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Improvement of an Existing Method of Asynchronous Sampling for Determining RMS Value

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Abstract: Measuring a value by definition seems to be the best solution for getting the most precise results. In this paper, it is shown that by changing the general definition of root mean square (RMS) and average value of a periodic signal, their measurement can be improved. When measuring RMS with asynchronous sampling, it was observed that results were scattered around two values, and it was found that the main cause for this was initial sampling time. Redefining the RMS value has been proposed in order to increase the efficiency of the asynchronous method. After comparing the results based on general and proposed definition for RMS value, it was observed that there was significantly less scattering of results and higher accuracy. The simulation proved that the application of the proposed definition of RMS gives more accurate and precise results than the general definition for analog to digital (AD) converters with both lower and higher resolution.

Keywords: Root mean square, Average, Asynchronous sampling, Measurements, Metrology.

1 Introduction

When measuring electrical values using an analog to digital conversion, the samples of the measured value are processed according to a certain mathematical equation in order to determine properties of the measured signal, and most often equations for determining these properties according to a definition are applied [1 − 6]. Although at first sight it may seem that measuring by definition gives the most precise results, but it does not have to be that way. Many examples of measurement in practice show that precision in metrology is very much dependent on the definition which must be understandable, logical and at the same time unequivocal. Often, the question of definition adequacy in imprecise results is not called into question, and such is a mistake that occurs

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often. A faulty definition does not represent a problem if there is no need for high accuracy measurements, but it is not the best solution for high-precision measurements. Measurement accuracy depends on the definition of the value. The reverse is true, that the condition of definition rigor depends on measurement accuracy. Metrology as a science faces a common problem - if it is possible to measure a value according to a particular definition. The value definition is acceptable if it can be measured appropriately. Although it can lead to high accuracy measurement, a very strict definition is not useful if the environment is such that it cannot be applied to it. This paper presents an important observation in determining the RMS value of periodic signal using asynchronous sampling, where definition rigor is of key importance.

2 Problem Description

The equation for the general definition based on which the RMS value of a signal has been determined is:

$$U_{RMS} = \sqrt{\frac{1}{T} \int_{t=0}^{t=T} u^2(t)\,\mathrm{d}t} \ . \qquad (1)$$

For discretization by time, including values of integral limits of general definition of RMS value, it was common practise to measure RMS according to the following definition:

The RMS value is equal to the square root of the average of the squared samples taken in the interval which starts when the sample becomes larger (alternatively smaller) than the zero volts and ends when it becomes, after one or several periods, again larger (alternatively smaller) than the zero volts, omitting the last sample.

In order to check the possibility for determining the RMS value of alternating signals with asynchronous sampling according to the general definition, it has been observed that the calculated RMS values in a series of successive measurements are not grouped around one value, as expected, but around two values. This means that samples were taken from the one which would be the first to have a higher value (or lower) than zero volts and the one that after one or more periods once again has a higher value (or lower) than zero volts, not taking the last sample into account.

Another thing that is noticed is that the number of results grouped around one and around another value are not the same. Due to the suspicion that such results are largely affected by the resolution of analog to digital converter and sampling rate, the simulation method was used, where the impact of discretization by value was ignored. Although the impact of discretization by value is now discarded from the potential causes of dissipation, scattering of results is still present. Since it was found that the resolution of the AD converter is not the main cause, and since asynchronous sampling cannot affect the initial sampling time compared to the sampling threshold, an increase in the sampling rate has been performed, and this is one of the steps that led to the conclusion. In this experiment, when changing the sampling rate, the results and their scattering were monitored, and it can be observed that:

1. For the defined frequency of periodic voltage signal f and sampling frequency f_s, in a series of successive measurements two sets of approximate values are obtained;
2. If the f_s/f ratio is closer to an even number, higher values are more frequently obtained, and vice versa, if the ratio f_s/f is closer to an odd number, lower values are more frequently obtained. When the ratio f_s/f is close to the arithmetic mean of the first adjacent even and odd number, the number of values that have a lower value and the number of values that have a higher value are approximately the same;
3. At first there was no clear reason for the observed phenomena. When the number of samples was drawn based on which the root mean square was calculated, a large correlation between the ratio f_s/f and the number of samples was observed:
   – When f_s/f is closer to an even number, more frequent results (larger results) correspond to a smaller number of samples based on which the root mean square is determined;
   – Smaller results of the RMS value correspond to situations where the number of samples was larger by one;
   – When f_s/f is closer to an odd number, more frequent results (smaller results) correspond to a greater number of samples based on which the RMS value is determined;
   – Larger results of the root mean square correspond to situations where the number of samples was smaller by one.

The following figures show the dependence of the calculated RMS value and the number of samples from f_s/f. Asynchronous sampling of pure sine signal without the influence of the AD converter resolution was simulated.

In a defined relationship f_s/f, no matter if it is closer to an even or an odd number, two groups of results are obtained, and lower values are always obtained in situations where the root mean square is determined on a larger set of samples.

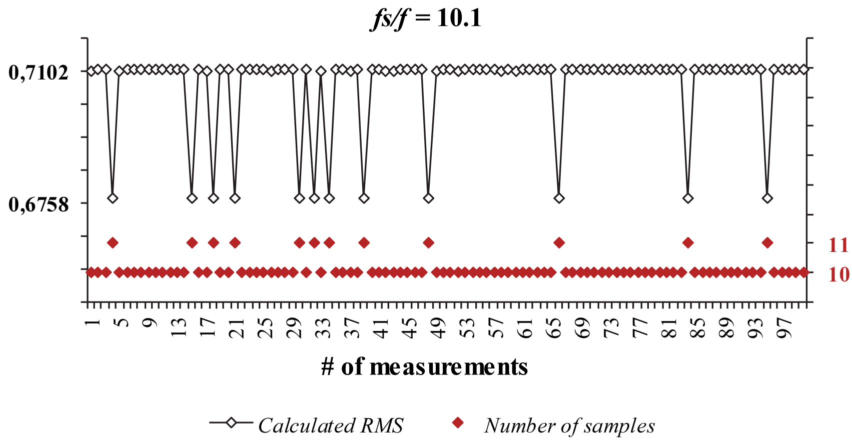

Fig. 1 – *Dependence of the calculated RMS and the number of samples when f_s/f is close to an even number.*

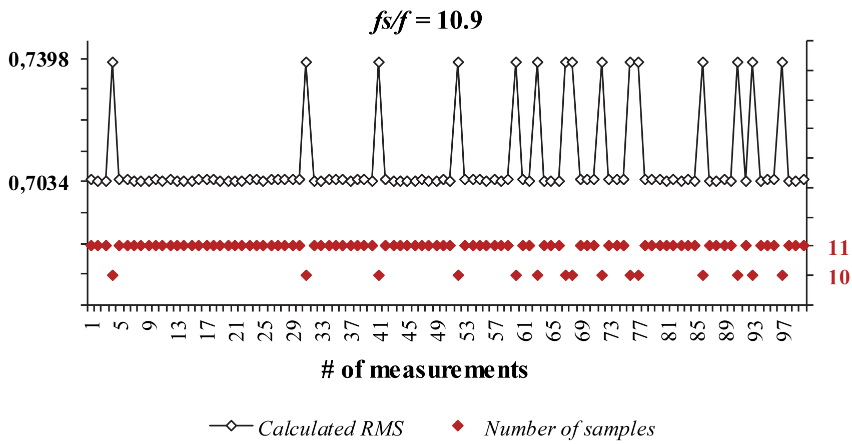

Fig. 2 – *Dependence of the calculated RMS and the number of samples when f_s/f is close to an odd number.*

The next logical step is to examine events at higher values of sampling rate. The following graph shows the relation between scattering and sampling rate. Simulations clearly show that when increasing the sampling rate, there are still two sets of results for the RMS value, but also that the limits in which these values appear have a decreasing tendency. A logical conclusion can be drawn from this about the problem of the error that can be reduced by increasing the sampling rate. Fig. 3 shows the effect of the sampling frequencies on the calculated RMS.

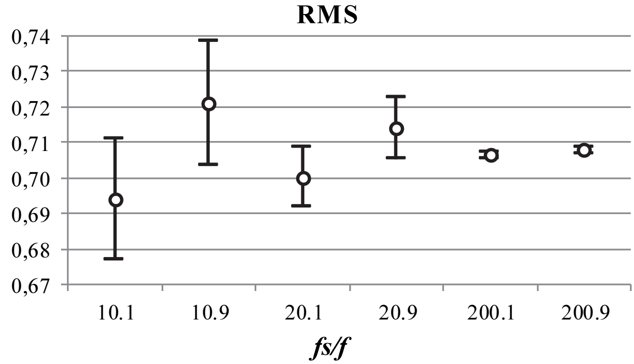

Fig. 3 – *Dependence of the calculated RMS from sampling frequency.*

Although the increase of f_s leads to a reduction in the boundaries of the calculated RMS, the question is raised as to whether, in addition to increasing the sampling rate, it can somehow affect the measurement result. The only thing that remains is the analysis of the starting moment in which the sampling begins. Since this moment in practice during asynchronous sampling is unknown, its impact on the result of the measurement is checked with the use of a simulation, that is, the influence of the time when the signal passes through zero to generating the next tact of the AD converter. The pure sine signal of the unit amplitude is simulated according to the following equation:

$$u_s(t) = \sin\left(\omega t - k\phi\right). \tag{2}$$

The time from when the signal passes through the sampling threshold to when the first tact is generated is unknown and theoretically there are infinitely many different times from the 0 to T_s interval, where T_s is the period of sampling tact. Since it is not possible to perform the simulation for an infinite number of values, a phase coefficient k is introduced which serves to transform an infinite interval to a set with a finite number of values. Fig. 4 shows characteristic conditions in which the measuring system can be found. The signals $u_1(t)$, $u_2(t)$ and $u_s(t)$ have the same frequency and amplitude. This shows one signal that can be found in different situations due to asynchronous sampling. The coordinate origin on Fig. 4 is defined by the transition of signal tact from lower to higher level, i.e. the moment of tact generation of the AD converter.

Fig. 4 also represents a graphical explanation of signal simulation. In the simulation by changing the coefficient k in the range from 0 to 1, different values of the samples are obtained, where the simulated signal is in fact the signal $u_s(t)$. Due to the periodicity of simulated and tact signal, it is sufficient for coefficient k to move in the given range, because for the values outside the range, the same situations will occur. Fig. 5 shows the relationship between the calculated RMS based on obtained samples and the factor k.

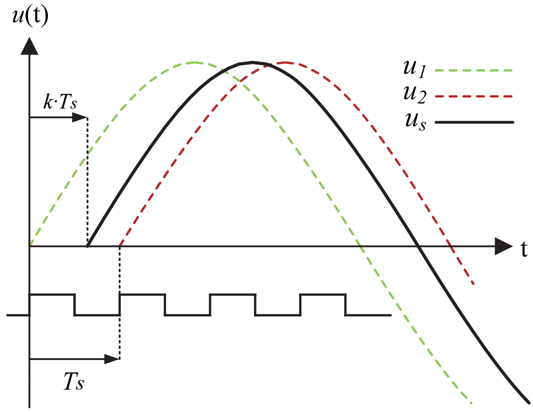

Fig. 4 – *Characteristic conditions in which the asynchronous sampling measuring system can be found.*

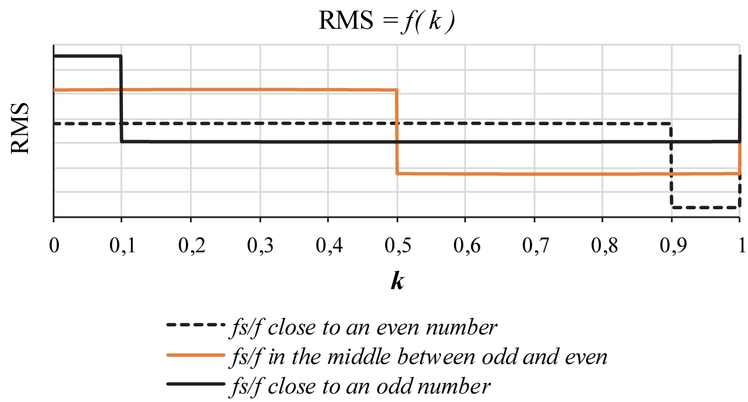

Fig. 5 – *Relationship between the calculated RMS and the phase coefficient k.*

Since k is completely unknown in practice and it is impossible to determine it, it is very important to analyze its impact which is of great importance. With Fig. 5 it can be seen that when f_s/f is close to an even number, for most values of factor k the calculated RMS has a smaller value, while for the remainder of values factor k has a higher value. Conversely, when the ratio is close to an odd number, the larger calculated RMS corresponds to the majority of factor k value. If this were to be seen from the statistical side, it could be concluded that:

given the fact that during asynchronous sampling, the initial sampling moment has a quasi-uniform distribution of occurrence, based on the relation of sampling rate and signal frequency for a series of measurements, it can be roughly predicted what is the number of results which have a lower and the number of results which have a higher value.

3 Proposed Method

The variation of the number of samples is the main cause for scattering of results, which leads to inaccurate measurement, and the question that arises is how to suppress scattering caused by variation of the number of samples. It was shown that the starting moment significantly affects the measurement result.

The mean value of the set $x_1,\ldots,x_n$ of *n* elements is:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i . \tag{3}$$

If an element which value is close to the mean value of the set is subtracted or added from the set, the mean value of that new set will not change significantly. In case that element has the mean value of the set, the mean value of the new set will not change. Using this fact, one must always know which values can be critical, since the occurrence or absence of such values will not significantly affect the mean value of the set.

If there is a possibility that for some reason there is a variation in number of members of a set, or until the critical members- x_k appear, for minimal deviations, critical members should have the value as close as possible to the mean value of the set:

$$x_k \cong \bar{x} . \tag{4}$$

If it is possible to influence the values of critical members, applying (4) the smallest influence of variation of member numbers on the mean value can be obtained.

According to the general definition of the RMS value, the interval of sampling ("window") begins in t_n when the signal takes a certain value and ends at t_m when after one period of signal it again takes the same value.

It is common to take samples at an interval defined by signal passing through zero, firstly because of easier hardware feasibility. Also, there was no particular attention paid on how such a sampling method affects the calculated RMS.

Due to the periodicity of the signal with period *T*, the valid equation is:

$$t_m = T + t_n . \tag{5}$$

With uniform asynchronous sampling of periodic signal with fixed frequency, the number of samples taken over one or more periods does not have to be the same for a certain number of measurements. Fig. 6 shows an example of variation in the number of samples.

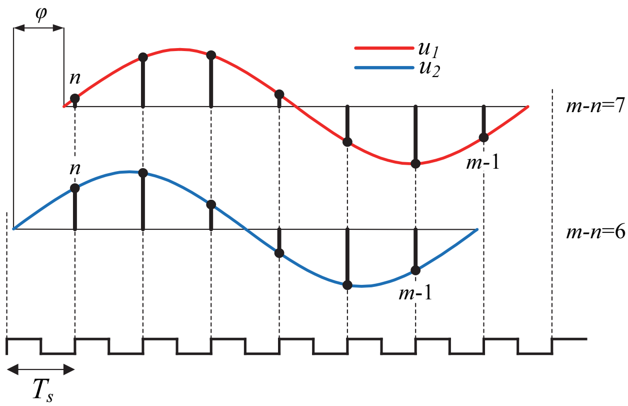

Fig. 6 – *Example of variation in the number of samples.*

The signals u_1 and u_2 have the same shape, period T, amplitude, and different phase shift φ. The rectangular signal represents the sampling rate with period T_s and at each transition from the smaller to the larger level the current signal value is taken. The state from Fig. 6 can be identified with an asynchronous sampling of a signal during one period, where during the first measurement a situation can be obtained as with signal u_1, and during second measurement of the same signal, a situation can be obtained as with signal u_2. The initial sampling time – t_n should be at the moment when the signal takes the value of the sampling threshold, which in this case is zero. In practice, this momentum varies in the range from t_n to $t_n + T_s$ with uniform distribution. When u_2 samples are taken, the number of samples is $(m - n) = 6$, while in the case of the signal u_1 the number of samples is $(m - n) = 7$, where the additional sample is taken close to the sampling threshold. The number n represents the ordinal number of the sample when the measured signal becomes larger than the sampling threshold, and the number m represents the ordinal number when the measured signal again becomes larger than the sampling threshold.

Samples of the measured signal taken around the sampling threshold can, but don't have to be found in a set of useful samples and they represent critical members of a useful set of samples, and as such are the main causes of result scattering.

The value of the critical members from a useful set of samples for asynchronous sampling is located in the vicinity of the sampling threshold value. Based on this, it has been concluded that a large standard deviation is a consequence of the sampling method.

By definition, the sine wave signal $u(t)$ with amplitude U_m and frequency f is defined by:

$$u(t) = U_m \sin(2\pi f t). \tag{6}$$

By definition, the equations for the mean and RMS value of the periodic signal are respectively:

$$U_{AVE} = \frac{1}{T-0}\int_0^T u(t)\,\mathrm{d}t\,, \tag{7}$$

$$U_{RMS} = \sqrt{\frac{1}{T-0}\int_0^T u^2(t)\,\mathrm{d}t}\,. \tag{8}$$

Due to the periodicity of the equations (7) and (8), they can be written in the following forms:

$$U_{AVE} = \frac{1}{(T+c)-(0+c)}\int_{0+c}^{T+c} u(t)\,\mathrm{d}t\,, \tag{9}$$

$$U_{RMS} = \sqrt{\frac{1}{(T+c)-(0+c)}\int_{0+c}^{T+c} u^2(t)\,\mathrm{d}t}\,, \tag{10}$$

where c is a time constant value.

By adding t_n instead of c, according to (5), (9) and (10), the following equations for the mean and RMS values are obtained:

$$U_{AVE} = \frac{1}{t_m - t_n}\int_{t_n}^{t_m} u(t)\,\mathrm{d}t\,, \tag{11}$$

$$U_{RMS} = \sqrt{\frac{1}{t_m - t_n}\int_{t_n}^{t_m} u^2(t)\,\mathrm{d}t}\,. \tag{12}$$

Since, in practice, the signal is sampled at discrete time units, the equations (11) and (12) will obtain the following form accordingly:

$$U_{AVE*} = \frac{1}{m-n}\sum_{i=n}^{m-1} u_i\,, \tag{13}$$

$$U_{RMS*} = \sqrt{\frac{1}{m-n}\sum_{i=n}^{m-1} u_i^2}\,, \tag{14}$$

where u_i is the i^{th} sample, m is ordinal number of sample when the measured signal becomes larger than the sampling threshold, n is ordinal number of sample when one or more periods earlier the measured signal has become larger than the sampling threshold.

According to (14), root mean square and mean values are defined as follows:

The RMS value is equal to the square root of the average of the squared samples taken in the interval which starts when the sample becomes larger (alternatively smaller) than the sampling threshold and ends when it becomes, after one or several periods, again larger (alternatively smaller) than the sampling threshold, omitting the last sample.

The average value is equal to the average of samples obtained in the interval which starts when the sample becomes larger (alternatively smaller) than the sampling threshold and ends when it becomes, after one or several periods, again larger (alternatively smaller) than the sampling threshold, omitting the last sample.

In addition to defining the RMS and mean value of the periodic signal for the newly introduced method, it is also necessary to define the sampling threshold, referring to the criterion for selecting critical members by the (4).

The expression (14) squared is:

$$\frac{1}{m-n}\sum_{i=n}^{m-1} u_i^2 = U_{RMS*}^2 . \tag{15}$$

For minimum deviations according to (4), the criterion set is:

$$u_k^2 \cong U_{RMS*}^2 , \tag{16}$$

or:

$$u_k \cong U_{RMS*} . \tag{17}$$

The value of the critical members of the useful set for determining the RMS value should be as near as possible to the RMS value of the signal.

Significant conclusion:

To determine the RMS value of a periodic signal, the sampling threshold should have a closest value to the RMS value of the measured signal.

To determine the mean value of a periodic signal, the sampling threshold should have a closest value to the mean value of the measured signal.

Proposed definitions for asynchronous sampling:

The RMS value is equal to the square root of the average of the squared samples taken in the interval which starts when the sample becomes larger (alternatively smaller) than the RMS and ends when it becomes, after one or several periods, again larger (alternatively smaller) than the RMS, omitting the last sample.

The average value is equal to the average of samples obtained in the interval which starts when the sample becomes larger (alternatively smaller) than the average and ends when it becomes, after one or several periods, again larger (alternatively smaller) than the average, omitting the last sample.

4 Simulation Results

Using a simulation the influence of the initial sampling time and sampling tact on the calculated RMS has been verified. The simulated signal of the unit amplitude is expressed in (2).

The calculated RMS values of the signal obtained when the sampling threshold is defined by passing of the signal through zero volts are compared to when the sampling threshold is defined by passing the signal through its RMS value (U_T). The calculated RMS values are compared with the root mean square of the simulated signal U_T:

$$U_T = \frac{1}{\sqrt{2}}. \tag{18}$$

For each k, samples were taken using the traditional method of 1.5 periods, and samples were taken for both methods from that set. For the traditional (suggested) method, samples were taken from the one that first became larger than zero (the actual RMS value – U_T) to the one that after another period again became larger than zero (the actual RMS value – U_T), omitting it. Simulation of the measurement of the sine wave signal with frequency $f = 50$ Hz was performed for several ranges of sampling rate – f_S with incrementation step of 1 Hz and division factor $q = 1000$.

$$k_i = \left[0, \frac{1}{q}, \ldots, \frac{q-1}{q}, 1\right]. \tag{19}$$

$$f_s \in \text{(1000 – 1050) Hz, (2500 – 2550) Hz, (5000 – 5050) Hz,}$$
$$\text{(7500 – 7550) Hz, (10000 – 10050) Hz}$$

or:

$$f_S / f = 20 - 21,\ 50 - 51,\ 100 - 101,\ 150 - 151,\ 200 - 201.$$

Over the same set of samples, RMS is calculated according to the general definition and proposed definition. The following figure shows the dependency of the maximum and minimum calculated RMS for the relationship f_s / f:

With the increase in sampling rate, it can be seen that the boundaries of appearance of the calculated RMS are reduced for both methods, but the preferred method remains (Fig. 8).

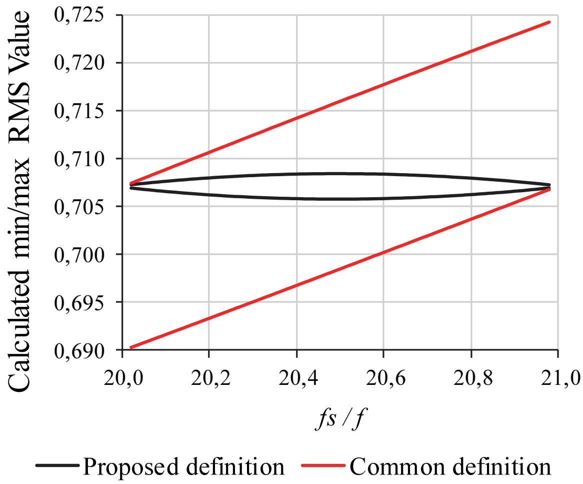

Fig. 7 – *Depedence of the maximum and minimum calculated RMS from the* f_s/f.

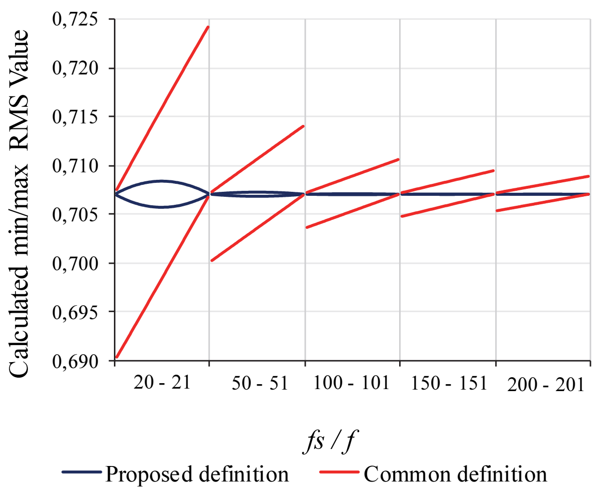

Fig. 8 – *Depedence of the maximum and minimum calculated RMS from the* f_s/f.

The error of determining the RMS value (20) is a representation in the form of a relative error of the calculated RMS value of the simulated signal – U_{MER} relative to the actual RMS value of the simulated signal – U_T:

$$\delta U = \frac{U_{MER} - U_T}{U_T} \cdot 10^6 \text{ [ppm]}. \tag{20}$$

For each value of sampling tact in a given interval, an error calculation was made for each k that depends on q. Because of the impossibility of knowing factor k in practice, the sign of error is also unknown. For this reason, for each value of the sampling tact, the absolute value of the error was taken, for k which error is highest in absolute value to ensure safe limits of the error. The following figures show the results for the sampling rate in the range (1001 – 1049) Hz. The limit values of 1000 Hz and 1050 Hz fall into the phase locked domain of signal frequency and sampling tact where the ratio of periods is an integer, which in practice is very rare in the case of asynchronous sampling and gives a small error. For this reason, the errors for these limit values are not displayed. In Fig. 9 the biggest absolute value errors in the traditional measurement method ($\delta U_{0,MAX}$) are shown, as well as the proposed measurement method ($\delta U_{RMS,MAX}$) for the same sampling rate.

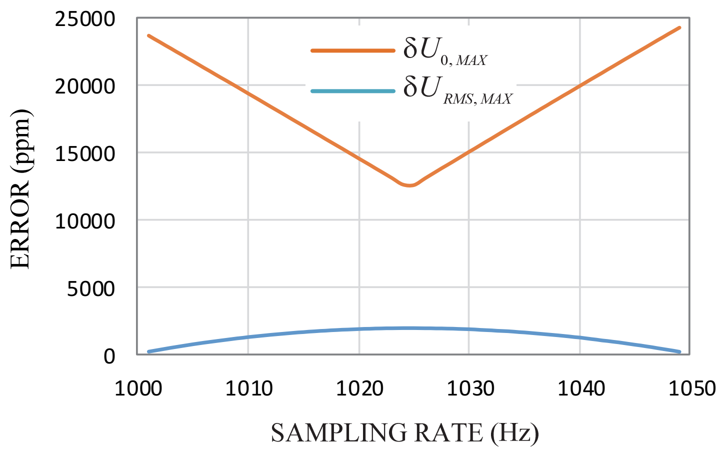

Fig. 9 – *The biggest absolute value errors in the traditional ($\delta U_{0,MAX}$) and proposed ($\delta U_{RMS,MAX}$) measurement method.*

However, it can be concluded that this is not a display of errors in the worst possible situation. The worst possible situation would be to compare the smallest absolute value error obtained with the traditional measurement method ($\delta U_{0,MIN}$) with the the largest error in absolute value for the proposed measurement method ($\delta U_{RMS,\ MAX}$). The proposed method also takes precedence in this situation, since at that time its maximum errors are less than the minimal errors of the traditional method for the same sampling rate. The largest errors of the proposed method ($\delta U_{RMS,\ MAX}$) were compared with the smallest errors in the traditional method ($\delta U_{0,\ MIN}$) and the following dependency was obtained:

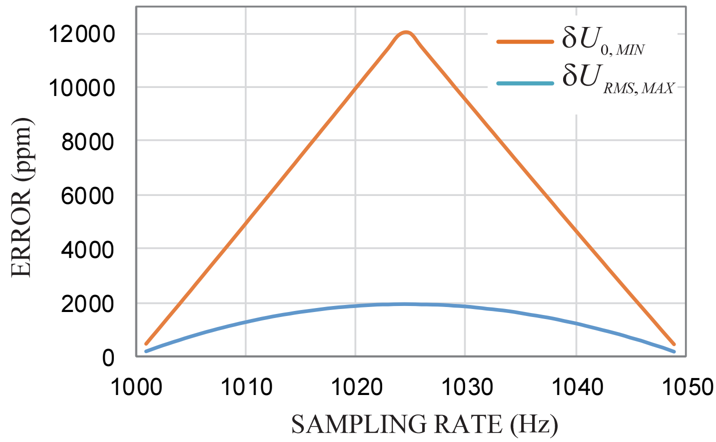

Fig. 10 – *The biggest absolute value error in the traditional* ($\delta U_{0,MAX}$) *method vs. smallest absolute value error in the proposed* ($\delta U_{RMS,MAX}$) *method.*

For all other processed sampling rate ranges, the proposed method also showed better results.

5 Conclusion

Quite large errors were observed due to the fact that the asynchronously taken samples used for determining RMS and mean values do not belong to exactly one or an integer number of periods of the measured signal. This paper shows that the determination of RMS and mean value using the asynchronous sampling method can be very precise if the newly introduced method is used.

The proposed method implies:

1. asynchronous sampling with a sampling threshold defined by passing of the signal through zero, lasting at least (p + 0.5) periods;
2. the calculation of the RMS value based on the recognition of p periods by comparing the samples with zero;
3. taking a predetermined root mean square as a criterion for selecting members from the starting set;
4. recalculating the RMS value over the samples starting from a sample that has a higher value than the calculated RMS value in the previous step, to the one that after the p periods again has a higher value than the calculated RMS value, omitting the last sample.

Synchronous sampling requires a phase synchronization of the instrument with a measured value. The imperfection of multiplicators of frequency and phase locked loops (PLL) that are used to synchronize, bring an error in the result of measurement and additional problems, while the hardware configuration of the asynchronous method is simpler. As with the synchronous method, the proposed method would not function in the case of a square wave signal, complex-periodic wave with multiple passing through zero or its own RMS value. The simulation has shown that for a 6-bit signal representation (peak-to-peak), a clear difference can be seen between the calculated RMS results in favor of the proposed definition.

However, the simulation study and the actual high precision measurements proved that using these definitions and up to date fast and precise AD conversion and data processing, the repeatability and reproducibility of the measurements of voltage, current and power, at power frequencies, are of the order of several ppm [7].

The simulation results are very encouraging, so the next step of checking the possibilities of the proposed method could be realized in practice.

5 Acknowledgement

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