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# Reducing of Noise Structure Influence on an Accuracy of a Desired Signal Extraction

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**Abstract:** In the paper, the issues regarding the analysis of the noise component structure are addressed and methods for reducing the error in estimating of the mathematical expectation of the noise component are proposed. The use of the proposed method of "noise purification" makes possibility to reduce the error introduced by the noise structure when estimating the mathematical expectation and dispersion of the noise component during research. The main scientific contribution in this paper in accuracy increasing of random processes parameters estimation. These theoretical results can be applied in different spheres of data analyzing and signal processing when random processes have some structure.

**Keywords:** Noise, Multiplication of estimates, Signal extraction, Mathematical expectation estimation, Signal smoothing.

## 1 Introduction

Currently, in the study of the efficiency of radio electronic systems, simulations are widely used. Using them, the accuracy of the desired signal extraction is estimated. To simulate the noise component, the built-in random number generator with uniform distributed law is used. With such generator, almost any law of distribution of random variables can be modeled on the basis of simple transformations [1]. However, the obtained set of random variables, for example for normal distribution, does not meet the requirements of stationarity and especially ergodicity. And the smaller the sample, the more pronounced this dependence is, which indicates the influence of the structure of the noise component on the basic of statistical characteristics of its model. The results of modeling [2] confirm the fact that the structure of the noise component influences the error in the extraction of the desired component. In fact, if we have a normalized dependence $\overline{m}_{RMS} = f(\sigma_{noise})$, i.e. divided by the value of the root mean square (RMS) deviation of the noise component, then the obtained dependence will have the form of a practically constant value of the additive noise component independent of the dispersion.

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## 2 Reducing of Noise Structure Influence Technique and Research

In this regard, let's consider in detail the structure of the noise component, which has always been given special attention in the solution of hydrolocational problems [1]. Since we are talking about the analysis of measurement results, which are represented by a finite number of samples (the volume of measurement results is limited), estimates of the mathematical expectation and variance essentially depend on the volume of the implementation being investigated.

To realize a random Gaussian process, the limit error in estimating the mathematical expectation can be determined from the expression [3 – 5]

$$\varepsilon = \frac{t_{\alpha,N-1} S}{\sqrt{N-1}}, \tag{1}$$

where $\alpha$ is the significance level, $t_{\alpha,N-1}$ - the Student's quantile distribution, $S$ - the RMS deviation estimate.

Thus, the confidence interval

$$\left( \bar{m} - \frac{t_{\alpha,N-1} S}{\sqrt{N-1}}; \quad \bar{m} + \frac{t_{\alpha,N-1} S}{\sqrt{N-1}} \right), \tag{2}$$

covers an unknown mathematical expectation with a given probability $\alpha$.

It is also possible to use more simple relations. For example, the relative inaccuracy of the estimation of the value $\sigma(\bar{m})$ is approximately $1/\sqrt{N-1}$, i.e. with a sampling length $N = 100$, the relative error is 10% [6].

Thus, the estimation of the mathematical expectation essentially depends on the volume of the sample being analyzed, with the growth of which the estimation error decreases. In order to reveal the reasons for this well-known fact, more than 1000 samples with a volume of 100 values of the model of the random Gaussian stationary noise component were analyzed, the results of which are given in **Table 1**.

**Table 1**

*The frequency of appearance of groups of different length in the sample N = 100.*

| M | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{m}_M$ | 5,934 | 2,927 | 1,468 | 0,732 | 0,384 | 0,21 | 0,122 | 0,059 | 0,027 | 0,012 | 0,005 | 0,002 | 0 |

In **Table 1**, by $M$ we mean the length of a group, i.e. the number of consecutive samples in a sample whose values are greater than or less than zero; by $\bar{m}_M$ we mean of the number of groups of duration $M$ in each sample of a volume $N$ averaged over 1000 implementations. The analysis of the results given in **Table 1** shows that in the sample volume $N = 100$ there are groups with a duration of 5 samples on average about 6 times; of 6 samples about 3 times, and so on. In particular, there are groups of up to 16 samples, although extremely rarely – 2 times per 1000 samples. Therefore, it can be assumed that the implementation of a random process of length $N$ contains a certain low-frequency function, which is determined by the random occurrence of groups of consecutive values greater than or less than zero. This explains the fact of the increase in the error in estimating the mathematical expectation for small samples and their difference in the analysis of different realizations, although they all belong to the same general population that satisfies the requirements of stationarity and ergodicity.

On the basis of the foregoing, it follows that the solution of the problem of minimizing the mathematical expectation estimation for small samples is a very urgent task, which will improve the accuracy of signal processing during simulation.

To solve the task an analysis of published sources was conducted in the paper, which showed that to isolate the noise structure it is necessary to use a method that allows processing the realization of a random process with a limited amount of a priori information about the statistical characteristics of the noise component. These conditions are fully satisfied by the method of multiplication of estimates (MoE), which is described in detail in [9 – 12]. In connection with this, during the research, the MoE-method was used with parameters that are recommended in [9]. The processing was performed on simulated samples of random Gaussian noise with $N = 100$ on 1000 implementations.

## 3 Results and Discussion

The obtained results of the research allow us to conclude that in the analysis of random stationary processes it is expedient to preliminarily use the MoE method in order to obtain more accurate values of statistical characteristics, especially for a limited volume of the results of measurements of the initial implementation. The use of the “cleaned” noise component in simulations, as can be seen from the results presented in Fig. 1, shows that the value decreases in 1.9 – 3.2 times depending on the dispersion of the additive noise component.

Thus, the influence of the structure of the noise component on the error in the separation of the desired signal is obvious. The MoE method partially “clears” the noise component from groups of 5 or more samples, but there remain groups of less than 5 counts, which cause an increase in the mean square error.

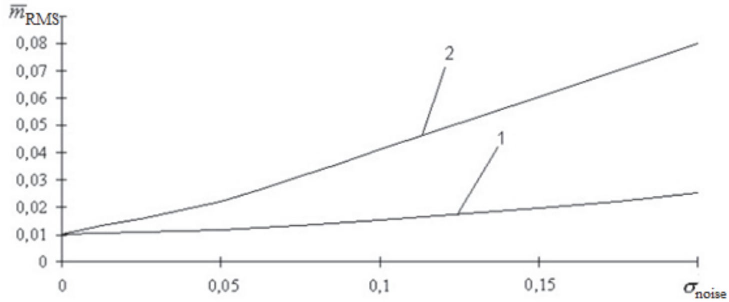


**Fig. 1** – *Dependence of* $\overline{m}_{RMS} = f(\sigma_ш)$ *on "cleaned" noise component (curve 1) and "not cleaned" (curve 2).*

It should also be noted the following regularity: if the value $\overline{m}_{RMS}$ (curve 1) is subtracted from the value $\overline{m}_{RMS}$ (curve 2), then the residual value of the error in estimating the mathematical expectation of the noise component can be written

$$\overline{m}_{RMS} \approx \overline{m}_{\sigma_{сиг}} + \overline{m}_{m_ш} , \qquad (3)$$

where $\overline{m}_{RMS}$ - the mean value of the RMS error of the desired signal allocation in the presence of an additive noise component, $\overline{m}_{\sigma_{sig}}$ - the mean value of the RMS error in the separation of the desired signal in the absence of an additive noise component, $\overline{m}_{m_{noise}}$ - the average value of the estimation of the mathematical expectation of the additive noise component.

Let us consider in more detail the physical processes that are taking place in the studies. Fig. 2 shows a segment of the realization of the simulated random process, an estimate of the mean, as a function of time, and a "cleaned" implementation.

As can be seen from the figure, the initial random process (solid line) after processing with the MoE, yielded the estimate $m(t)$ (dashed line), and finally the difference process, i.e. $\mu(t) - \overline{m(t)}$ where $\mu(t)$ is the initial model of the random process (dash-dotted line). An analysis of the above fragment shows that consecutive values greater than or less than zero are allocated and the resulting difference process has a better structure for a more accurate estimate of the mathematical expectation. When comparing the estimation of the mathematical expectation of the initial implementation and the processed one, the fragment of which is presented in Fig. 3, it follows that only in 2% of 1000 realizations the value of the ratio $\overline{m(t)} / \overline{m_1(t)}$ is less than one (straight line on Fig. 3), and in some cases reaches hundreds and thousands of times.

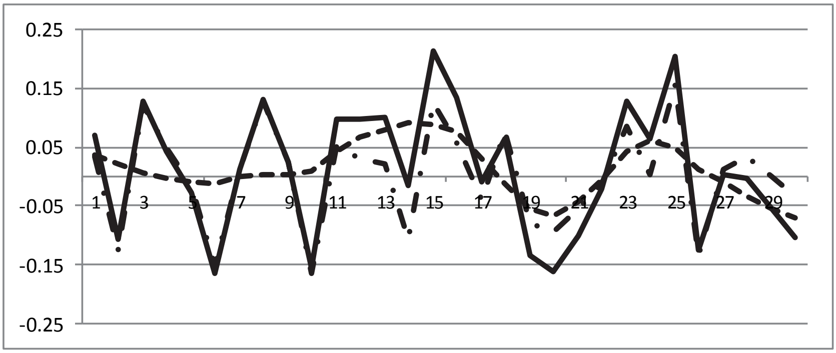


**Fig. 2** – *Fragment of a random set processing.*

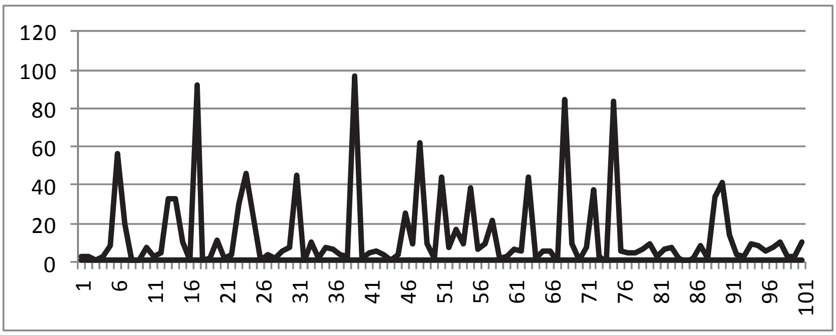


**Fig. 3** – *A fragment of comparison of the mathematical expectation estimation of the original implementation and the processed one.*

A comparative analysis of 1000 realizations of a length of 100 points shows that the average value of the ratio $\overline{m(t)}/\overline{m_1(t)}$ is more than 33. Thus, using filtering by the multiplication of estimates method, it is possible to substantially approximate the parameters of the realization of a random process with a small number of samples to the parameters of realization hundreds of times larger than the initial one.

Thus the scientific contribution of proposed method of estimates multiplication consist in increasing the accuracy of random processes parameters estimation in conditions of limited data set. These theoretical results can be applied in different spheres of data analyzing and signal processing when random processes have some structure. For instance for pseudo random processes.

The results of the conducted studies showed that when comparing the estimation of the mathematical expectation of the initial implementation and the processed value, only in 2% of the 1000 realizations have the value of the ratio

$\overline{m(t)} / \overline{m_1(t)}$ less than one, and usually the ratio reaches hundreds and thousands. The comparative analysis has shown that the average value of the ratio $\overline{m(t)} / \overline{m_1(t)}$ exceed 33.

## 4 Conclusion

In the paper we have shown the method for reducing the error in estimating of the mathematical expectation of the noise component. Proposed method of "noise purification" powerful reduces the error introduced by the noise structure when estimating the mathematical expectation and dispersion of the noise component during its research. The accuracy increasing of random processes parameters estimation was shown in this paper. Different spheres of data analyzing and signal processing can use proposed method in condition when random processes have some structure.

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