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Low-Frequency Plane Wave Diffraction On a Two-Layer Grating

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Abstract: In this paper the problem of TE and TM plane wave diffraction from a two-layer grating in low-frequency case is presented. The problem is solved using the network model. Numerical results for the reflection coefficient of the lowest (n=0) mode versus frequency are given and compared to available results from literature.

Key words: Diffraction, Two-layer grating, Network model.

1 Introduction

Diffraction of a plane wave on single-layer and multi-layer gratings has been attracting scientists for a long time and it is still actually in present days. One of the most powerful tools for analysis of this problem is the Riemann-Hilbert method, which is presented in detail in monograph [1]. In the paper [2], the so-called network model for diffraction of a plane wave on a single-layer grating is presented, and in paper [3] simple analytical expressions for elements of that model are found. In this paper the network model is applied to the analysis of the diffraction of a plane wave on a two-layer grating in the low frequency case (grating period $\ll \lambda$). In that case network model is very simplified, because it does not take into account higher harmonics.

2 Network Model of the Problem

The geometry of the problem is shown on Fig. 1.

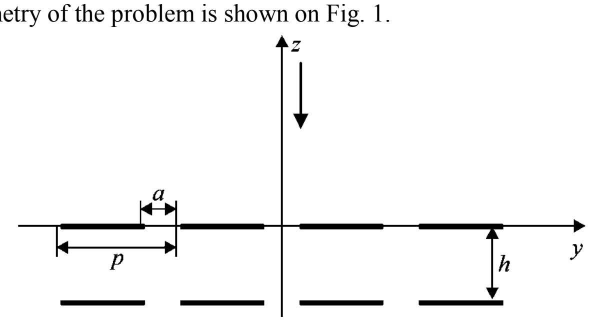

Fig. 1 - *Geometry of the problem.*

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Gratings are identical, and their position is such that they would overlap when translated along the *z*-axis. Grating period is *p*, the distance between them is –*h*, and the distance between the strips is *a*. The medium is vacuum. Plane TM (or TE) polarized wave is incident perpendicularly on the gratings. The network model [2], which takes only the lowest ($n = 0$) mode into account, is the same in both cases and it is shown on Fig. 2.

The quantities on this figure are [3]:

$$Y_{00} = j \frac{Y_0}{\frac{p}{\lambda} \ln \cos \frac{\pi a}{2p}} \tag{1}$$

for TE case and

$$Y_{00} = -j4 \frac{p}{\lambda} Y_0 \ln \sin \frac{\pi a}{2p} \tag{2}$$

for TM case, while $Y_0 = 1/120\pi$ is the wave admittance for vacuum.

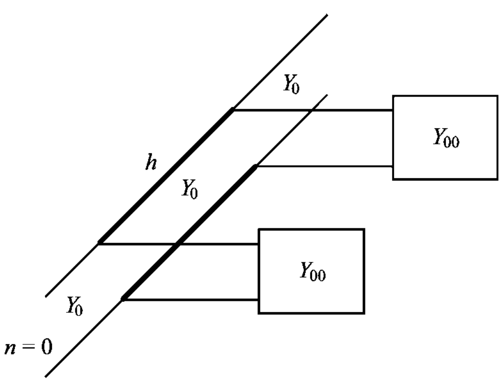

Fig. 2 - *Equivalent network model.*

Reflection coefficient (for the lowest mode) can be obtained from Fig. 2,

$$S_{11} = \frac{-\hat{Y}_{00}\left(2 + j\hat{Y}_{00}\ \mathrm{tg}\,\beta h\right)}{2 + 2j\ \mathrm{tg}\,\beta h + 2\hat{Y}_{00} + 2j\hat{Y}_{00}\ \mathrm{tg}\,\beta h + j\hat{Y}_{00}^{2}\ \mathrm{tg}\,\beta h}, \tag{3}$$

where

$$\hat{Y}_{00} = \frac{Y_{00}}{Y_0} = 120\pi \cdot Y_{00} \tag{4}$$

and

$$\beta = \frac{2\pi}{\lambda}. \tag{5}$$

3 Numerical Results

Diagrams of the reflection coefficient magnitude for the zero-mode in the TM case, for $h/\lambda = 0.5$ and for different values of a/p are shown on Figures 3-5 by solid lines. Calculations are made by using eqns. (2), (3), (4) and (5).

Dashed lines on the same figures show reflection coefficient magnitude calculated by the approximate low-frequency formulas for the transmission coefficient [4]

$$S_{21} = \frac{e^{j\beta h}}{1 + 4j\frac{p}{\lambda}\ln\sin\frac{\pi a}{2p} - \left(\frac{p}{\lambda}\right)^2 \left(1 - e^{j2\beta h}\right)\ln^2\sin\frac{\pi a}{2p}}. \tag{6}$$

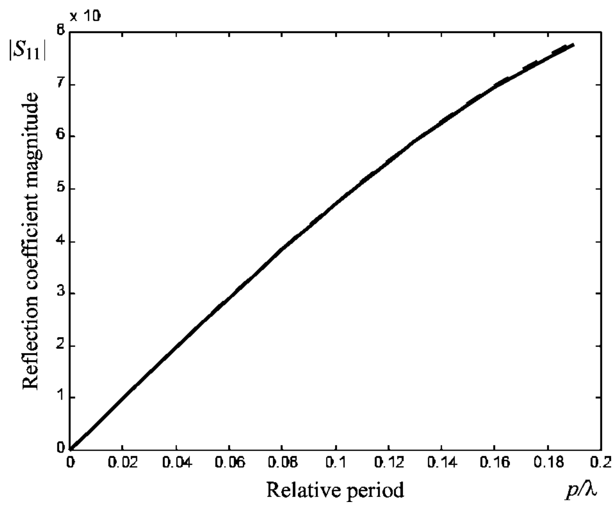

Fig. 3 - *Reflection coefficient magnitude* for zero-mode in *TM case,* $a/p = 0.5$.

Diagrams of the reflection coefficient magnitude for the zero-mode in the TE case, for $h/\lambda = 0.5$ and for different values of a/p are shown of Figures 6-8 by solid lines. Calculations are made by using eqns. (1), (3), (4) and (5).

Dashed lines on the same figures show reflection coefficient magnitude calculated by the approximate low-frequency formulas for the transmission coefficient [4]

$$S_{21} = \frac{4\left(\frac{p}{\lambda}\right)^2 \ln^2\cos\frac{\pi a}{2p}\, e^{j\beta h}}{4\left(\frac{p}{\lambda}\right)^2 \ln^2\cos\frac{\pi a}{2p} + e^{2j\beta h}}. \tag{7}$$

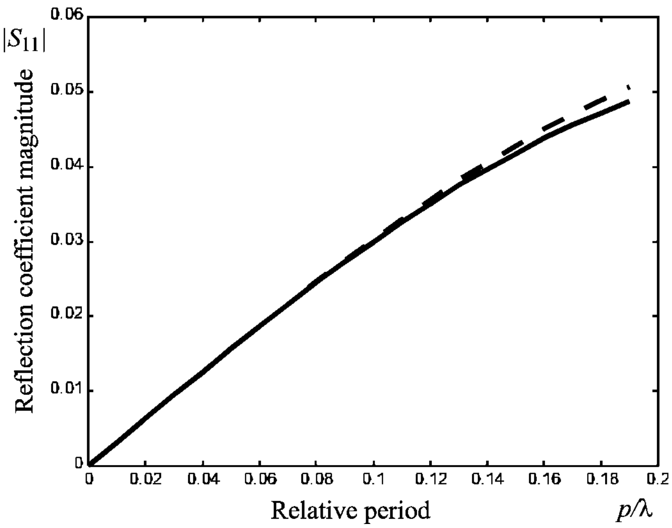

Fig. 4 - *Reflection coefficient magnitude for zero-mode in TM case,* $a/p = 0.75$.

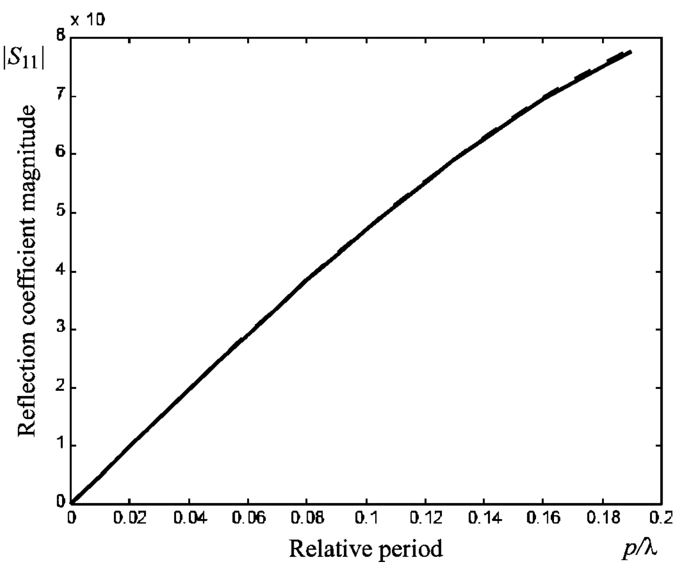

Fig. 5 - *Reflection coefficient magnitude for zero-mode in TM case,* $a/p = 0.9$.

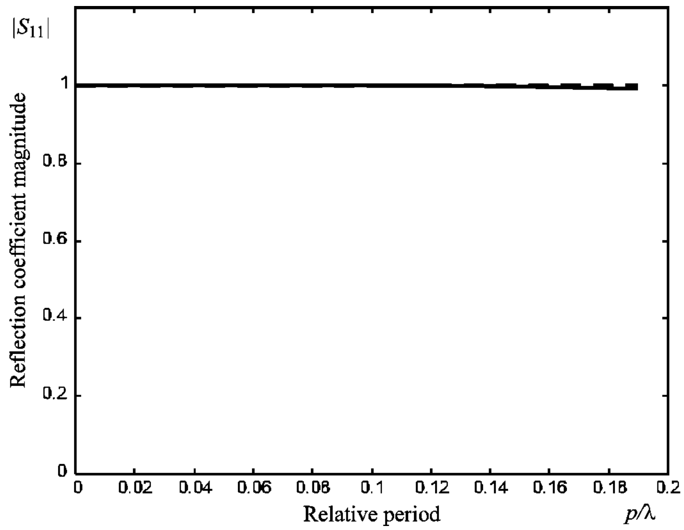

Fig. 6 - *Reflection coefficient magnitude for zero-mode in TE case,* $a/p = 0.25$.

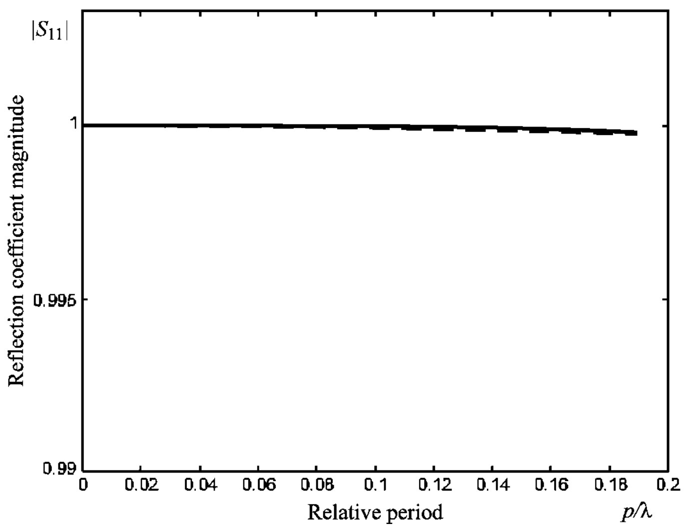

Fig. 7 - *Reflection coefficient magnitude for zero-mode in TE case,* $a/p = 0.5$.

As one can see from Figures 3-8, in the low-frequency case ($p/\lambda << 1$), when higher harmonics can be neglected, our results based on the network model are in agreement with available results from the literature.

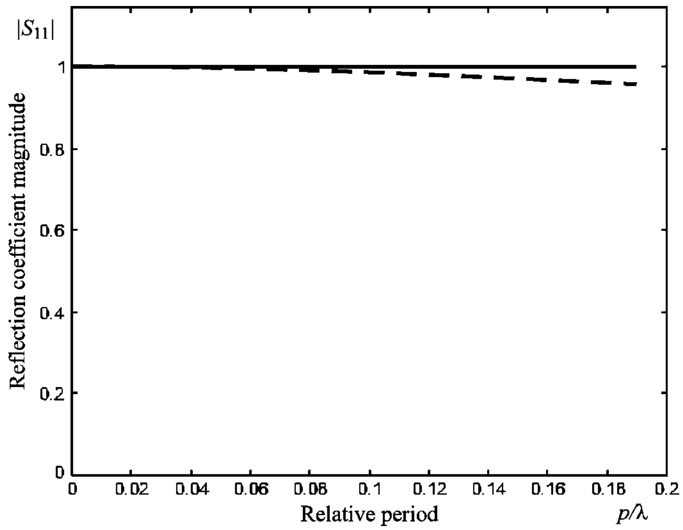

Fig. 8 - *Reflection coefficient magnitude for zero-mode in TE case,* $a/p = 0.75$.

4 Conclusion

In this paper diffraction problem of TE and TM polarized plane wave on a two-layer grating for perpendicular incidence and the low-frequency case ($p << \lambda$) is discussed. The problem is solved using network model, which does not take into account higher harmonics. Reflection coefficient magnitude is calculated and shown versus the relative period p/λ.

The obtained results are in good agreement with the available results from the literature.

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