is a convergence factor, $e(n)$ is an error signal between desired signal $d(n)$ and the actual adaptive filter output signal $y(n)$; $U_r(n)$ is the input signal of *r*-th sparse subfilter and $R_r(n)$ is autocorrelation matrix of the *r*-th subfilter input vector $U_r(n)$, which is given by:

$$U_r(n) = c_r(Ln), \tag{11}$$

where L is sparsity factor of subfilter.

On the other hand, the *k*-th output of the continuous Hopfield network, shown in Fig. 4, is given by:

$$c\frac{dw_k(t)}{dt} = -\frac{1}{r}w_k(t) + \sum_{m=0}^{N-1} p_{m,k} w_m(t) + b_k, \tag{12}$$

where $w_k(t)$ is output signal of *k*-th node, r and c are positive constants, $p_{m,k}$ is connection weight from *m*-th node to *k*-th node and b_k is *k*-th bias input. Using the Laplace transform and Final Value Theorem, after some manipulations, we get:

$$W(\infty) = [I - rP]^{-1} rB\ . \tag{13}$$

This equation shows that after convergence of Hopfield network, output signal of the network W is a vector, which is obtained by multiplication of an inversion matrix of $[I - rP]$ and a vector rB. Now compared with (10) and set positive constant $r = 1$, we assume that:

$$R = I - P \tag{14}$$

and

$$U = B. \tag{15}$$

In the second term of eq.(10) is rewritten as:

$$\nabla g = \mu e(n) W^* \tag{16}$$

and equation (10) is presented by:

$$g_r(n) = g_r(n-1) + \nabla g(n), \tag{17}$$

where W^* is output of the Hopfield network, after the network is converged.

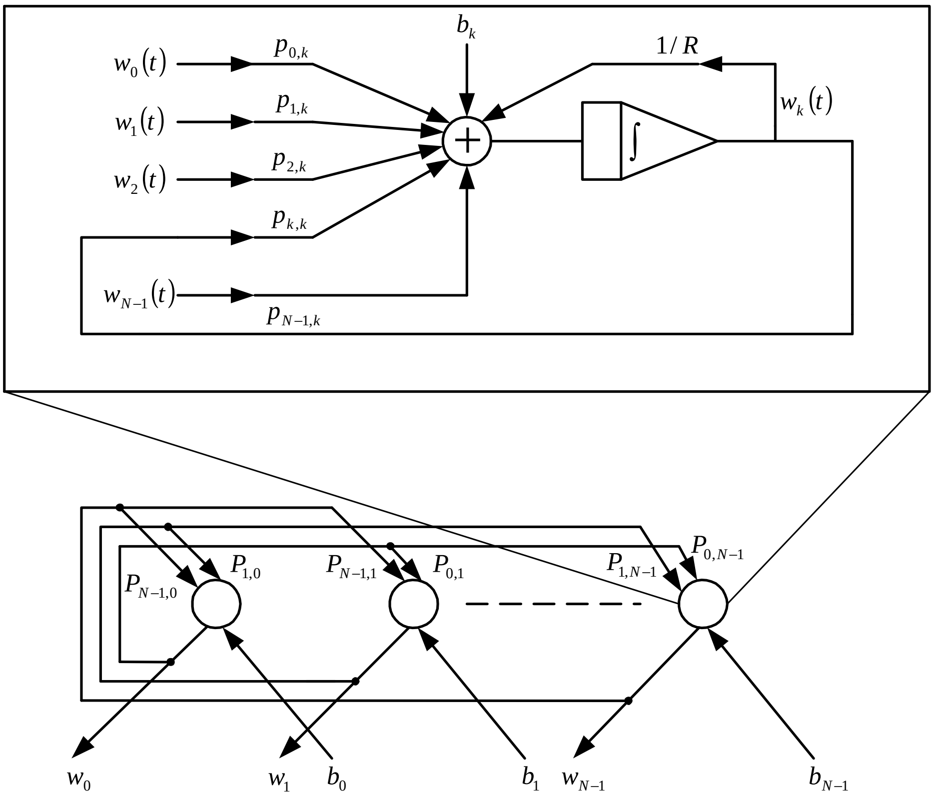

Fig. 4 – *Modified continuous time Hopfield network.*

4 Computer Simulations

It is evaluated the convergence performance of proposed structure using a system identification configuration with an unknown system of order $N = 20$. The proposed system has 4 parallel **FIR** subfilters, such that M is equal to 4 and sparsity factor L is 5.

Fig. 5 shows the convergence performance of the proposed structure and the conventional **RLS** structure. In both cases the input signal is white noise sequence and the signal-to-noise ratio (**SNR**) is equal to 20dB. Fig. 6 shows the convergence performance of the proposed structure and the conventional one. In this case the **SNR** is equal to 40dB and the input signal is a 5th order **AR** (autoregressive) process.

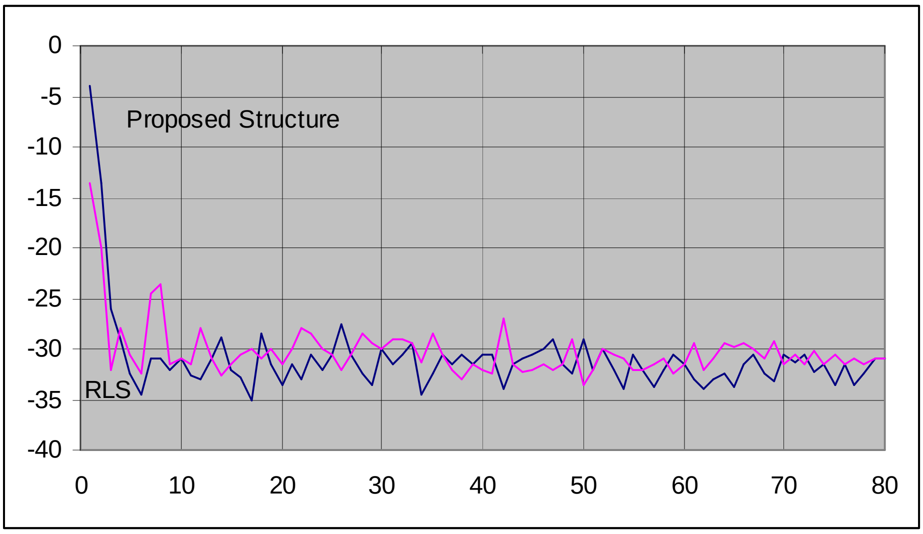

Fig. 5 – *Convergence performance of proposed and conventional **FIR** adaptive filter structure. The input signal was a white noise sequence.*

Simulations results show that proposed structure has similar convergence performance that the conventional full band **FIR** adaptive filters structure with a much lower computational cost.

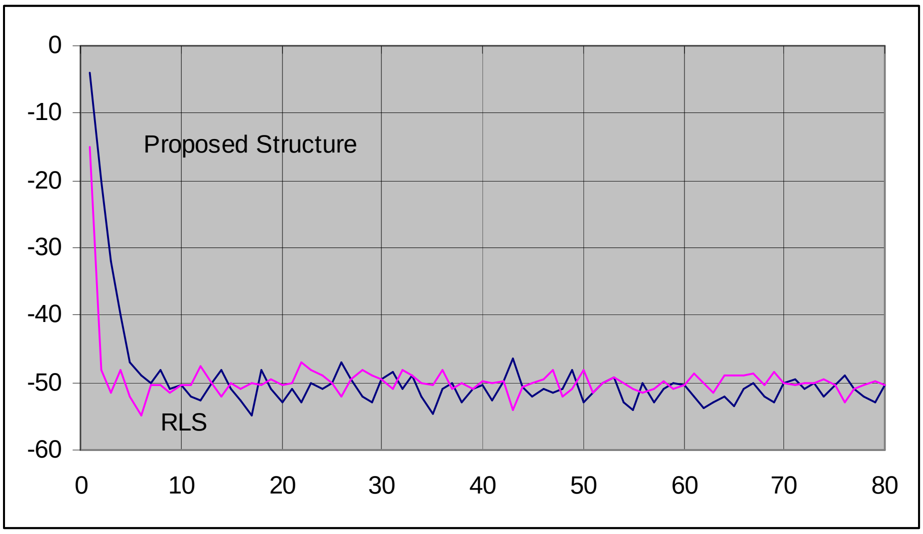

Fig. 6 – *Convergence performance of proposed and conventional **FIR** adaptive filter structure. The input signal is* 5^{th} *order **AR** process.*

5 Conclusion

This paper proposed a parallel form **FIR** adaptive filter structure in which the input signal is decomposed into *M* orthogonal components by using an orthogonal transformation. Subsequently each orthogonal component is feed into a sparse subfilter whose coefficient vector is updated by using a modified Hoipfield network. Computer simulations show that the proposed structure has similar convergence performance that the conventional full band **RLS** structure, with a much less computational cost. For examples, the number of multiplication to update the coefficient in the proposed structure is approximately $M \times N$, where *M* is number of subfilters and *N* is order of the unknown system, while the computational cost of conventional **RLS** algorithm is $O(N^2)$.

6 Reference

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