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Wave-based Digital Models of Different Branch-Line Couplers

Biljana P. Stošić[1]

Abstract: Microwave and millimeter-wave communication systems require couplers in different applications. The aim of this paper is to construct simple wave digital models of directional couplers with two or three parallel lines. The idea behind this aim is to be able to model a microstrip structure and to simulate it using Simulink platform which has already been used extensively by researchers. Advanced Design System (ADS) software and MATLAB/Simulink environment are used to design, implement and simulate the investigated microstrip circuits and their models. Validity and accuracy of the presented digital models are fully demonstrated through several benchmark problems.

Keywords: Coupler, Branch-line coupler, Microstrip circuit, Wave digital model, Circuit simulator.

1 Introduction

There is a large range of commercially available radio frequency (RF)/microwave computer-aided design (CAD) tools. Since most microwave circuits are comprised of linear elements, linear simulations based on the network analyses, are simple and fast for CAD. In linear simulations, frequency responses are calculated in the equivalent circuit models. Recently, a significant work has been concentrated on the development of wave digital approach for modelling and analysis of different physical systems. A detailed review of an application of wave digital structures for electromagnetic (EM) field simulation is given in [1, 2]. A basic of wave digital (WD) filters is presented in [3, 4].

Basically, it is important to analyze a microstrip structure quickly, to get proper information in a short period of time. In practice, many models have been in use: the EM models, the electric circuit models, etc. The EM models are highly accurate, as they aim to model interactions in a whole structure; however this makes them extremely complex and time consuming. The electric circuit models however are less complex and provide sufficient representation of a structure.

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[1] University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, Niš, Serbia;
E-mail: biljana.stosic@elfak.ni.ac.rs

In this paper, an existing approach based on WD theory is extended to be used in a border class of microstrip circuits. The idea behind this paper is to develop digital models that one can implement to model a real multi-port microstrip structure and to simulate it using Simulink platform that researchers have already used intensively.

Three kinds of couplers are particularly interesting for RF applications because of their performances and their compatibility with the technologies: the branch-line coupler, the coupled lines coupler and the Lange coupler which is an extension of the coupled lines proficiencies. One of the most popular passive circuits used for various microwave and millimeter-wave applications is a branch-line directional coupler. In this paper, digital models of different coupler structures are presented and discussed. Simple wave digital models are generated and simulated by usage of mentioned MATLAB/Simulink environment.

The rest of the paper is organized as follows. Section II is devoted to the preliminaries of a wave-based modeling technique, more specifically to the basic wave digital elements used for device modeling. Different branch-line couplers, their models and simulation results along with discussion are addressed in Section III, while Section IV concludes the paper.

2 Preliminaries of a Wave-Based Modeling

A starting point for creation of a WD model of the observed four-port structure is to look on that structure as a connection of several individual arms. The equivalent circuit parameters are not quoted at the beginning, therefore the initial step in the proposed technique is to synthesize the values of inductors and capacitors based on the frequency and the characteristic impedances of the individual coupler branches. The lumped equivalent circuit model for the conventional branch-line coupler is proposed in [5]. During modeling process, i.e. model synthesis, parts of the system representing multiport networks with inductors and capacitors in the equivalent circuit are replaced with their wave digital counterpart. One sub-block in the generated WD counterpart models a specific part of coupler layout; it represents one wave digital element (WDE). The main attention in the papers [6, 7] is focused on generating so-called basic two-dimensional symmetric and asymmetric WDEs. So, a WD model of the device under consideration is based on basic multi-port WDEs and two-port adaptors for port impedance matching. Therefore, the developed WD model will be completely characterized by its parameters: port resistances and adaptor coefficients.

The five-port parallel adaptor with dependent port five [1, 3, 4] is used for modeling of capacitor in parallel branch. For the four-port wave digital network of a capacitor in parallel branch, the equations for wave variables are:

$$A_5 = z^{-1} B_5 , \tag{1}$$

$$A_0 = 2A_5 + \sum_{k=1}^{4} \alpha_k (A_k - A_5) , \tag{2}$$

$$B_5 = A_0 - A_5 , \tag{3}$$

$$B_k = B_5 + A_5 - A_k , \quad k = 1,2,3,4 , \tag{4}$$

where A_k and B_k are incident and reflected wave variables. The multiplier coefficients are calculated by expressions

$$\alpha_k = 2G_k \Big/ \sum_{i=1}^{5} G_i , \quad k = 1,2,3,4 . \tag{5}$$

The inductors in series branches are modelled by unit wave digital elements. The basic asymmetric WDEs that will be used widely in this paper are given closely in Fig. 1 [7].

In general, Simulink forms the core environment for model-based design and creation of accurate models of different systems. The graphical, block-diagram let one drag-and-drop predefined model elements, connect them together and create model. User can create its own customized element library in Simulnik by collecting basic symmetric and asymmetric WDEs. The library is suitable for use in the complete model of a device, and can represent a starting point for creation of a new model.

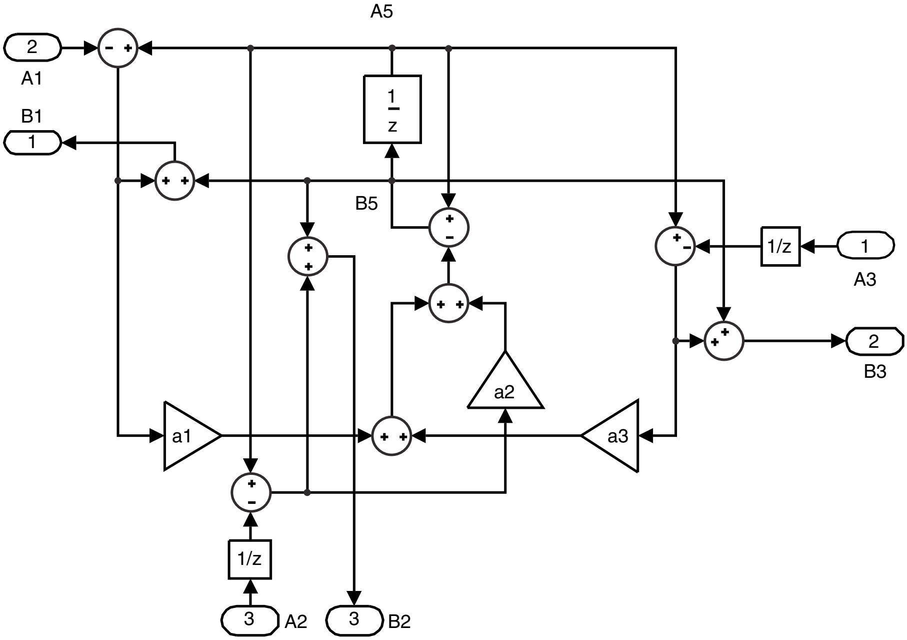

Fig. 1a – *Three-port asymmetric WDEs: Left-Up (WDE-LU).*

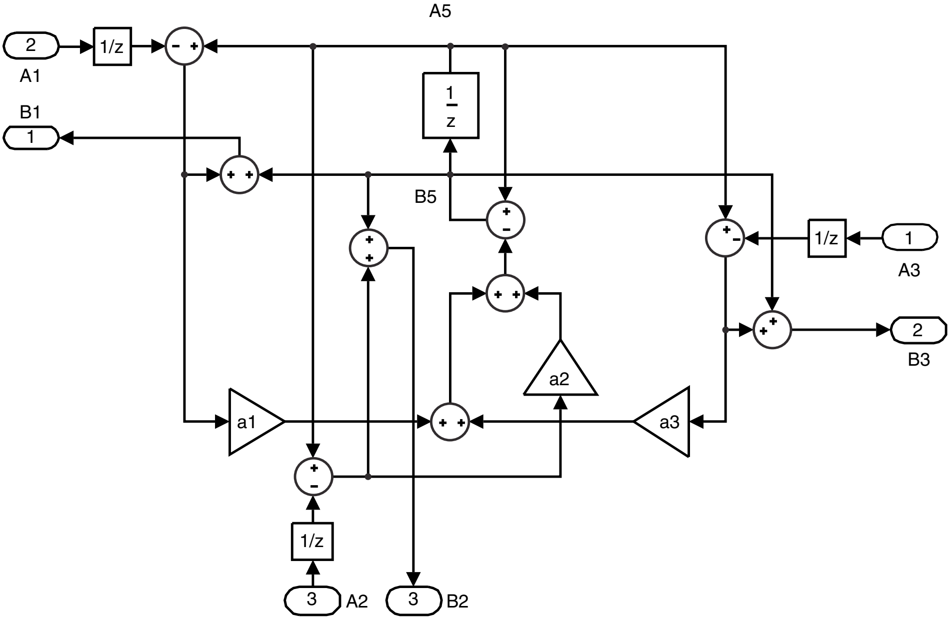

Fig. 1b – *Three-port asymmetric WDEs: Center-Up (WDE-CU).*

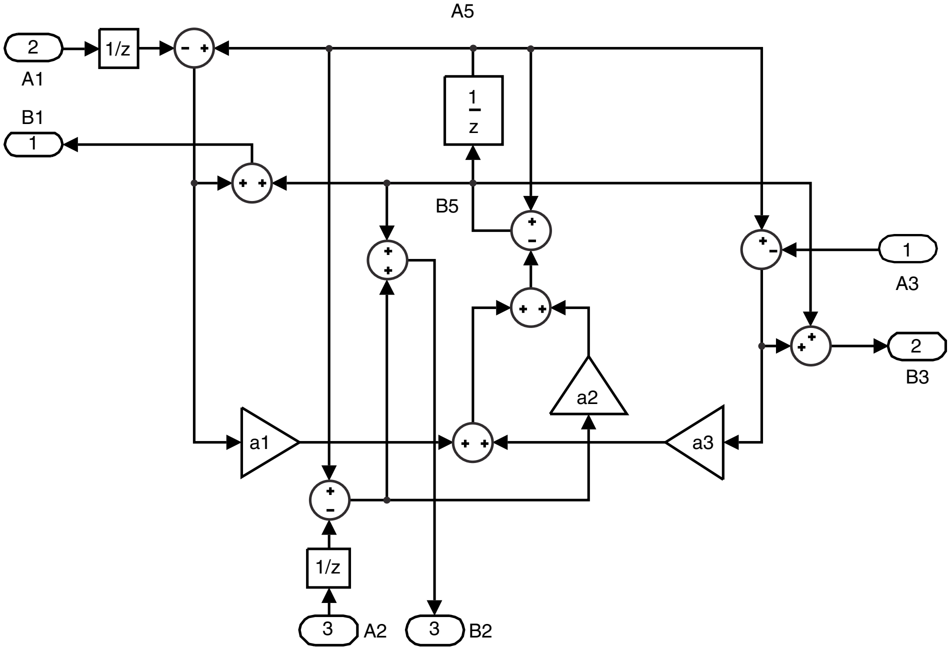

Fig. 1c – *Three-port asymmetric WDEs: Rigth-Up (WDE-RU).*

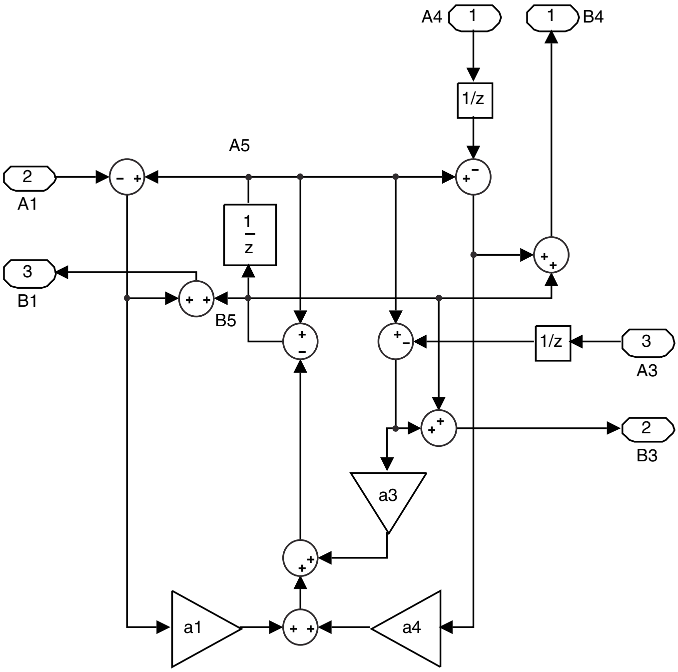

Fig. 1d – *Three-port asymmetric WDEs: Left-Down (WDE-LD).*

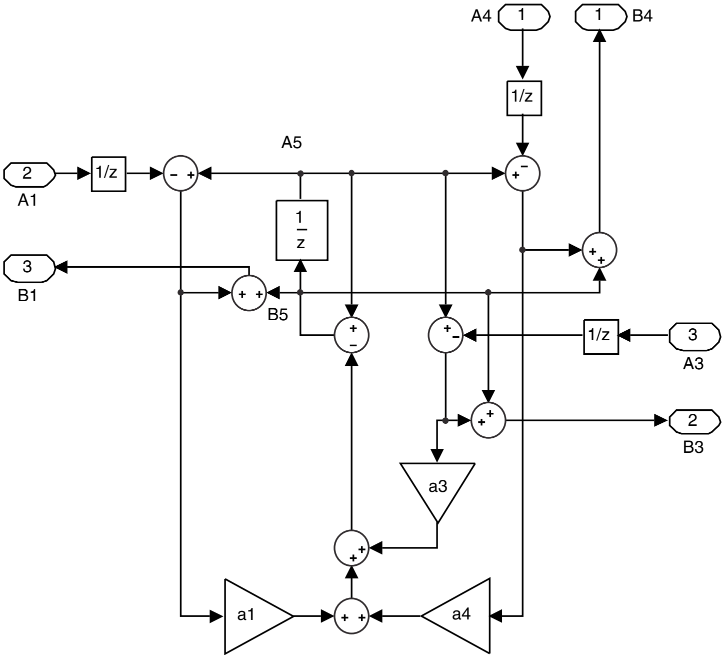

Fig. 1e – *Three-port asymmetric WDEs: Center-Down (WDE-CD).*

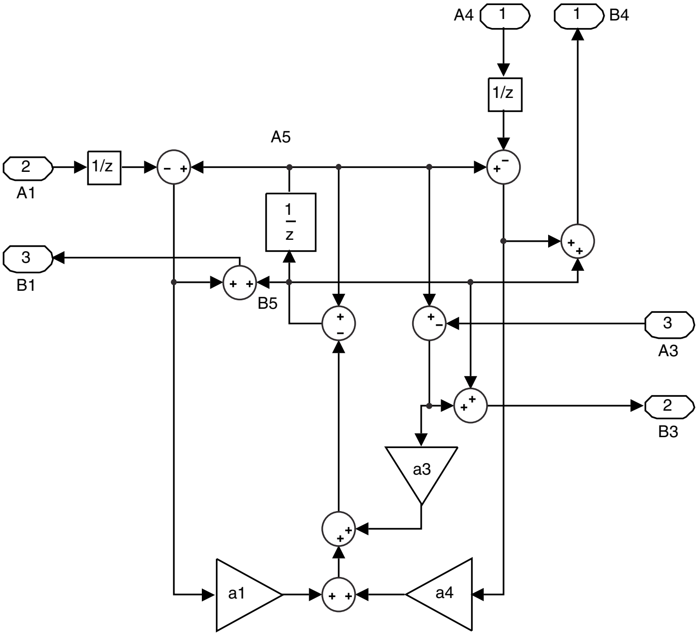

Fig. 1f – *Three-port asymmetric WDEs: Rigth-Down (WDE-RD).*

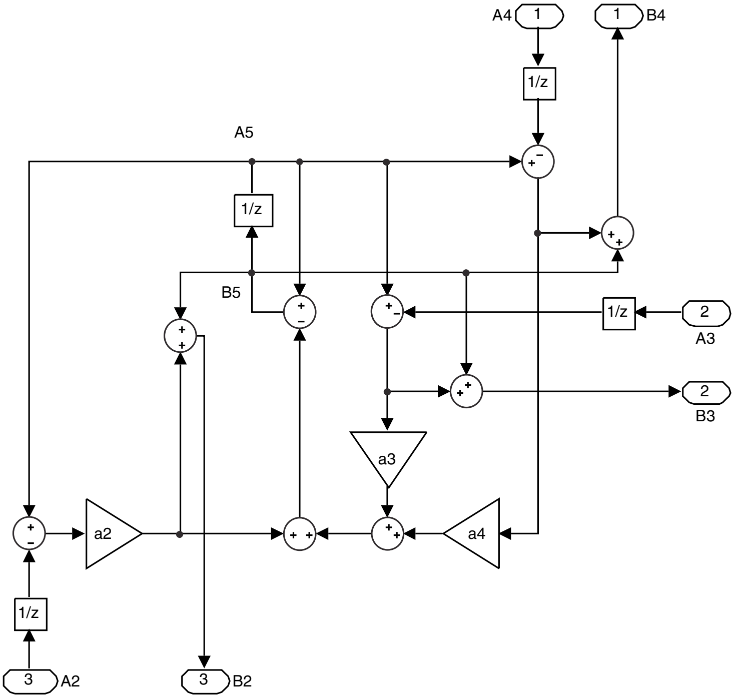

Fig. 1g – *Three-port asymmetric WDEs: Left-Center (WDE-LC).*

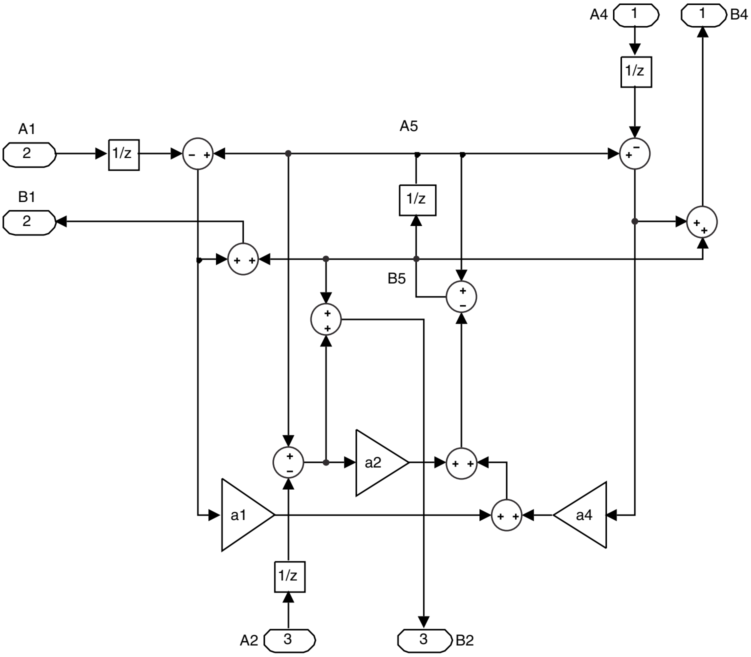

Fig. 1h – *Three-port asymmetric WDEs: Rigth-Center (WDE-RC).*

3 Wave-based Models of Different Branch-line Couplers

A wave-based modelling technique is applied on four-port devices such as single-band and dual-band branch-line couplers with two and three parallel lines. These examples show how to implement the basic WDEs in order to generate the WD models of different multi-port devices. A comparison, on the basis of efficiency and accuracy, with the ADS (Advanced Design System) software results is made. A brief overview as well as discussion of the different coupler structures and their performance is made in the next subsections.

3.1 A WD model of single-band directional coupler – Case 1 [8]

A branch-line directional coupler, also known as a quadrature $(+90^{O})$ hybrid coupler, is one of the most fundamental components used in microwave circuits. Conventionally, it is a four-port network consisting of four quarter-wavelength lines, which form a loop with appropriate impedances determined by the power division. Its schematic diagram is shown in Fig. 2. It can be used as a single antenna Transmitter/Receiver system or an I/Q signal splitter/ combiner.

A symmetric single-band coupler, shown in Fig. 2 with characteristic impedance of the main line $Z_1 = Z_0/2$, and for the branch line $Z_2 = Z_0$, for

$50\,\Omega$ port impedance (Z_0) is observed. The electrical lengths of the transmission line segments are $\theta = \pi / 2$. The design of this coupler is the simplest one. The MATLAB code behind WD model depicted in Fig. 3 uses these line parameters to calculate the parameter values of used two-port adaptors and three-port WDEs.

With a small modification of the MATLAB code, the same WD model from Fig. 3 can also be used for the modeling purpose of the coupler with variable load impedances.

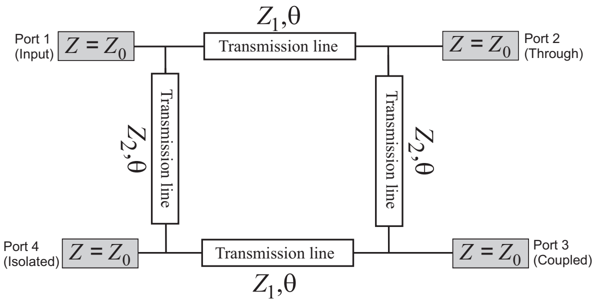

Fig. 2 – *Case 1: Circuit diagram of a single-band directional coupler.*

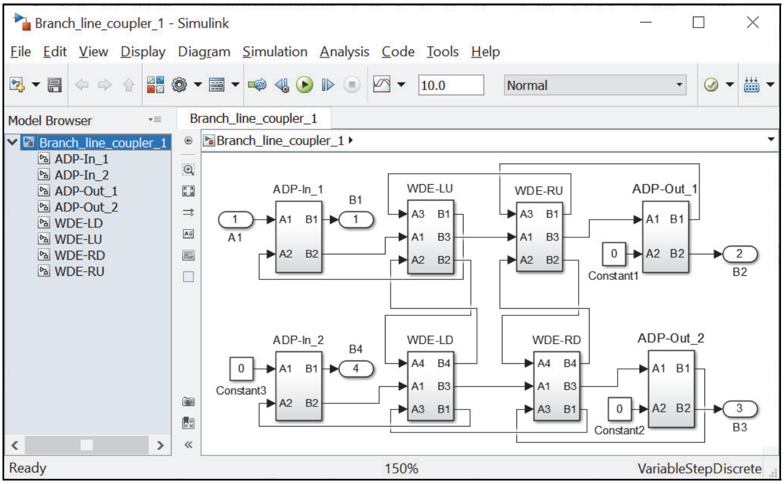

Fig. 3 – *Case 1: WD model of directional coupler with two parallel lines.*

Validation of the proposed wave-based modeling technique is demonstrated and discussed in this section through: simulations of different models (ADS models, developed wave digital model), extraction of model S-parameters (magnitudes and phases), and graph plotting and comparing parameters.

The equivalent network of the coupler [5] depicted in Fig. 4 is simulated in ADS, and the results of magnitudes and phases of S-parameters are shown in Fig. 5.

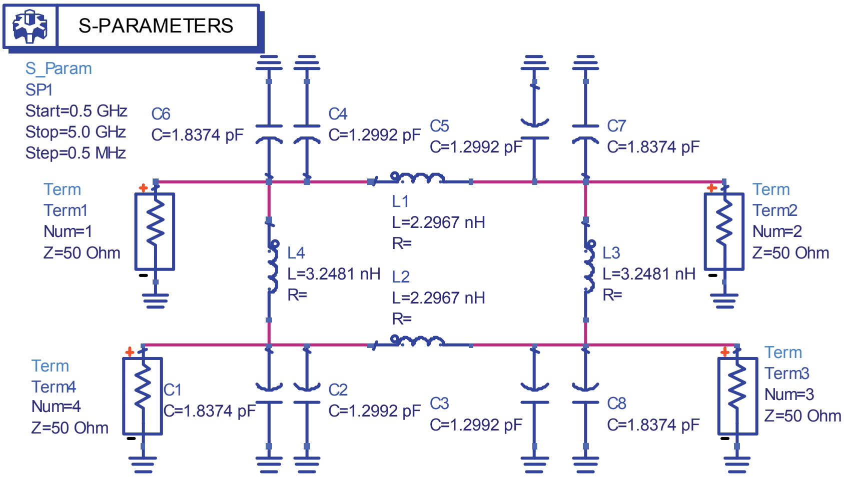

Fig. 4 – *Case 1: Equivalent network of 3-dB coupler in ADS.*

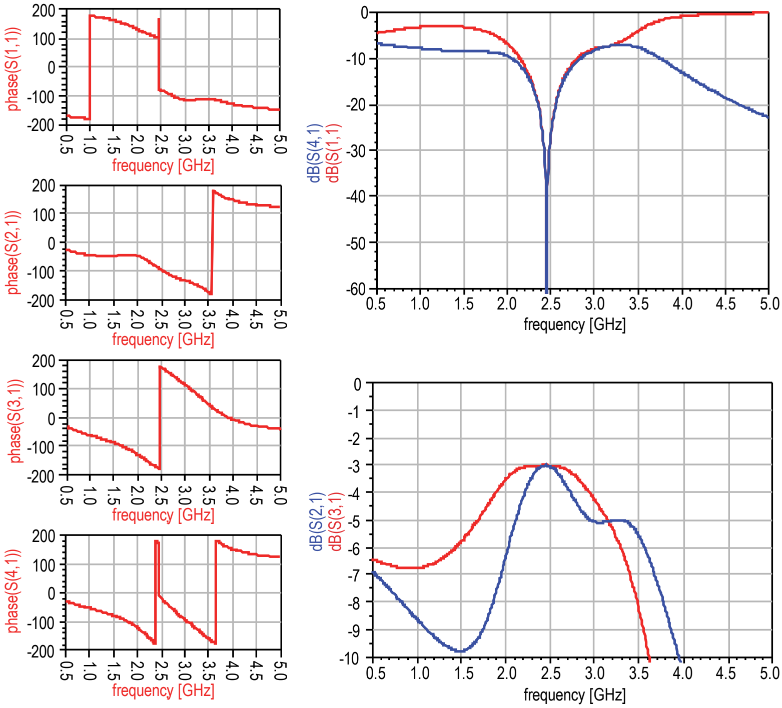

Fig. 5 – *Case 1: Simulation results of the equivalent circuit model.*

Also, different models of a single-band branch-line directional coupler based on ideal transmission lines, as well as on microstrip technology are built-in and simulated in ADS. Different models are shown in Figs. 6, 8 and 9. Simulation results of ADS model based on ideal transmission lines are presented in Fig. 7.

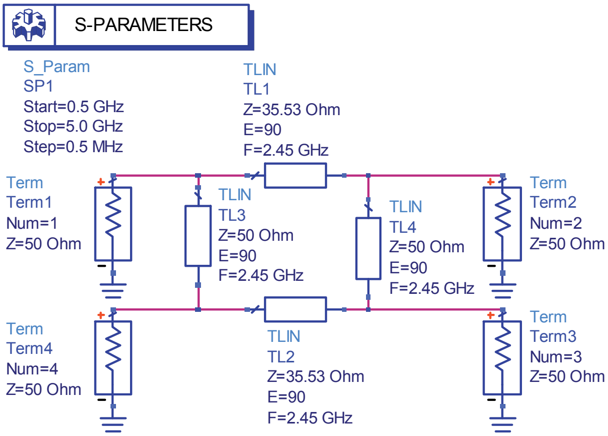

Fig. 6 – *Case 1: Coupler model in ADS – Ideal transmission lines.*

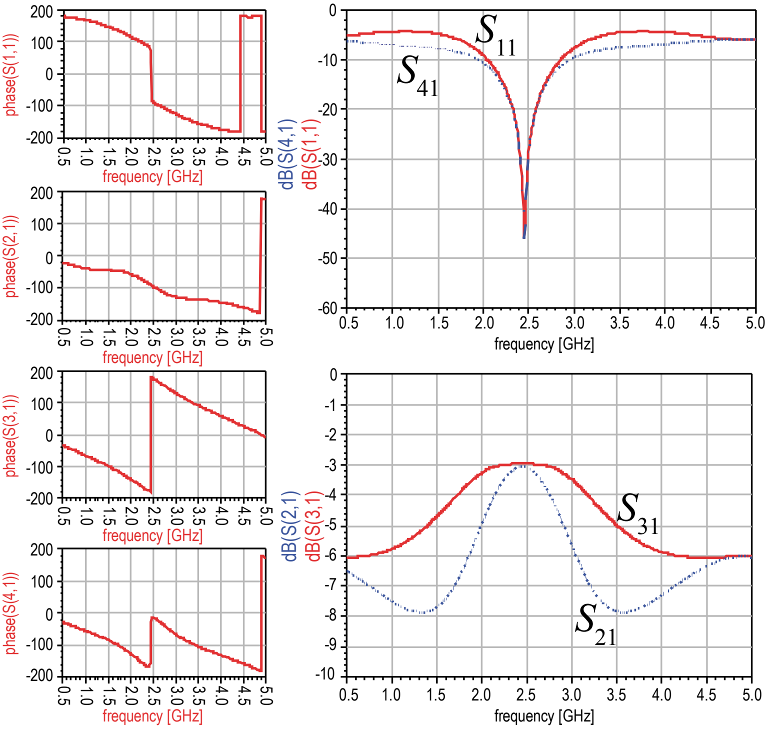

Fig. 7 – *Case 1: Simulation results of ADS model from Fig. 6.*

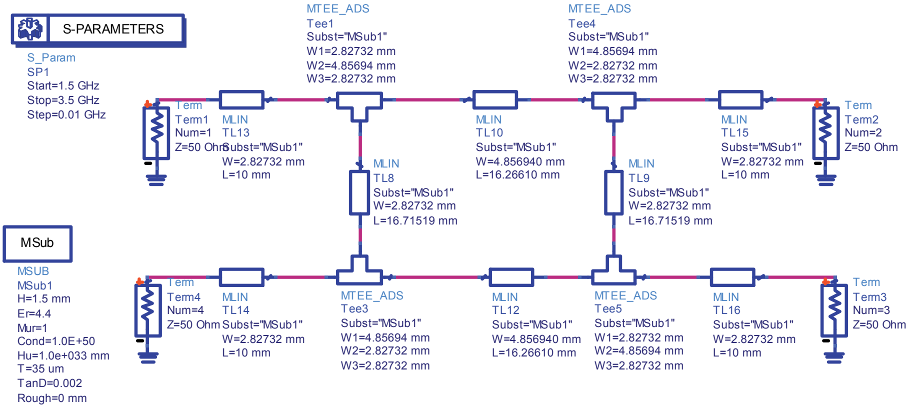

Fig. 8 – *Case 1: Coupler model in ADS – Microstrip lines.*

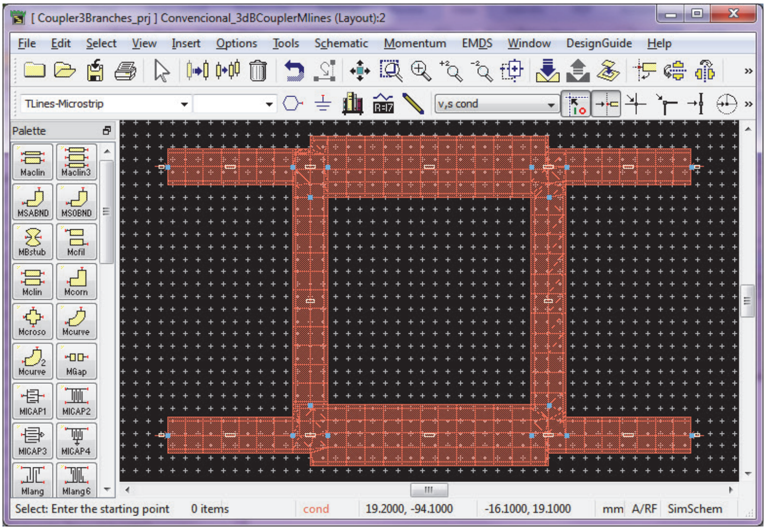

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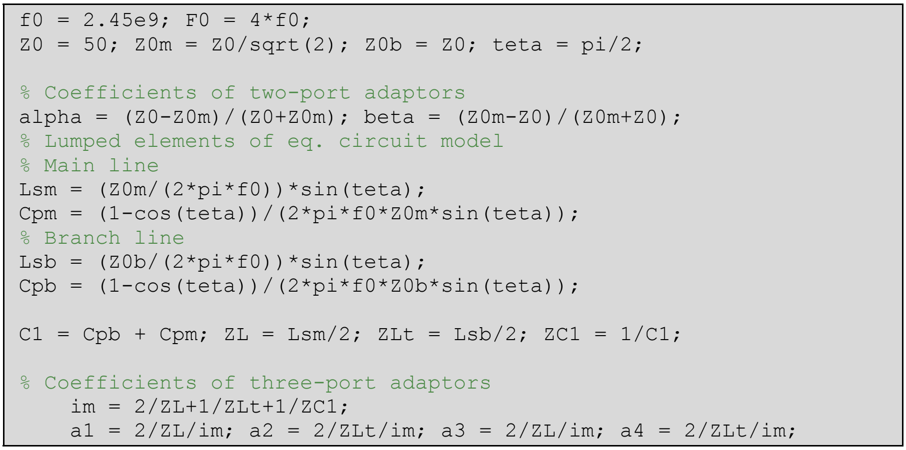
Fig. 9 – *Case 1: Coupler layout in ADS.*

As stated before, for WD-based modelling and simulation purposes, MATLAB code and WD model in Simulink (Fig. 3) are generated. A part of the MATLAB code used to calculate necessary parameters for the WD model is illustrated in Fig. 10. It shows how to model coupler using the characteristic impedance of the main line to implement the lumped elements of an equivalent circuit, which are used further to calculate the coefficient values of three-port adaptors.

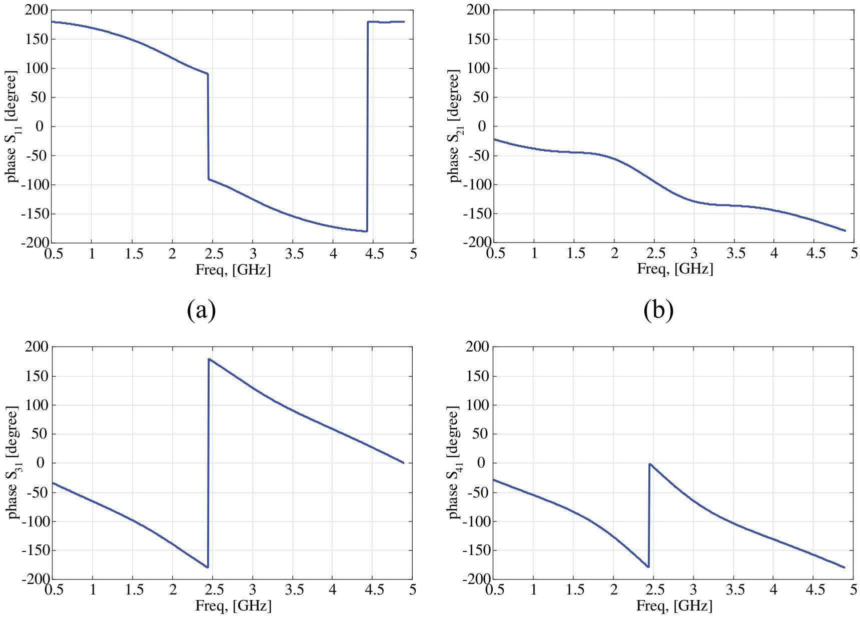
The WD model is then simulated, and plots of S-parameters versus frequency are generated. Figs. 11 and 12 present the phases and magnitudes of the generated S-parameters.



```
f0 = 2.45e9; F0 = 4*f0;
Z0 = 50; Z0m = Z0/sqrt(2); Z0b = Z0; teta = pi/2;

% Coefficients of two-port adaptors
alpha = (Z0-Z0m)/(Z0+Z0m); beta = (Z0m-Z0)/(Z0m+Z0);
% Lumped elements of eq. circuit model
% Main line
Lsm = (Z0m/(2*pi*f0))*sin(teta);
Cpm = (1-cos(teta))/(2*pi*f0*Z0m*sin(teta));
% Branch line
Lsb = (Z0b/(2*pi*f0))*sin(teta);
Cpb = (1-cos(teta))/(2*pi*f0*Z0b*sin(teta));

C1 = Cpb + Cpm; ZL = Lsm/2; ZLt = Lsb/2; ZC1 = 1/C1;

% Coefficients of three-port adaptors
    im = 2/ZL+1/ZLt+1/ZC1;
    a1 = 2/ZL/im; a2 = 2/ZLt/im; a3 = 2/ZL/im; a4 = 2/ZLt/im;
```

Fig. 10 – *One segment of MATLAB code.*

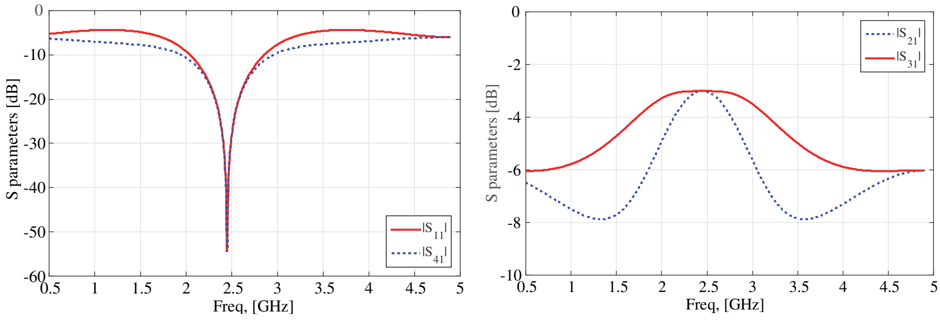
The simulation results obtained from both wave digital (Figs. 11 and 12) and circuit models with ideal transmission lines (Fig. 7) are in good agreement with one another. This is expected because the parameters in WD model are calculated at the coupler center frequency. From these models, it can be observed also how the characteristic impedances of lines of the circuit geometry contribute to the respective resonant frequency.



(a) (b)

(c) (d)

Fig. 11 – Case *1: Phase responses calculated from WD model.*



(a) (b)

Fig. 12 – *Case 1: Magnitudes of S-parameters calculated from WD model.*

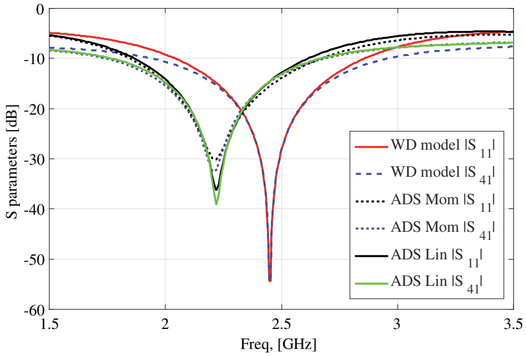


Fig. 13 – Case *1: Comparison of magnitudes of* S_{11} *and* S_{41} *parameters.*

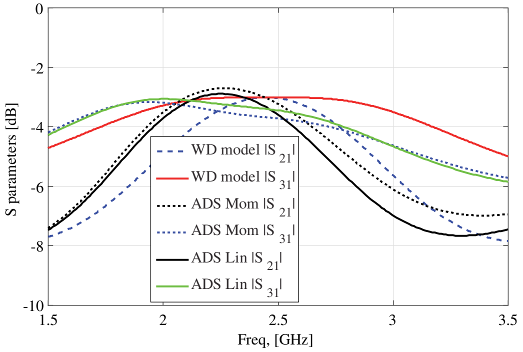


Fig. 14 – Case *1: Comparison of magnitudes of* S_{21} *and* S_{31} *parameters.*

In order to evaluate the results, a comparison is carried out and the magnitude responses in dBs are shown in Figs. 13 and 14.

It is clear from these figures that there is frequency shift between the results of linear and momentum simulation of ADS model based on microstrip technology and the simulation results obtained from generated WD model.

The difference is due to dimensions of microstrip lines and included tee-junction elements in the models. One can design coupler in microstrip technology on the exact frequency by reducing length of microstrip lines in order to shift the center frequency of coupler.

Fig. 15 shows an important parameter which has to be consider for the coupler hybrid, i.e. the phase difference between the outputs of the through and coupled ports. At the center frequency of 2.5 GHz, the phase difference between ports 2 (through port) and 3 (coupled port), calculated from the WD model, is obtained to be $+90^{\mathrm{O}}$ as expected.

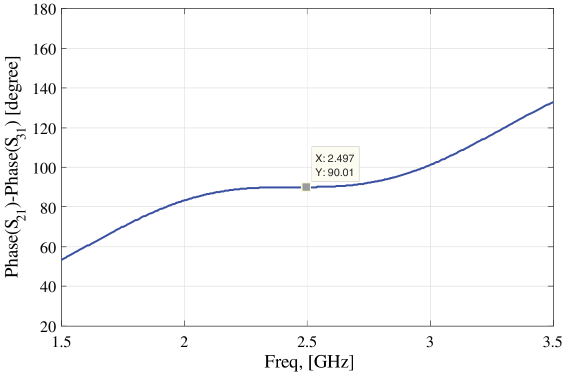


Fig. 15 – *Case 1: Phase difference.*

3.2 A wave digital model of coupler – Case 2

This example shows how a WD model of more complex device can be built from the generated and saved basic WDEs. Modelling technique is applied here on a single-band coupler which circuit diagram is shown in Fig. 16 [9]. The electrical lengths of the transmission line segments are $\theta = \pi / 2$.

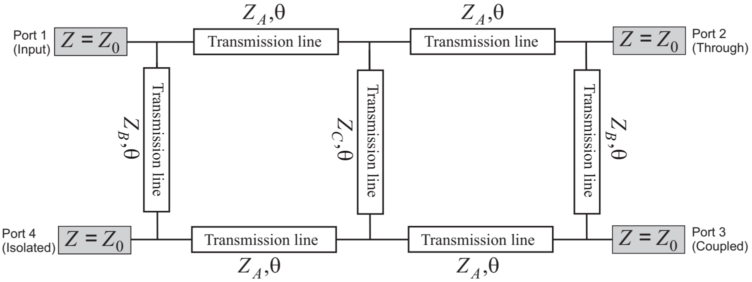


Fig. 16 – *Circuit diagram of a directional coupler.*

The coupler is modeled as a device whose equivalent circuit parameters depend on the characteristic impedance of main branch Z_A. In the model, the characteristic impedances of the other coupler lines are calculated according to value of Z_A like $Z_B = Z_A/0.41$ and $Z_C = 1.41Z_A$.

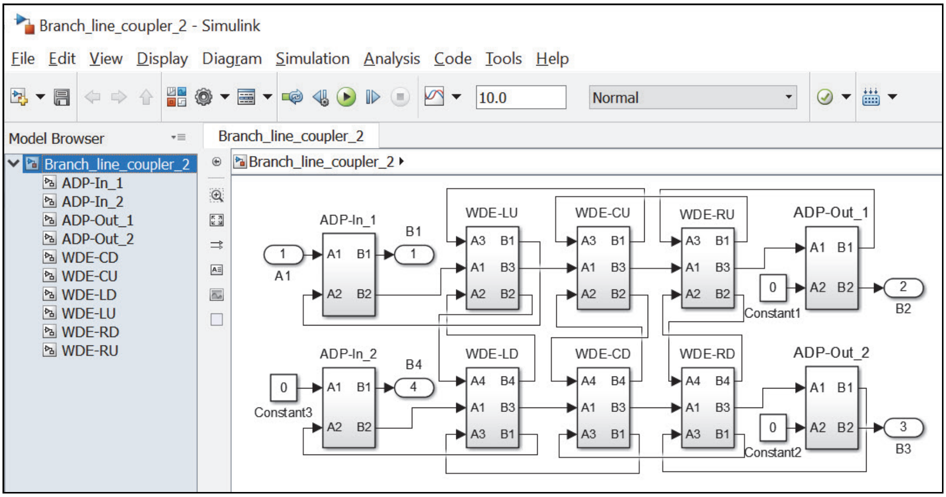


Fig. 17 – *Case 2: WD model of directional coupler with three parallel lines.*

The simulation results using MATLAB based on the proposed wave digital model from Fig. 17 are presented in Figs. 18 and 19, for the impedances beening calculated for $Z_A = 50\ \Omega$ devoted for applications in Ku-band. For K-band applications, circuit parameters are calculated for the impedance $Z_A = 70.50\ \Omega$. The value of chosen characteristic impedance Z_A is allowed to vary, without changing the generated WD model of this coupler.

By observing Figs. 18 and 19, it can be found that all *S*-parameters obtained from the WD model agree very well with the relevant *S*-parameters from ADS simulation, not just at the center frequency but also in wide frequency range. As a result of comparison, the WD results are in great agreement with the ideal simulated results: the return loss $|S_{11}|$ and the isolation loss $|S_{41}|$ remain below -10 dB over the entire frequency band and less that -50 dB at the center frequency of 15 GHz; the insertion losses $|S_{21}|$ and $|S_{31}|$ are about -3 dB at the center frequency, more specifically -2.985 dB and -3.036 dB.

The phase responses are given in Fig. 20, and the phase difference calculated from the WD model is depicted in Fig. 21.

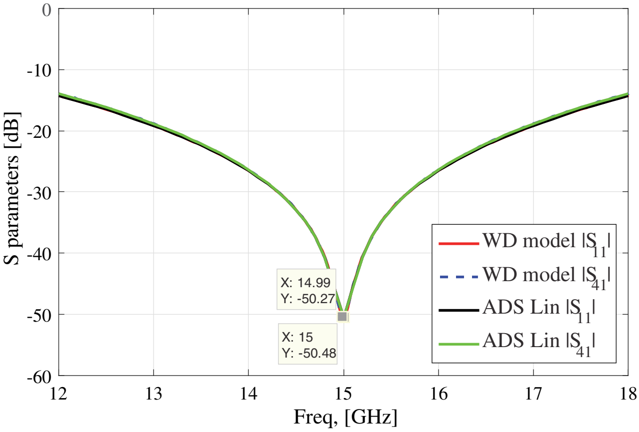


Fig. 18 – *Case 2: Comparison of magnitudes of* S_{11} *and* S_{41} *parameters.*

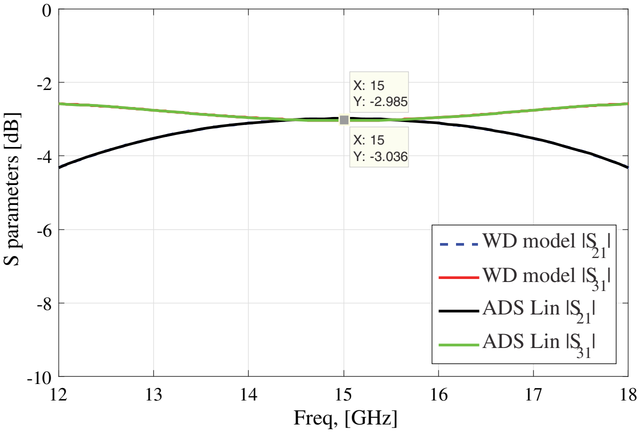


Fig. 19 – *Case 2: Comparison of magnitudes of* S_{21} *and* S_{31} *parameters.*

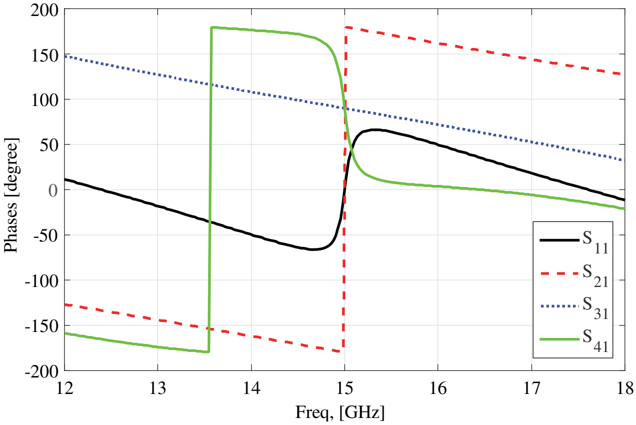


Fig. 20 – *Case 2: S-parameter phases.*

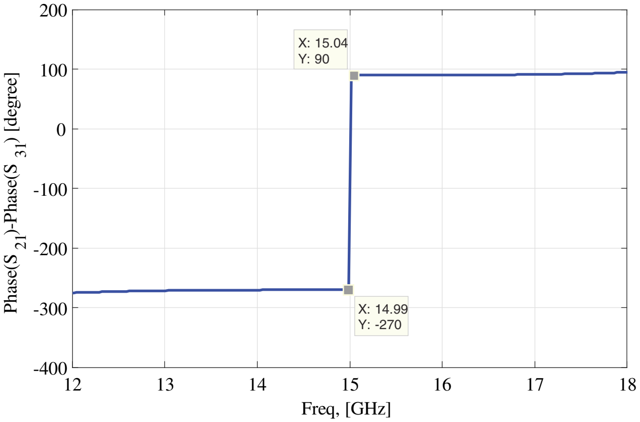


Fig. 21 – *Case 2: Phase difference.*

3.3 A wave digital model of coupler – Case 3

The third benchmark problem in this paper is a dual-band branch-line coupler structure with three parallel lines which schematic diagram can be found in Fig. 22 [10]. It is design to operate at 1.0/2.5 GHz. Compared with the conventional branch-line coupler, an additional parallel line (Z_3) connects the centers of two branch lines (Z_2).

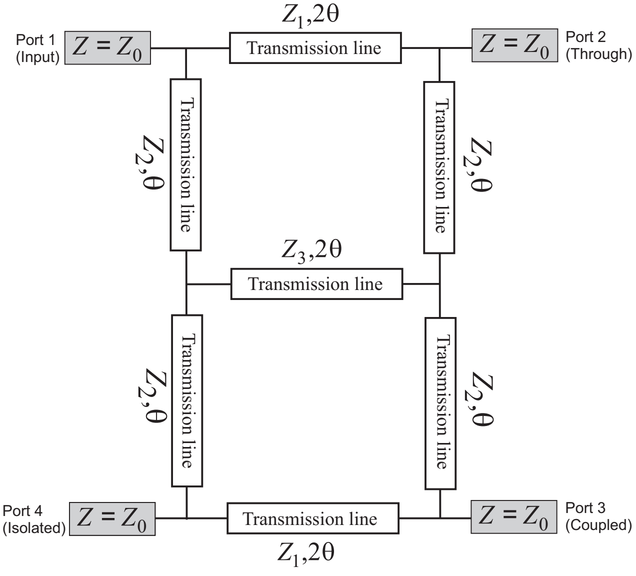


Fig. 22 – *Circuit diagram of a directional coupler.*

For $50\ \Omega$ characteristic impedance Z_0, the calculated line impedances are $Z_1 = 49.48\ \Omega$, $Z_2 = 79.21\ \Omega$, and $Z_3 = 132.66\ \Omega$ having the power ratio of $3\,\mathrm{dB}$. This example is specific, because of different electrical lengths of transmission lines, $\theta = 51.43^\circ$ and 2θ. This leads to an increase in the number of unit wave digital elements [4] for representation of the transmission lines of 2θ lengths.

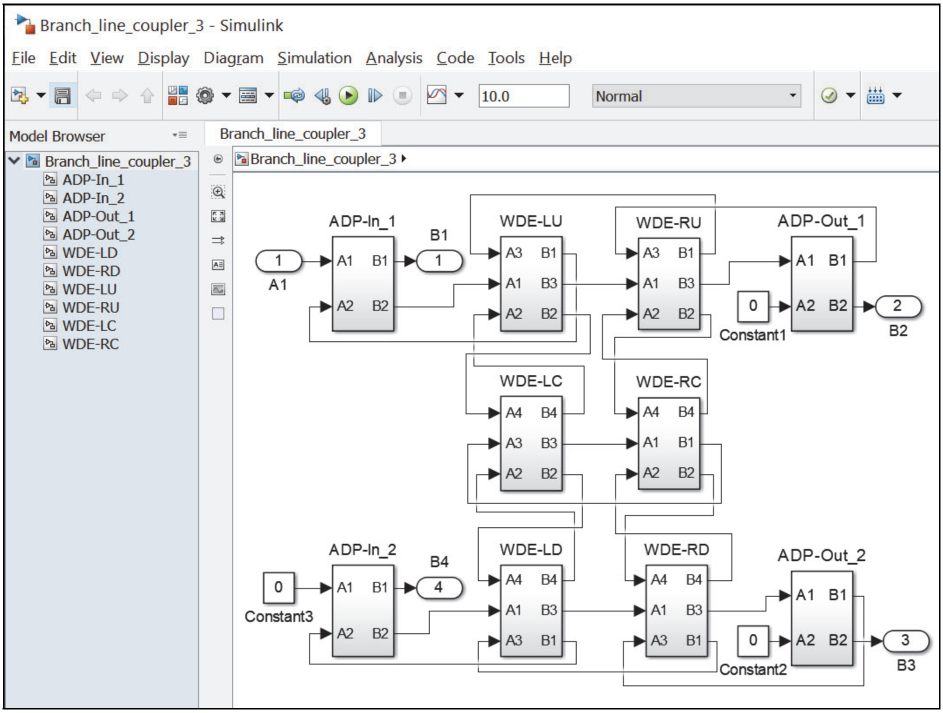


Fig. 23 – *WD model of directional coupler with three parallel lines.*

The simulation results obtained in MATLAB based on the proposed wave digital model given in Fig. 23 are presented in Figs. 24 and 25. **Table 1** compares the simulated (ADS) and modeled (WD model) results for center frequencies of both operating bands. Return loss and isolation loss better than 18 dB are found.

The phase differences at center frequencies of both operating bands, calculated from WD model, are quoted in Fig. 26.

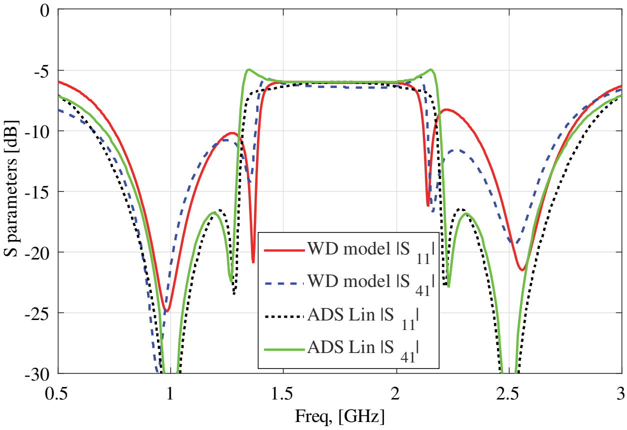


Fig. 24 – *Case 3: Comparison of magnitudes of* S_{11} *and* S_{41} *parameters.*

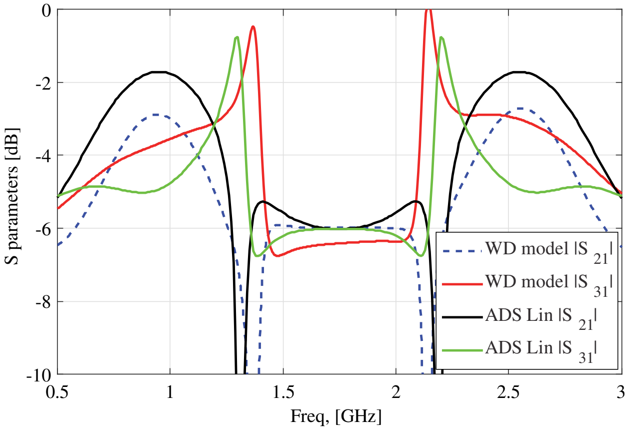


Fig. 25 – *Case 3: Comparison of magnitudes of* S_{21} *and* S_{31} *parameters.*

Table 1
Comparative study.

Result type	ADS simulated	WD modeled
Return loss S11 [dB]	< -30 @ both bands	-24.13 @ 1 GHz -18.28 @ 2.5 GHz
Isolation loss S41 [dB]	< -30 @ both bands	-21.21 @ 1 GHz -19.04 @ 2.5 GHz
Insertion loss S21 [dB]	-1.76 @ both bands	-2.99 @ 1 GHz -2.93 @ 2.5 GHz
Insertion loss S31 [dB]	-4.76 @ both bands	-3.55 @ 1 GHz -2.93 @ 2.5 GHz

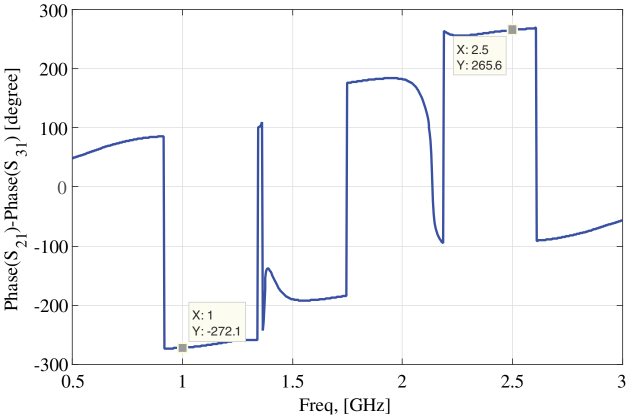


Fig. 26 – *Case 3: Phase difference.*

3.4 WD models: Derived conclusions

The generated WD models allow engineers to quickly gain experience and to test the changes in the structure performance when varying its characteristics.

The presented coupler structures confirm possibility to utilize a wave digital approach for the modeling and simulation purposes of the branch-line couplers. The theory about development of WDEs and a WD approach itself is taken from the previous author’s publications [6 − 8, 11]. The results of ADS simulations and WD models are compared with very good correspondence.

The next step in research could be replacement of ideal element values with values obtained from some EM simulator in order to observe the response shifts in the scattering parameters of interest.

4 Conclusion

In consideration of the structure performance, one should performed, whenever possible, a set of analyzes involving theoretical background, simulations, practical experiments and measurements. Sometimes it can be complicated, as well as time-consuming to provide an evaluation that includes all the above-mentioned methods. Developed WD models enable researchers to test structures with different parameters very quickly and without an additional expense. The development of the model would aid in reducing the time taken to design microstrip circuit in an electromagnetic simulator, due to the fact that the developed WD elements can be built-in MATLAB as additional library elements. Synthesis of wave digital model of structure with more or less elements is easy task with simple addition or subtraction of existing blocks, i.e. library elements.

Through comparative study, the proposed wave-based models of different coupler structures and the results of their analyses can be utilized for many other applications.

In addition, as it deals with signal flow graphs, it is much closer to the software implementation of the considered EM problem model compared to some classical full-wave EM modelling techniques.

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