

# Optimal Placement and Sizing of Shunt Capacitors in Distorted Distribution Systems Using a Hybrid Algorithm

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**Abstract:** In this paper, a novel hybrid phasor particle swarm optimization and gravitational search algorithm, namely the hybrid PPSOGSA algorithm, is proposed to find the optimal size and location of shunt capacitors in distribution systems with non-linear loads. The performance of PPSOGSA are studied and evaluated using the IEEE 9- and 85-bus test systems with the objective of minimizing the total annual cost of the system. The procedure is conducted taking into account effects of harmonic distortion and discrete size of capacitors available in the market. Simulation results are compared with those obtained by other optimization techniques, and verified using the Electrical Transient Analysis Program (ETAP). It was established that that the hybrid PPSOGSA algorithm provides better solutions in terms of convergence and accuracy.

**Keywords:** Capacitor placement, Distribution system, Harmonic distortion, Hybrid algorithm.

## 1 Introduction

Determination of optimal placement and size of capacitor banks is a complex and up-to-date optimization problem that attracts the attention of researchers for decades [1]. A proper compensation of reactive power can significantly improve the voltage conditions in a system, reduce power losses, and therefore improve the transmission capabilities of the system. The traditional approach to solving this problem is based on the assumption that the loads are linear and includes only the fundamental frequency, ignoring the higher harmonic frequency components. However, the increasing presence of non-linear loads that generate harmonic currents in distribution power systems has imposed a need for developing new tools and techniques, as well as adapting existing ones, for the purpose of taking into account the influence of harmonics. Harmonic currents caused by non-linear loads pass through the impedance of the system and cause voltage distortions that may have a detrimental effect on the power system and equipment. Adverse

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effects of harmonics are manifested by increased power losses in lines, machines and transformers, dangerous resonance conditions, heating of cables, capacitors and equipment, vibrations and noise, interference on communication circuits, etc. [2, 3]. Researches show that most of the problems associated with the harmonic distortion in electric power systems result due to resonance. The use of shunt capacitors can cause resonant conditions which magnify the harmonic currents and create high harmonic voltage distortion levels. The presence of resonance in the electric power system is unacceptable and must be avoided. It is therefore very important to take the constraint on the harmonic distortion into account in the process of finding the optimal location and size of capacitor banks in systems with non-linear loads.

In the past few years, many population-based metaheuristic optimization algorithms, such as Particle Swarm Optimization (PSO) [4], Plant Growth Simulation Algorithm (PGSA) [5], Whale Optimization Algorithm (WOA) [6], Bacterial Foraging Optimization Algorithm (BFOA) [7], Fuzzy Genetic Algorithm (GA) [8], and Improved Harmony Algorithm (IHA) [9] have been successfully applied to find the optimal solution of the capacitor placement and sizing problem in distribution systems with different objective functions that reflect power losses minimization, voltage profile improvement, cost reduction, net saving maximization and system stability enhancement. In these papers, it is assumed that the loads are linear and the effects of harmonics are not taken in consideration. Particularly, the optimal solutions of the problem are found under the assumption of the sinusoidal condition and they may not be optimal due to the additional costs of losses or high harmonic levels caused by resonance. Accordingly, consideration of the harmonics effect is very essential in formulating the capacitor placement problem. In the scientific literature, there are few documents that considered the effect of harmonics on the optimal solution of the capacitor placement and sizing problem [10 – 17], such that the root-mean-square (RMS) voltages and total harmonic distortions of voltage ( $THD_V$ ) are within permissible limits specified in the IEEE-519 standard [18].

This paper proposes a metaheuristic algorithm, called PPSOGSA, based on the phasor particle swarm optimization (PPSO) [19] and gravitational search algorithm (GSA) [20] for solving the optimization problem of capacitor placement and sizing in radial distribution systems with non-linear loads. The proposed algorithm is tested on the IEEE 9- and 85-bus test systems with the objective of minimizing the total annual cost of the system. The constraints of the problem include bus RMS voltages,  $THD_V$  levels and distribution line loading. The results are compared with those obtained by other metaheuristic methods like WOA [6], PPSO [19], GSA [20], PSOGSA [21, 22], Fuzzy Logic Technique [23], Hybrid FUZ-DP-GA [24], MINLP [25], and verified using the Load Flow Analysis and Harmonic Analysis modules of the Electrical Transient Analysis Program (ETAP) [26].

## 2 Problem Formulation

In this paper, the problem of optimal placement and sizing of fixed capacitors in distorted radial distribution systems is considered as a constrained non-linear and non-differentiable optimization planning problem with the objective of minimizing the annual operating cost of the system. The control variables of the problem are locations and sizes of fixed shunt capacitors, while the dependent variables contain RMS bus voltages ( $V_{RMS}$ ), THD<sub>V</sub> levels and distribution line loading ( $S_l$ ).

### 2.1 Objective function

The objective cost function contains two parts. The first one is the cost of active power losses, and the second one is the cost of the reactive power provided by capacitors. It can be defined as [11]:

$$F = K_p P_{loss} + \sum_{i=1}^{N_C} K_{C,i} Q_{C,i}, \quad (1)$$

where:

- $P_{loss}$  is total active power losses in the system [kW],
- $K_p$  is the annual cost per unit of power losses [\$ /kW],
- $K_{C,i}$  is the capacitor annual cost [\$ /kVAr],
- $Q_{C,i}$  is the size of capacitor at bus  $i$  [kVAr]; and
- $N_C$  is the total number of shunt capacitors.

### 2.2 Control variables

As aforementioned, the control variables of this problem are locations and sizes of fixed capacitors. Each bus in a network represents a potential location for placement of a capacitor. Only a capacitor can be connected in a bus. The commercially available capacitors are discrete and herein they are taken into account. Some of the commercially available capacitors with their corresponding costs are shown in **Table 1**.

As in [5, 6, 23 – 25], the bus reactive compensation power is limited to

$$Q_{C,i} \leq \sum_{i=1}^{N_{bus}} Q_{L,i}, \quad (2)$$

where  $Q_{L,i}$  is the reactive load power at bus  $i$ , and  $N_{bus}$  is the total number of buses in the system.

**Table 1**  
Commercially available capacitors and their costs [6].

$Q_C$ [kVAr]	$K_C$ [\$/kVAr]	$Q_C$ [kVAr]	$K_C$ [\$/kVAr]	$Q_C$ [kVAr]	$K_C$ [\$/kVAr]	$Q_C$ [kVAr]	$K_C$ [\$/kVAr]
150	0.5	1200	0.17	2250	0.197	3300	0.174
300	0.35	1350	0.207	2400	0.17	3450	0.188
450	0.253	1500	0.201	2550	0.189	3600	0.17
600	0.22	1650	0.193	2700	0.187	3750	0.183
750	0.276	1800	0.187	2850	0.183	3900	0.182
900	0.183	1950	0.211	3000	0.18	4050	0.179
1050	0.228	2100	0.176	3150	0.195	-	-

### 2.3 Constraints

Equality constraints include power balancing at the fundamental frequency defined by (3) and (4), and constrains related to the harmonic power flow (HPF) represented by (5). On the other hand, inequality constraints are RMS bus voltage limits, branch loading limits and voltage distortion limits described by (6), (7) and (8), respectively.

$$P_{G,i} - P_{D,i} = |V_i^{(1)}| \left| \sum_{j=1}^{N_{bus}} |V_j^{(1)}| |Y_{i,j}^{(1)}| \cos(\theta_{i,j}^{(1)} - \delta_i^{(1)} + \delta_j^{(1)}) \right|, \quad (3)$$

$$Q_{G,i} - Q_{D,i} = |V_i^{(1)}| \left| \sum_{j=1}^{N_{bus}} |V_j^{(1)}| |Y_{i,j}^{(1)}| \sin(\theta_{i,j}^{(1)} - \delta_i^{(1)} + \delta_j^{(1)}) \right|, \quad (4)$$

$$V^{(h)} = [Y_{BUS}^{(h)}]^{-1} I^{(h)}, \quad (5)$$

$$V_{RMS,i}^{\min} \leq \sqrt{\sum_{h=1}^{h_{\max}} |V_i^{(h)}|^2} \leq V_{RMS,i}^{\max}, \quad (6)$$

$$S_{l,i} \leq S_{l,i}^{\max}, \quad (7)$$

$$THD_{V,i}(\%) = \frac{1}{|V_i^{(1)}|} \cdot \sqrt{\sum_{h \neq 1}^{h_{\max}} |V_i^{(h)}|^2} \times 100(\%) \leq THD_{V,i}^{\max}. \quad (8)$$

In equations (3) – (8), the variables have the following meaning:  $P_{G,i}$  and  $Q_{G,i}$  are active and reactive power generations at bus  $i$ , respectively;  $P_{D,i}$  and  $Q_{D,i}$  are active and reactive load demands at bus  $i$ , respectively;  $V_i^{(1)}$  and  $V_j^{(1)}$  are

voltages at buses  $i$  and  $j$ , respectively;  $Y_{i,j}^{(1)}$  is the  $(i,j)$ -th element of the admittance matrix;  $\theta_{i,j}^{(1)}$  is the angle of the  $(i,j)$ -th element of the admittance matrix;  $\delta_i^{(1)}$  and  $\delta_j^{(1)}$  are voltage angles at buses  $i$  and  $j$ , respectively;  $\mathbf{V}^{(h)}$  is the bus voltage vector at the  $h$ -th harmonic;  $\mathbf{Y}_{\text{BUS}}^{(h)}$  is the bus admittance matrix at the  $h$ -th harmonic;  $\mathbf{I}^{(h)}$  is the bus injected current vector at the  $h$ -th harmonic;  $V_{RMS,i}^{\min}$  and  $V_{RMS,i}^{\max}$  are the minimum and maximum bus voltage limits at bus  $i$ , respectively;  $S_{l,i}^{\max}$  is the maximum power flow in line/branch  $i$ , and  $THD_{V,i}^{\max}$  is the maximum acceptable level of the  $THD_V$  at any bus  $i$ .

To calculate the parameters of the considered distribution systems at the fundamental frequency, the Backward-Forward Sweep (BFS) method [27, 28] was used. On the other hand, the HPF solutions are obtained by using the Decoupled Harmonic Power Flow (DHPF) method [11, 12, 29].

## 2.4 Expanded objective function

The inequality constraints of the dependent variables such as RMS and THD for all bus voltages, as well as line flows, are added to the objective function as quadratic penalty terms, i.e.

$$F_e = F + \lambda_V \sum_{i=1}^{N_{\text{bus}}} (V_{RMS,i} - V_{RMS,i}^{\text{lim}})^2 + \lambda_{THD} \sum_{i=1}^{N_{\text{bus}}} (THD_{V,i} - THD_{V,i}^{\text{lim}})^2 + \lambda_S \sum_{i=1}^{N_l} (S_{l,i} - S_{l,i}^{\text{lim}})^2, \quad (9)$$

where:

- $F_e$  is the expanded objective function;
- $\lambda_V$ ,  $\lambda_{THD}$  and  $\lambda_S$  are the corresponding penalty factors; and
- $x^{\text{lim}}$  is the limit value of dependent variable  $x$ , which is given by:

$$x^{\text{lim}} = x^{\text{max}} \text{ if } x > x^{\text{max}} \text{ and } x^{\text{lim}} = x^{\text{min}} \text{ if } x < x^{\text{min}}.$$

In this study, the penalty factor of  $10^6$  is selected for all the inequality constraints.

## 3 Solution Method

In this section, the hybrid PPSOGSA algorithm is proposed to solve the problem of optimal placement and sizing of capacitors in distorted distribution systems.

The PPSOGSA approach consists of a combination of PPSO [19] and GSA [20]. The improvement of this algorithm in relation to the original PSOGSA [21, 22] is based on modelling the algorithm parameters with a phase angle ( $\theta$ ), which converts the standard PSO to a self-adaptive and parametric independent algorithm. Namely, the acceleration coefficients  $c_1$  and  $c_2$ , which are the main control parameters of PSOGSA, are fixed during iteration process, and the PSOGSA's searching ability and efficiency depend largely on the values of these factors. Instead of using fixed values, in this paper the periodic nature of sine and cosine functions is used to represent control parameters through phase angles.

The velocity and position of particle  $i$  at iteration  $t$  are given by the equations:

$$\begin{aligned} V_i(t+1) = & r_1 V_i(t) + r_2 \left| \cos(\theta_i(t)) \right|^{2\sin(\theta_i(t))} \mathbf{ac}_i(t) + \\ & + r_3 \left| \sin(\theta_i(t)) \right|^{2\cos(\theta_i(t))} (\mathbf{gbest}(t) - X_i(t)), \end{aligned} \quad (10)$$

$$X_i(t+1) = X_i(t) + V_i(t+1). \quad (11)$$

In equation (10), the phase angle of particle  $i$  ( $\theta_i$ ) is calculated as [19]:

$$\theta_i(t+1) = \theta_i(t) + \left| \cos(\theta_i(t)) + \sin(\theta_i(t)) \right| 2\pi, \quad (12)$$

where:

–  $V_i(t)$  and  $X_i(t)$  are the velocity and the position of agent  $i$  at iteration  $t$ , respectively;

–  $r_1$ ,  $r_2$  and  $r_3$  are random numbers between 0 and 1;

–  $\mathbf{ac}_i(t)$  is the acceleration of agent  $i$  at iteration  $t$ , and

–  $\mathbf{gbest}(t)$  is the best position of all particles in the group at iteration  $t$ .

The values of  $\mathbf{ac}_i$  and  $\mathbf{gbest}$  in equation (10) are obtained as in [21, 22].

### 3.1 PPSOGSA implementation

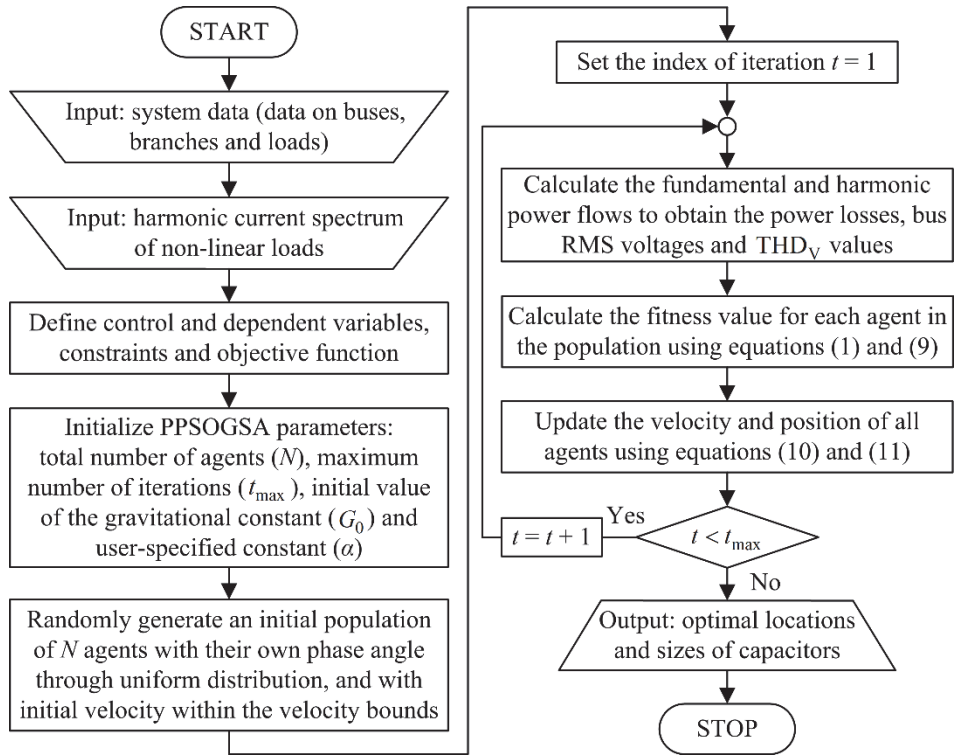
In this study, a potential solution can be presented by a vector consisting of a combination of locations and rated power of capacitors in these locations. The position of agent  $i$  can be defined as follows:

$$X_i = \left[ Bus_1^1, \dots, Bus_{N_c}^d, Q_{C,1}^{d+1}, \dots, Q_{C,N_c}^n \right], \quad (13)$$

where  $n$  indicates the number of control variables.

The flowchart of the PPSOGSA algorithm is presented in Fig. 1.

More details on PPSOGSA and its application in solving various problems in electrical engineering can be found in [30 – 33].



**Fig. 1** – Flowchart illustrating the PPSOGSA algorithm to optimize the locations and sizes of shunt capacitors in the presence of harmonics.

## 4 Simulation Results

The proposed approach was implemented in MATLAB and tested on the IEEE 9- and IEEE 85-bus systems. For both test systems, the substation voltage is set to 1 p.u. In addition to this, it is assumed that the voltage at the utility connection point does not contain any harmonics. The equivalent impedance of the system was determined from short-circuit power of 1000 MVA and reactance-to-resistance ratio of 22.2.

Both sinusoidal and non-sinusoidal operating conditions were taken into account. In every bus, the loads of the systems comprise a linear part and a non-linear part. At the fundamental frequency (i.e., 50 Hz), all loads are represented by the constant power model, while capacitors are represented by the constant impedance model. At higher frequencies, linear passive loads are modelled using the parallel RL model [12], and non-linear loads are modelled as decoupled harmonic current sources.

The harmonic spectrum of currents injected from non-linear loads is presented in **Table 2**.

**Table 2**  
*The harmonic spectrum of non-linear loads.*

Harmonic order	Magnitude [%]	Phase angle [degree]
1	100	0
5	60.82	-175
7	33.42	-172
11	3.84	166
13	7.74	-177
17	1.27	32
19	1.54	179
23	1.08	38
25	0.16	49

The harmonic spectrum has harmonic characteristics of a six-pulse converter of a pulse-width modulation (PWM) adjustable-speed drive (ASD) and only contains odd harmonics, except triplen harmonics.

For all test systems, the yearly cost of losses ( $K_p$ ) is selected to be 168 \$/kW [5], the maximum acceptable level of the THD<sub>v</sub> is 5%, and the voltage limits are 0.9 p.u and 1.1 p.u., according to the limits specified by the IEEE-519 standard [18].

In order to prove the efficiency of the proposed algorithm, the same problem was solved using PPSO, GSA and PSO<sub>GSA</sub> and the results are compared with those obtained by other techniques from the literature [6, 23 – 25]. The setting parameters of PPSO, GSA, PSO<sub>GSA</sub> and PPSO<sub>GSA</sub> used in the calculation are provided in **Table 3**. The results presented herein are the best values obtained over twenty consecutive test runs.

**Table 3**  
*Parameter setting used in PPSO, GSA, PSO<sub>GSA</sub> and PPSO<sub>GSA</sub>.*

Control parameters	PPSO	GSA	PSO <sub>GSA</sub>	PPSO <sub>GSA</sub>
Population size ( $N$ )	50	50	50	50
Maximum number of iterations ( $t_{max}$ )	250	250	250	250
Acceleration constants ( $c_1, c_2$ )	-	-	2, 2	-
Initial value of the gravitational constant ( $G_0$ )	-	100	1	1
User-specified constant ( $\alpha$ )	-	2	20	20



#### 4.1 IEEE 9-bus test system

The algorithms are first tested on the IEEE 9-bus radial distribution system [12] with total active and reactive power loads of 12368 kW and 4186 kVAr, respectively. As in [23, 24], it was decided that the maximum capacitor size should not exceed the total reactive load (i.e., 4186 kVAr).

##### 4.1.1 Sinusoidal condition

Under the sinusoidal operating condition all loads in the system are treated as linear and the calculation is performed at the fundamental frequency. The cost function and total active power losses in the initial case before compensation are 131676 \$ and 783.787 kW, respectively. The simulation results obtained by different optimization techniques are shown in **Table 4**.

**Table 4**  
*Simulation results of the IEEE 9-bus system under the sinusoidal condition.*

Bus number	Without capacitors	With capacitors					
		Ref. [23]	Ref. [24]	PPSO	GSA	PSOGS A	PPSOGS A
1	-	-	-	-	-	-	-
2	-	3600	3600	4050	3900	4050	4050
3	-	-	4050	4050	-	-	-
4	-	4050	450	-	3000	3450	3600
5	-	1650	1200	2100	1650	1650	-
6	-	-	-	-	-	-	1350
7	-	-	-	-	-	-	-
8	-	600	150	-	-	-	-
9	-	-	600	600	900	750	750
Minimum voltage [p.u.]	0.8375	0.9003	0.9001	0.9001	0.9001	0.9002	0.9001
Maximum voltage [p.u.]	1	1.007	1.007	1.0081	1.0052	1.0062	1.0059
Losses [kW]	783.787	686	681.28	681.31	682.03	680.55	680.75
Cost [\$]	131676	11703 5	11631 7	11641 1	11631 3	116231	116189
Benefits [\$]	-	14641	15359	15265	15363	15445	15487

As can be seen from **Table 4**, the minimum losses and cost of the system found by the PPSOGSA are, respectively, 680.748 kW and 116189 \$, showing a reduction of 13.15% in power losses and 11.76% in cost of the system. Also, it can be observed that the cost of the system obtained with the proposed PPSOGSA method is less than that one obtained with other techniques.

From **Table 4**, it can be seen that the bus voltages after capacitor placement meet the defined limits.

To establish the accuracy of the BFS method, the power flow calculations were also performed using the Load Flow Analysis module of the Electrical Transient Analysis Program (ETAP). The results obtained by the ETAP software in the initial case and the case when the capacitors' parameters are generated by PPSOGSA are presented in Figs. 2 and 3, respectively.

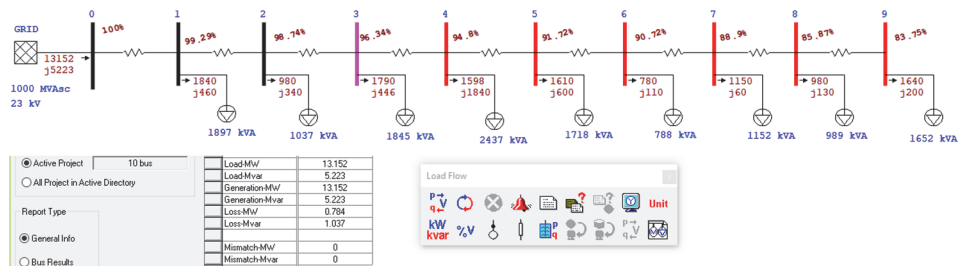


Fig. 2 – Results obtained with ETAP program for the IEEE 9-bus test system in the case of the sinusoidal operation without capacitors.

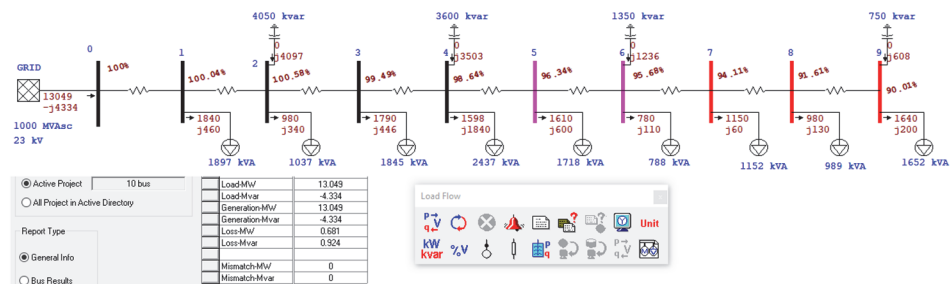


Fig. 3 – Results obtained with ETAP program for the IEEE 9-bus test system in the case of the sinusoidal operation with capacitors.

By comparing the results from Figs. 2 and 3 with the corresponding results from the second and eighth columns of **Table 4**, it can be observed that the results calculated by the BFS method are almost equal to those obtained with ETAP program.

#### 4.1.2 Non-sinusoidal condition

In order to demonstrate the performance of the hybrid PPSOGSA algorithm under non-sinusoidal conditions, the optimal capacitor placement and sizing problem is expanded for higher harmonic frequencies. In every bus, it is assumed that the loads of this test system comprise a linear part of 80% and a non-linear

part of 20%. All non-linear loads have the same harmonic spectrum from **Table 2**. The optimal results are given in **Table 5**.

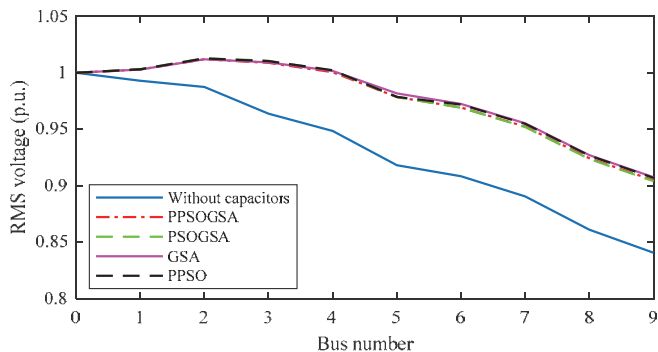
**Table 5**

*Simulation results of the IEEE 9-bus system under the non-sinusoidal condition.*

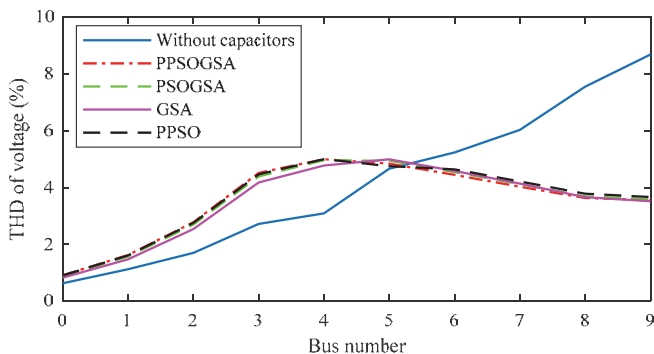
Bus number	Without capacitors	With capacitors			
		PPSO	GSA	PSOGSA	PPSOGSA
1	-	-	-	-	-
2	-	4050	3900	4050	4050
3	-	3600	3000	3300	4050
4	-	3600	3000	3300	3000
5	-	-	2700	2100	1800
6	-	1650	-	-	-
7	-	-	-	-	-
8	-	-	-	-	-
9	-	-	-	-	-
Min. $V_{RMS}$ [p.u.]	0.8406	0.9065	0.9070	0.9050	0.9041
Max. $V_{RMS}$ [p.u.]	1.0000	1.0128	1.0121	1.0125	1.0129
Max. $THD_V$ [%]	8.685	5.000	4.998	4.966	4.998
Fund. freq. losses [kW]	783.787	702.733	706.639	702.392	701.773
Harmonic losses [kW]	6.860	8.567	8.579	8.8307	8.828
Total losses [kW]	790.647	711.3	715.218	711.223	710.601
Total cost [\$]	132828	121765	122451	121728	121707
Benefits [\$]	-	11063	10377	11100	11121

From the second column of **Table 5**, it can be seen that the total active power losses of the system are 790.647 kW, while the cost of the system is 132828 \$. In addition, it is evident that the minimum RMS voltage and the maximum  $THD_V$  violate the permissible limits of 0.9 p.u. and 5%, respectively. The results from the last row of **Table 5** show that the proposed PPSOGSA algorithm is better than PPSO, GSA, PSOGSA algorithms in terms of achieving the benefits. Compared to the initial case, the best solution generated by PPSOGSA provides reductions of 10.12% and 8.37% in power losses and cost, respectively.

Figs. 4a and 4b respectively show the comparisons between RMS voltage profiles and  $THD_V$  levels in the IEEE 9-bus system for cases without and with capacitors. From **Table 5** and Fig. 4, after capacitor placement, it can be seen that the  $V_{RMS}$  voltages, as well as  $THD_V$  values meet limits specified in [18].



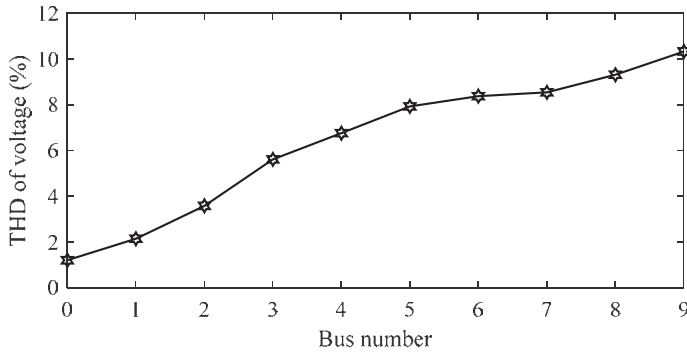
(a)



(b)

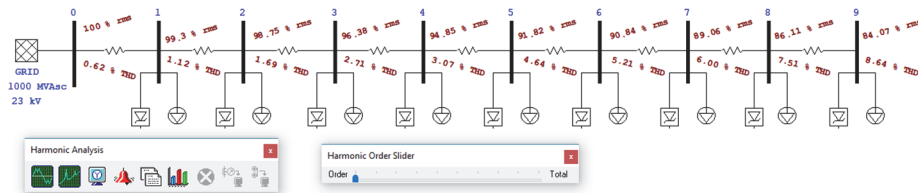
**Fig. 4** – Comparisons between: (a) voltage profiles; (b)  $THD_V$  levels of the IEEE 9-bus system.

By comparing the power losses at the fundamental frequency from **Table 5** obtained after implementing the optimization procedure with those from **Table 4**, it can be seen that the losses from **Table 5** are higher by about 3%. To demonstrate the effect of the installation of shunt capacitors on the harmonic distortion voltage, the calculation of the HPF was performed. Fig. 5 illustrates the  $THD_V$  at each bus in the case when capacitors' parameters are obtained by PPSOGSA under the sinusoidal operation condition. It is clear that the  $THD_V$  at buses 3 to 9 reaches the level higher than 5%. In relation to the initial case from **Table 5**, the maximum value of  $THD_V$  increases 18.9% and reaches 10.33%. This increase in voltage distortion levels is due to harmonic resonant conditions caused by capacitors in combination with load and feeder reactance. From the results in Fig. 5, it is obvious that the optimal solution obtained in the case of the sinusoidal operation, lead to unacceptable  $THD_V$  levels at system buses. Based on that, it can be said that the optimal results from **Table 5** are more appropriate, because they take the harmonic constraint into consideration.

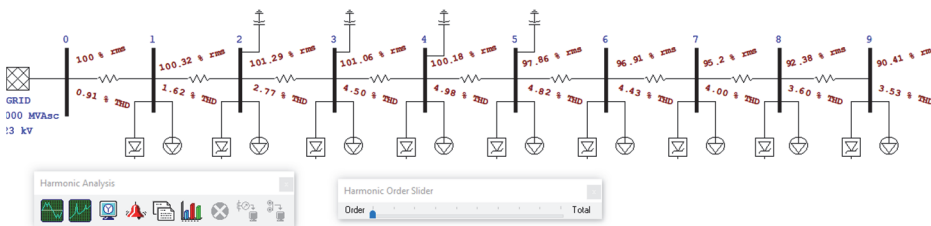


**Fig. 5** – THD<sub>V</sub> levels of the IEEE 9-bus test system with capacitors’ parameters obtained by PPSOGSA under the sinusoidal condition.

The harmonic analysis was also performed using the ETAP software. The simulation results generated by the ETAP software for the initial case and the case when the capacitors’ parameters are obtained by the PPSOGSA algorithm are presented in Figs. 6 and 7, respectively.



**Fig. 6** – Results obtained with ETAP program for the IEEE 9-bus system in the case of non-sinusoidal operation without capacitors.



**Fig. 7** – Results obtained with ETAP program for the IEEE 9-bus test system in the case of non-sinusoidal operation with capacitors.

Based on the comparisons of  $V_{RMS}$  and THD<sub>V</sub> values from Figs. 6 and 7 with the corresponding results from Fig. 4, it can be observed that the results agree well.

## 4.2 IEEE 85-bus test system

The second test system that is used to demonstrate the performance of the proposed algorithm is the standard IEEE 85-bus system [6]. The total active and reactive powers of the system are 2570.3 kW and 2622.1 kVAr, respectively.

### 4.2.1 Sinusoidal condition

The simulation results of the IEEE 85-bus system under the sinusoidal condition are presented in **Table 6**.

**Table 6**  
*Simulation results of the IEEE 85-bus system under the sinusoidal condition.*

Case	Method	Optimal location	Optimal size [kVAr]	Min. $V_{fund}$ [p.u.]	Total losses [kW]	Total cost [\$]	Benefits [\$]
Without capacitors	-	-	-	0.8713	316.116	53107	-
With capacitors	PPSOGSA	12	450	0.9226	149.216	25610	27497
		25	900				
		48	600				
		67	600				
	PSOGSA	9	1050	0.9227	148.978	25627	27480
		30	450				
		48	450				
		67	600				
	PPSO	12	600	0.9206	149.341	25674	27433
		29	600				
		48	450				
		67	750				
GSA	9	1200	0.9232	149.240	25627	27480	
	31	600					
	52	300					
	68	450					
Ref. [25]	7	300	0.9176	159.410	27354	25753	
	8	700					
	29	900					
	58	500					
Ref. [6]	25	490	0.9214	149.520	25652	27455	
	34	709					
	67	566					
	80	417					

As can be seen in **Table 6**, the minimum voltage, total power losses and total cost of the system are 0.8713 p.u., 316.116 kW and 53107 \$, respectively. By adjusting the control variables to the optimal values obtained using PPSOGSA, the minimum voltage is improved to 0.9226 p.u., while power losses and system cost are decreased to 149.216 kW and 25610 \$, respectively. This apparently indicates that the proposed PPSOGSA algorithm can be successfully used for large-scale power systems.

From **Table 6**, it can be seen that the proposed approach outperforms PPSO, GSA, PSOGSA, WHO [6] and MINLP [25], because the results obtained using PPSOGSA are better than those obtained using other methods.

#### 4.2.2 Non-sinusoidal operating condition

As well as in the previous test system, it is assumed that the loads comprise a linear part of 80% and a non-linear part of 20% in every bus. Also, all non-linear loads have the same harmonic spectrum from **Table 2**. The optimal results are given in **Table 7**.

**Table 7**  
Simulation results of the IEEE 85-bus system under the non-sinusoidal operating condition.

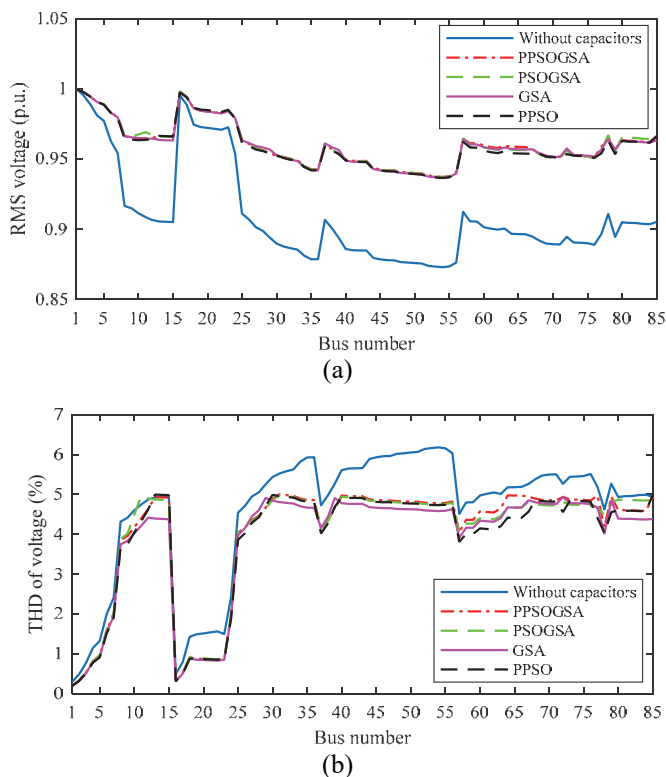
Case	Method	Optimal location	Optimal size [kVAr]	Min. $V_{RMS}$ [p.u.]	Max. $THD_V$ [%]	Total losses [kW]	Total cost [\$]	Benef. [\$]
Without capacitors	-	-	-	0.8730	6.179	321.327	53982	-
With capacitors	PPSOGSA	13	750	0.9372	5.000	190.127	32661	21321
		25	300					
		31	1200					
		64	1200					
With capacitors	PSOGSA	11	1350	0.9371	4.971	190.937	32830	21152
		16	300					
		31	1200					
		67	900					
With capacitors	PPSO	13	900	0.9365	4.995	191.422	32864	21118
		20	150					
		30	1500					
		68	900					
With capacitors	GSA	12	750	0.9366	4.929	196.911	33845	20137
		23	0					
		29	1650					
		72	1050					

The power losses of the system in the initial case are 321.327 kW of which 5.21 kW represents the harmonic power losses. The annual cost of the system,

minimum RMS voltage and maximum  $THD_V$  level are 53982 \$, 0.873 p.u. and 6.179%, respectively. After applying the optimization procedure, it appears from **Table 7** that the power losses, and therefore the annual cost of the system, are significantly reduced in comparison to the initial case. In addition, voltage profile and power quality are improved.

Based on the comparison of the results from **Table 7**, it can be noted that the hybrid PPSOGSA approach outperforms PPSO, GSA and PSOGSA providing the highest benefits.

Figs. 8a and 8b, respectively, illustrate the comparisons between RMS voltage profiles and between  $THD_V$  levels in the IEEE 85-bus system for cases without and with capacitors.



**Fig. 8** – Comparisons between: (a) voltage profiles; (b)  $THD_V$  levels of the IEEE 85-bus system.

The system parameters computed by BFS and DHPF methods, including voltages, power losses and harmonic levels, were also verified using the Load Flow Analysis and Harmonic Analysis modules of the ETAP software, but these



results are not displayed due to the limited space. It was also found that there is a high degree of compatibility between them.

### 4.3 Statistical parameters and convergence profiles

The statistical parameters of the results obtained by PPSO, GSA, PSO GSA and PPSOGSA over twenty independent runs for the IEEE 9- and 85-bus test systems are presented in **Tables 8** and **9**, respectively.

**Table 8**

*Statistical parameters of different algorithms for the IEEE 9-bus system.*

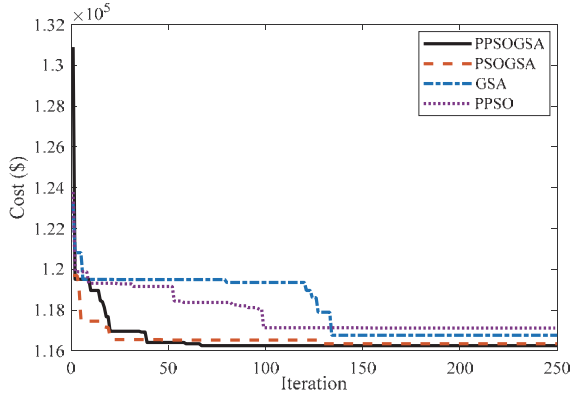
Operating condition	Method	Minimum solution [\$]	Maximum solution [\$]	Average solution [\$]	Standard deviation [\$]
Sinusoidal	PPSO	116410	117141	116575	465
	GSA	116313	117650	117030	582
	PSOGSA	116231	116982	116645	361
	PPSOGSA	116189	116845	116420	265
Non-sinusoidal	PPSO	121765	129340	123762	1570
	GSA	122451	129847	126196	2051
	PSOGSA	121728	129715	123851	1805
	PPSOGSA	121707	125601	123283	1118

**Table 9**

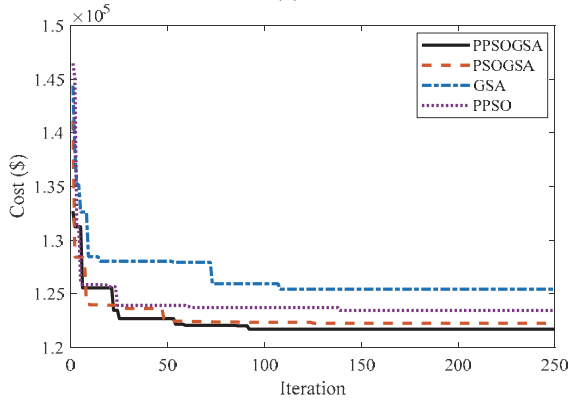
*Statistical parameters of different algorithms for the IEEE 85-bus system.*

Operating condition	Method	Minimum solution [\$]	Maximum solution [\$]	Average solution [\$]	Standard deviation [\$]
Sinusoidal	PPSO	25674	29171	25845	808
	GSA	25627	27068	26301	560
	PSOGSA	25627	26512	26283	251
	PPSOGSA	25610	26144	25864	226
Non-sinusoidal	PPSO	32864	38229	34177	2171
	GSA	33845	37665	35927	1571
	PSOGSA	32830	35898	34094	1027
	PPSOGSA	32661	35668	33936	821

The results from **Tables 8** and **9** show that the proposed algorithm has better performance in comparison to PPSO, GSA and PSO GSA. The convergence profiles of all algorithms are shown in Figs. 9 and 10. The figures indicate that the proposed algorithm can converge to its optimal solutions in lower iterations compared to the other algorithms. The authors have also performed several test runs and it was found that the proposed PPSOGSA in most cases converges to a solution after 50-100 iterations.

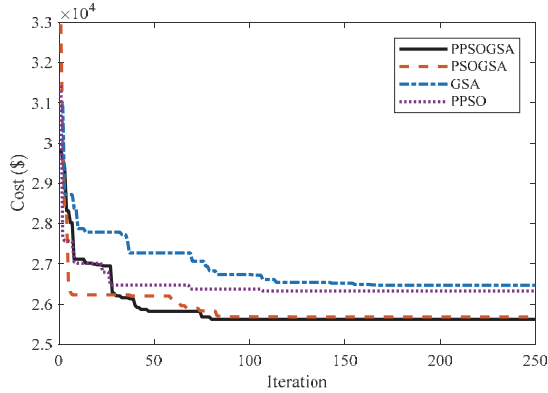


(a)

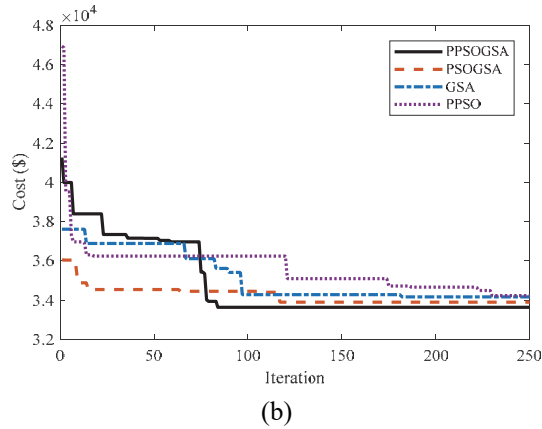


(b)

**Fig. 9** – Convergence profiles for the IEEE 9-bus system under: (a) sinusoidal condition; (b) non-sinusoidal condition.



**Fig. 10a** – Convergence profiles for the IEEE 85-bus system under sinusoidal condition.



**Fig. 10b** – Convergence profiles for the IEEE 85-bus system under non-sinusoidal condition.

## 5 Conclusion

In this paper, a new hybrid and efficient metaheuristic algorithm is proposed to find optimal sizing and placement of shunt capacitors with the objective of minimizing the annual operating cost of the system. The proposed algorithm is tested on the IEEE 9- and 85-bus systems with non-linear loads. The solutions obtained by the proposed algorithm are compared with those obtained with other optimization techniques. Simulation results are shown that the hybrid PPSOGSA algorithm provides efficient, robust and high-quality solutions. Furthermore, by comparing the optimal values of the results, standard deviations and convergence profiles, it is found that the hybrid PPSOGSA algorithm gives better results than the other methods applied to solve this problem.

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