Fast Computation of the Iteration-Based Simulation Method - Case Study of Non-coherent ASK with Shadowing

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Abstract: This article presents a new method of fast symbolic computations of very complex calculations, which are necessary for the analysis, simulation, and design of wireless communication systems using CAS (Computer Algebra System). CAS is applied to formulate hypotheses and define joint probability density functions of certain modulation technique. This is used to prepare for the semi-symbolic calculation to complete specify wireless system by using CAS. We have developed an iteration-based simulation method that aids to solve semi-symbolic expressions and gives closed form solutions (with some parameters specified as numbers and some as symbols). So far, they are solved by numerical methods. Students can perform performance analysis and understand the processes in the data transfer. Engineers and researchers may have a better insight into the impact of the important parameters necessary to properly transmit and detect information unlike traditional numerical methods. The main contribution is to obtain solutions for the probability density function, and outage probability where no solution can be obtained using numerical methods. A strong emphasis is placed on very fast calculations that significantly save the time of analysis, simulation, and design.

Keywords: Iteration-Based Simulation Method, Kummer's transformation, Closed-form solution, Closed-form expression.

1 Introduction

Traditional numeric tools provide an enormous amount of data in the analysis of data transmission. However, obtained results illustrate quantitative indicators on graphics as one variable in term of the second variable. The

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engineers and researchers may represent a high time consumption due to the inability to have an insight into certain parameters impact on system performance. In addition, all they can represent the gap between theoretical knowledge and expert skills to the final stage of implementation. As opposed to numerical tools, symbolic computation provides final analytical solutions of observed data transmission and their statistical parameters. Computer algebra system plays a central role of these analyzes. Students or engineers use it to formulate statistical equations, to prepare it for symbolic solving, and thus completely specify performance analysis by using software package Mathematica as CAS. Nowadays, a large number of numerical tools are implemented in analysis and simulation software, to help in the design of the system. Most of these numerical tools include complex calculations, such as differential and integral equations, algebraic structures, using numerical mathematics algorithms such as Romberg integration, Gauss-Christoffel, Newton-Cotes, trapezium rule, Gauss formulas and special functions [1 – 2]. A large number of scientific disciplines within the IT field has requirements for fast calculations [3 – 5]. Some problems encountered by students and engineers are derivations of mathematical expressions. They can sometimes be cumbersome and tiresome and often do not lead to a solution if there are performed by hand. Even if we use numerical tools, it takes a lot of time to get the final result due to a large number of iterations, frequent parameter settings, and so on.

For this reason, we use a developed iterative method [6 – 7] that helps us to get complete solutions in a closed form. Such solutions can be simplified, further manipulated, and we use them to have a clear insight into the process of analysis and speed up of the calculation. In that sense, we use that CAS delivers more capabilities and it is used for deeper understanding and preparing students to learn and engineers to simulate and design. It takes place in several kinds of software. One of them is software package Mathematica with Wolfram language as in [8 – 9].

In particular, it is present in the wireless communications when integrals appear. Their solutions are usually performed numerically due to the very high complexity and the inability to come up to analytical solutions. Matlab and MathCAD are software packages that help in solving them [10 – 11].

We introduce a new method for fast computation of symbolic solving all statistical expressions used in wireless communications. The proposed method leads to complete performance analysis of system to the final analytical formulas. It is developed using Wolfram language as one of the most complete symbolic tools. Non-coherent ASK in the presence of shadowing and correlated noise were chosen as an example for observation. Its specificity is that performance of system cannot obtain traditional numerical methods.

This paper is structured as follows. In Section 2, the overviews of non-coherent ASK in the presence of shadowing and correlated noise. In section 3, the iteration-based simulation method and Wolfram language modeling for semi-symbolic analysis are presented. In Section 4, speeding up calculation procedure using Kummer's transformation is described, as well as the symbolic derivations of transformation and its application. The experimental results are shown in Section 5. In Section 6, the overview of related works is given. Finally, Section 7 concludes this paper.

2 Review of Non-coherent ASK in the Presence of Shadowing and Correlated Noise

One of the most common models used in wireless communications to describe phenomenon of multiple scattering is shadowing as in [12 – 15]. It causes strong fluctuations in the amplitude of the useful signal. It is present in urban areas and described in form lognormal distribution. In our analysis, we performed an outage probability of non-coherent ASK system in presence of shadowing and correlated noise.

Transmitting signals by two symbols are observed in the non-coherent ASK system as in [6 – 7]. The noise is correlated and its coefficient of correlation is marked by R ($R \neq 1$). The noise is narrowband stochastic process. Mathematically, it can be described in the form of $n_i(t) = x_i(t)\cos(\omega t) - y_i(t)\sin(\omega t)$.

The receiver is sheltered; the optical visibility does not exist towards transmitter. If the system sends logical zero then the signal $s_0(t) = a_0(t)\cos(\omega t)$ has been sent, but if the system sends logical unit then the signal $s_1(t) = a_1(t)\cos(\omega t)$ has been sent. Combining these signals, the codeword from two symbols have been obtained.

Passing through a transmitting channel receiver detects information signal $b_0\cos(\omega t)$ and $b_1\cos(\omega t)$ with envelops z_0 and z_1, respectively. In addition, the correlated narrowband noise is present apart from the useful signal. The receiver system is consisted of filters, detector envelops, and it is shown on Fig. 1. In the receiver input, the signal is the form:

$$r_m(t) = b_m\cos\omega t + x_m\cos\omega t - y_m\sin\omega t = z_m\cos(\omega t + \varphi_m), \ m = 0,1 \qquad (1)$$

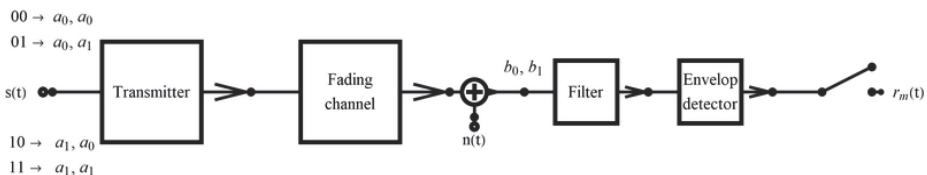with envelops z_0 and z_1, and phases φ_0 and φ_1, respectively.

Fig. 1 – *Non-coherent ASK system.*

Due to the presence of shadowing the form of a joint probability density function has lognormal distribution described as in [13]:

$$p_{ij}(b_0,b_1) = \frac{1}{2\pi\sigma^2 b_0 b_1 \sqrt{1-R^2}} \times$$

$$\times \exp\left\{-\frac{\left(10\log b_0 - a_i\right)^2 + \left(10\log b_1 - a_j\right)^2}{2\sigma^2\left(1-R^2\right)}\right\} \times \quad (2)$$

$$\times \exp\left\{\frac{2R\left(10\log b_0 - a_i\right)\left(10\log b_1 - a_j\right)}{2\sigma^2\left(1-R^2\right)}\right\}.$$

For $i=j=0$ the codeword 00 was sent. Similarly, for $i=1$ and $j=0$ the codeword 01 was sent, for $i=0$ and $j=1$ the codeword 10 was sent, and for $i=1$ and $j=1$ the codeword 11 was sent. The R is a coefficient of correlation.

From equations (1), it follows:

$$\begin{aligned}
x_0 &= z_0 \cos\varphi_0 - b_0, \\
y_0 &= -z_0 \sin\varphi_0, \\
x_1 &= z_1 \cos\varphi_1 - b_1, \\
y_1 &= -z_1 \sin\varphi_1,
\end{aligned} \quad (3)$$

moreover, substituting in (2), it is obtained

$$p(r_0,r_1/b_0,b_1,\varphi_0,\varphi_1) = p(x_0,y_0,x_1,y_1)|J|, \quad (4)$$

where $|J| = z_0 z_1$ is Jacobian. Applying expression (3) in (2), we obtain

$$p(z_0,z_1/b_0,b_1,\varphi_0,\varphi_1) = \frac{z_0 z_1}{\left(2\pi\right)^2 \sigma^4 \sqrt{1-R^2}} \times$$

$$\times \exp\left\{-\frac{z_0^2 + z_1^2 - 2Rb_0 b_1 - 2Rz_0 z_1 \cos(\varphi_0 - \varphi_1)}{2\sigma^2\left(1-R^2\right)}\right\} \times \quad (5)$$

$$\times \exp\left\{\frac{2(1+R)(b_0+b_1)(z_0 \cos\varphi_0 + z_1 \cos\varphi_1)}{2\sigma^2\left(1-R^2\right)}\right\}.$$

Before deriving closed form solutions, it is needed to transform last expression using transformation for modified Bessel function as in [17]

$$e^{x\cos\alpha} = \sum_{n=-\infty}^{\infty} I_n(x)\cos n\alpha, \quad (6)$$

and applying trigonometric transformation:

$$\cos\alpha\cos\beta\cos\gamma =$$

$$= \frac{1}{4}[\cos(\alpha+\beta+\gamma) + \cos(\alpha-\beta+\gamma) + \cos(\alpha+\beta-\gamma) + \cos(\alpha-\beta-\gamma)] \tag{7}$$

The expression (5) is simpler

$$p(z_0, z_1/b_0, b_1, \varphi_0, \varphi_1) =$$

$$= Ce^{C_0}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} I_n(C_1)I_m(C_2)I_k(C_3)\times\cos[n\varphi_0]\cos[m\varphi_1]\cos[k(\varphi_0-\varphi_1)], \tag{8}$$

where:

$$C = \frac{z_0 z_1}{(2\pi)^2 \sigma^4 \sqrt{1-R^2}},$$

$$C_0 = -\frac{z_0^2 + z_1^2 - 2Rb_0 b_1}{2\sigma^2(1-R^2)},$$

$$C_1 = \frac{b_0 + b_1}{\sigma^2(1-R)}z_0, \tag{9}$$

$$C_2 = \frac{b_0 + b_1}{\sigma^2(1-R)}z_1,$$

$$C_3 = \frac{Rz_0 z_1}{\sigma^2(1-R^2)}.$$

Distribution is obtained by averaging with φ_0 and φ_1 for all values between $-\pi$ and π.

$$p(z_0, z_1/b_0, b_1) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} p(z_0, z_1/b_0, b_1, \varphi_0, \varphi_1)\,d\varphi_0\,d\varphi_1. \tag{10}$$

The integral (10) is solved using integral:

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\cos n\varphi_0 \cos m\varphi_1 \cos k(\varphi_0-\varphi_1)\,d\varphi_0\,d\varphi_1 = \begin{cases} \pi^2, & |n|=|m|=|k| \\ 0, & n \neq m \neq k \end{cases} \tag{11}$$

and using Bessel identity $I_n(x) = I_{-n}(x)$, it follows

$$p(z_0, z_1/b_0, b_1) =$$

$$= 2\pi^2 C\, e^{C_0(b_0, b_1)}\times\sum_{n=0}^{\infty} I_n\big[C_1(b_0, b_1)\big] I_n\big[C_2(b_0, b_1)\big] I_n\big[C_3\big]. \tag{12}$$

The integration for all values b_0 and b_1 is necessary to obtain distributions of envelops for all combinations of codeword. We calculate the true probability of detection that is used to calculate the outage probability. So, when the

codeword |**ij**| ($i = 0, 1; j = 0, 1$) has been sent and when the same are detected on the input of receiver, the detection of the signals is described in the form:

$$P(D_i D_j / H_i H_j) =$$

$$= p_{ij}(z_0, z_1) = \int\limits_0^\infty \int\limits_0^\infty p(z_0, z_1 / b_0, b_1) p_{ij}(b_0, b_1) \, \mathrm{d} b_0 \, \mathrm{d} b_1, \tag{13}$$

where $i = 0,1$ and $j = 0,1$. The case $D_i D_j$ represent the situation when the code word is detected at the moment when it is with $H_i H_j$ hypothesis. The outage probability is obtained as

$$P_e = 1 - \sum_{i=0}^1 \sum_{j=0}^1 P(H_i H_j) P(D_i D_j / H_i H_j), \tag{14}$$

where $P(H_i H_j) = P(H_i) P(H_j) = \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4}$, $i = 0,1;$ $j = 0,1$.

The expression (14) for the outage probability represents closed form expression and very often cannot be present in the form of closed form solution. Closed-form expression represents an implicit solution that is contained in a mathematical expression. Closed-form solution provides solved problem in terms of functions and mathematical operations from a given generally-accepted set [18]. In other words, closed-form solution provides explicitly solution of observed problem, whereas the closed-form expression shows implicitly or insufficient solution. In this paper, we just derive a closed form solution.

3 Iteration-Based Simulation Method and Wolfram Language Modeling

The Wolfram language is the concept to encapsulate expert knowledge in software [9]. For better understanding of how Wolfram language is executed, it is necessary to provide background for writing own code in an optimum manner. One of the main performances is that Wolfram language provides simplification, calculation, rearrange, analysis and simulation systems and processes based on expressions. According to this, it is very easy to check that this automated derivation is generating the same result.

The iteration-based simulation method is designed to simplify complex algebraic expressions, expressed as a closed form expression, and offers acceptable analytical and reduced form for further manipulation or simulation as a closed form solution published as in [7]. Basically, many integrals exist in the most of analyses; the iteration-based simulation method introduces iteration as elementary parameter for calculating using Riemann sum. In this case, the iteration seems as a parameter and it can be manipulated. Riemann sum in Wolfram language defined as follows:

```
RiemannSumDouble[f_,{x_,a_,b_},{y_,c_,d_},q_]:=
  Sum[Evaluate[(f/.{x→(a+(b-a)k/q),y→(c+(d-c)k/q)})],{k,1,q}]*
(b - a) (d - c)
─────── ───────
  q       q
```

Fig. 2 – *Riemann sum defined using Wolfram language.*

The function name `RiemannSumDouble` defines double integrals over Riemann sum. Double integral where f is marked as function, x and y are variables, a, b, c, d are below the upper limits of integral, and q is parameter of iteration. The closed form solutions define the integral as complex mathematical formulas in this way.

To perform expressions of closed form solutions using Wolfram language, we write expressions in the same way as defined in the closed form expressions. So, the expression (3) the joint probability density function is described as:

$$JPDF[x\_,y\_,s\_,R\_]:=\frac{1}{2*\pi*(s)^2*b0*b1*\sqrt{1-R^2}}*$$

$$Exp\left[-\frac{(10*Log[10,b0]-x)^2+(10*Log[10,b1]-y)^2}{2(s)^2(1-R^2)}\right]*$$

$$Exp\left[\frac{2*R*(10*Log[10,b0]-x)*(10*Log[10,b1]-y)}{2(s)^2(1-R^2)}\right]$$

Fig. 3 – *Condition joint probability density function
using Wolfram language for shadowing.*

After manipulations of expression (5) and substituting of expressions (9)

$$Cc=\frac{z0*z1}{(2\pi)^2(s)^4\sqrt{1-R^2}};$$

$$C0=-\frac{(z0)^2+(z1)^2-2R*b0*b1}{2(s)^2(1-R^2)};$$

$$C1=\frac{b0*b1}{(s)^2(1-R)}z0;$$

$$C2=\frac{b0*b1}{(s)^2(1-R)}z1; C3=\frac{R}{(s)^2(1-R^2)}z0*z1;$$

Fig. 4 – *Substitutions of elements in Bessel functions.*

The general form of the condition joint probability density function defined in (12), it is described as:

```
LogNormalP[v_,s_,R_]:=
                    v
2 π² *Cc* Exp[C0] * ∑ (BesselI[k,C1]*BesselI[k,C2]* BesselI[k,C3])
                   k=1
```

Fig. 5 – *Lognormal distribution for non-coherent ASK in presence of shadowing.*

With s is marked variance σ, the R is correlation coefficient and v is order of the iterations.

The key place in the paper is to obtain closed form solutions of probability density function, outage probability versus iterations. The calculation of the original probability of detection is performed using preview defined function of Riemann and we obtain closed form solution of original probability density function:

In obtained closed form solution, the h_0 and h_1 present the borders of integration and q is iteration parameter, and can be observed as iteration's resolution. This expression is ready for further manipulations, calculations and simulations. The next step is to obtain outage probability marked as P_{outage}, so:

$$\frac{1}{q^2}(h_0 - h_1)^2 \sum_{k=1}^{q} \left(z_0 z_1 I_1\left(\frac{R z_0 z_1}{(1-R^2)\sigma^2}\right) I_1\left(\frac{\left(h_0 + \frac{k(h_1-h_0)}{q}\right)^2 z_0}{(1-R)\sigma^2}\right) \right.$$

$$I_1\left(\frac{\left(h_0 + \frac{k(h_1-h_0)}{q}\right)^2 z_1}{(1-R)\sigma^2}\right) \exp\left(-\frac{\left(\frac{10\log\left(\frac{(h_1-h_0)k}{q}+h_0\right)}{\log(10)} - a_0\right)^2 + \left(\frac{10\log\left(\frac{(h_1-h_0)k}{q}+h_0\right)}{\log(10)} - a_1\right)^2}{2(1-R^2)\sigma^2}\right. +$$

$$\frac{R\left(\frac{10\log\left(\frac{(h_1-h_0)k}{q}+h_0\right)}{\log(10)} - a_0\right)\left(\frac{10\log\left(\frac{(h_1-h_0)k}{q}+h_0\right)}{\log(10)} - a_1\right)}{(1-R^2)\sigma^2} -$$

$$\left.\left.\frac{-2R\left(\frac{(h_1-h_0)k}{q}+h_0\right)^2 + z_0^2 + z_1^2}{2(1-R^2)\sigma^2}\right)\right) \bigg/ \left(4\pi(1-R^2)\sigma^6\left(\frac{(h_1-h_0)k}{q}+h_0\right)^2\right)$$

Fig. 6 – *Closed form solution of probability density function of non-coherent ASK system.*

$$\frac{1}{q^2} \left| \sum_{k=1}^{q} \frac{q^2 z_0 z_1 I_1\left(\frac{R z_0 z_1}{(1-R^2)\sigma^2}\right) I_1\left(\frac{k^2 z_0}{q^2(1-R)\sigma^2}\right) I_1\left(\frac{k^2 z_1}{q^2(1-R)\sigma^2}\right) \exp\left(-\frac{-\frac{2k^2 R}{q^2}+z_0^2+z_1^2}{2(1-R^2)\sigma^2} + \frac{R\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}-1\right)^2}{(1-R^2)\sigma^2} - \frac{\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}-1\right)^2}{(1-R^2)\sigma^2}\right)}{4\pi k^2 (1-R^2)\sigma^6} + \right.$$

$$\sum_{k=1}^{q} \frac{q^2 z_0 z_1 I_1\left(\frac{R z_0 z_1}{(1-R^2)\sigma^2}\right) I_1\left(\frac{k^2 z_0}{q^2(1-R)\sigma^2}\right) I_1\left(\frac{k^2 z_1}{q^2(1-R)\sigma^2}\right) \exp\left(-\frac{-\frac{2k^2 R}{q^2}+z_0^2+z_1^2}{2(1-R^2)\sigma^2} + \frac{R\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}+1\right)^2}{(1-R^2)\sigma^2} - \frac{\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}+1\right)^2}{(1-R^2)\sigma^2}\right)}{4\pi k^2 (1-R^2)\sigma^6} +$$

$$2\sum_{k=1}^{q} \frac{1}{4\pi k^2 (1-R^2)\sigma^6} q^2 z_0 z_1 I_1\left(\frac{R z_0 z_1}{(1-R^2)\sigma^2}\right) I_1\left(\frac{k^2 z_0}{q^2(1-R)\sigma^2}\right) I_1\left(\frac{k^2 z_1}{q^2(1-R)\sigma^2}\right)$$

$$\left. \exp\left(-\frac{-\frac{2k^2 R}{q^2}+z_0^2+z_1^2}{2(1-R^2)\sigma^2} + \frac{R\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}-1\right)\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}+1\right)}{(1-R^2)\sigma^2} - \frac{\left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}-1\right)^2 + \left(\frac{10\log\left(\frac{k}{q}\right)}{\log(10)}+1\right)^2}{2(1-R^2)\sigma^2}\right) \right|$$

Fig. 7 – *The outage probability of non-coherent ASK for shadowing.*

4   Speeding up Calculation Procedure

The resulting P_{outage} shows all the variables affecting the final result in a closed form. Such an expression can be further used to manipulate and obtain desired results that quantitatively and qualitatively describe the observed system. It is easy to notice that the new parameter is present in an expression that allows outage probability to be obtained in a closed form, which is a step of iteration. However, a large number of iterations is required, which depends on a given accuracy error, for calculating the specific numerical values of the outage probability. In order to obtain a result with a great time saver, it is necessary to speed up the expression in Fig. 7. Since this expression has its own structure of the sum, it can be viewed as a series. In this regard, we approach the development of an algorithm that will speed up the calculation of this series. For this purpose, we use Kummer's transformation that serves to speed up the series.

4.1 Kummer's transformation

Kummer's transformation (more famous as Kummer's acceleration method) is used to accelerate the convergence of many series. The basic concept is to subtract from a given convergent series $\sum a_k$, and another equivalent series $\sum b_k$ whose sum $C = \sum_{k \geq 0} b_k$ is well known and finite. The first transformation condition must be fulfilled

$$\lim_{k \to \infty} \left(\frac{a_k}{b_k} \right) = \rho \neq 0 , \tag{15}$$

then the transformation is given by

$$\sum_{k=0}^{\infty} a_k = \rho \sum_{k=0}^{\infty} k_k + \sum_{k=0}^{\infty} \left(1 - \rho \frac{b_k}{a_k} \right) a_k = \rho C + \sum_{k=0}^{\infty} \left(1 - \rho \frac{b_k}{a_k} \right) a_k \tag{16}$$

The convergence of the right hand side of equation (16) is faster because $1 - \rho b_k / a_k$ tends to 0 as k tends to infinity [19].

Two theorems about series are important for Kummer's transformation. The first one is: if sum $\sum_{k \geq 0} a_k$ converges then

$$\lim_{k \to \infty} a_k = 0 . \tag{17}$$

The other one let us $\sum_{k \geq 0} a_k$ and $\sum_{k \geq 0} b_k$ are positive series. If $\lim_{k \to \infty} \left(\frac{a_k}{b_k} \right) = \rho \neq 0$ $(b_k \neq 0)$ where $\rho \neq 0$ is finite value, $\sum_{k \geq 0} a_k$ and $\sum_{k \geq 0} b_k$ converge or diverge in the same time.

4.2 Symbolic derivations of transformation and its application

Now, the expression (16) needs to be applied to the expression for the outage probability shown in Fig. 7. The first step is to segregate general term from outage probability P_{outage}, marked by a_k. The term is shown on Fig. 8.

$$-\frac{1}{4\pi k^2 (R^2 - 1)\sigma^6} z^2 I_1\left(\frac{R z^2}{(1 - R^2)\sigma^2}\right) \left(2 e^{\frac{2R}{(R^2-1)\sigma^2}} \left(\frac{k}{q}\right)^{\frac{20}{(R+1)\sigma^2 \log(10)}} + \left(\frac{k}{q}\right)^{\frac{40}{(R+1)\sigma^2 \log(10)}} + 1\right)$$

$$\left(\frac{k}{q}\right)^{-\frac{20}{(R+1)\sigma^2 \log(10)}} I_1\left(\frac{k^2 z}{q^2 (1 - R)\sigma^2}\right)^2 \exp\left(-\frac{\frac{k^2 R + q^2 (R - z^2 - 1)}{q^2 (R-1)} + \frac{100 \log^2\left(\frac{k}{q}\right)}{\log^2(10)}}{(R + 1)\sigma^2}\right)$$

Fig. 8 – *General term in series of outage probability marked as a_k.*

After simplification we perform convergence testing of series:

$$\lim_{\substack{k \to q \\ q \to \infty}} -\frac{z^2\left(e^{-\frac{2R}{(R^2-1)\sigma^2}}+1\right)e^{-\frac{z^2+1}{(R^2-1)\sigma^2}} I_1\left(\frac{z}{(1-R)\sigma^2}\right)^2 I_1\left(\frac{Rz^2}{(1-R)\sigma^2}\right)^2}{2\pi q^2(R^2-1)\sigma^6} = 0 . \quad (18)$$

Convergence testing is performed with assumptions that $0 \leq R < 1$, $\sigma > 0$, $z \geq 0$, and $q \geq 1$. One of the most important points of this method is adjustment of an auxiliary series. The series

$$C = \sum_{k=0}^{\infty} \frac{1}{k^5 2^{k-1}} \quad (18)$$

provides best features for speeding up. This auxiliary series converges and tends to $Li_5(1/2)$, where $Li_n(z)$ gives the polylogarithm function as

$$Li_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n} . \quad (19)$$

For creating of the transformation, we use a small modification of Kummer's transformation members in order to provide all the conditions for accelerating the convergence of the newly established series. So,

$$s = \rho C + \sum_{k=0}^{\infty}\left(1 - \rho \frac{a_k + c_k}{a_k}\right)a_k , \quad (20)$$

where

$$\rho \to \frac{a_k}{a_k + c_k}, \qquad c_k = \frac{1}{k^5 2^{k-1}} . \quad (21)$$

Such modification provides a new symbolic close-form expression of ρ, so,

$$\rho = \frac{1}{\dfrac{\pi 2^{2-q} R\left(R^2-1\right)\sigma^6 e^{-\frac{-2R+z^2+1}{(R^2-1)\sigma^2}}}{q^3 Rz^2\left(e^{-\frac{-2R+z^2+1}{(R^2-1)\sigma^2}}+1\right)I_1\left(\frac{z}{(R-1)\sigma^2}\right)^2 I_1\left(\frac{Rz^2}{(R^2-1)\sigma^2}\right)}+1} . \quad (22)$$

After convergence testing the value ρ tends to 1. The speeding up of the computation is performed by assuming how much iteration is required to calculate the outage probability P_{outage} obtained by the iteration-based simulation method. Otherwise, a large number of iterations are required to calculate the closest exact values of P_{outage}, but computation lasts to long time.

Then, such the resulting close form of P_{outage} equalizes with a new series obtained by the Kummer's transformation, and performs point matching for the various values of the envelops, followed by a new reduced number of iterations. After that, the verification of the obtained results is performed by checking the relative error, which determines the degree of adjustability of the algorithm. In the next section, the algorithm is represented through the experiment and experimental results are presented.

5 The Experimental Results

Let us assume that the closest exact value is obtained after 500 iterations by using the outage probability P_{outage} in Fig. 7. After transforming P_{outage}, we get the series that is marked with s, and we equate these two series. For various values of the envelop z we calculate the new values of the number of iterations that are reduced. This is performed by the command of `FindRoot[s==Poutage,{q,1}]`. For a concrete case, we take the range of values $z = \{1, 17\}$.

It should also be noted that we calculate the reduced number of iterations for the different values of the coefficient of correlation R and the variance σ. All calculations were performed with a precision of 10^{-6}.

All tests are performed on PC with: Intel® Core™ i5-6500 CPU@ 3.2GHz, 8 GB RAM, 64-bit Operating System, Windows 10, and Mathematica Wolfram 11.1. The reduced number of iterations is shown in **Table 1**.

The Fig. 9. depicts dependence of the number of iterations q in term of envelop z. The obtained values of a reduced number of iterations may now serve to draw outage probability P_{outage}. Fig. 10 shows the comparative features of P_{outage} with 500 iterations and the same parameter with a reduced number of iterations.
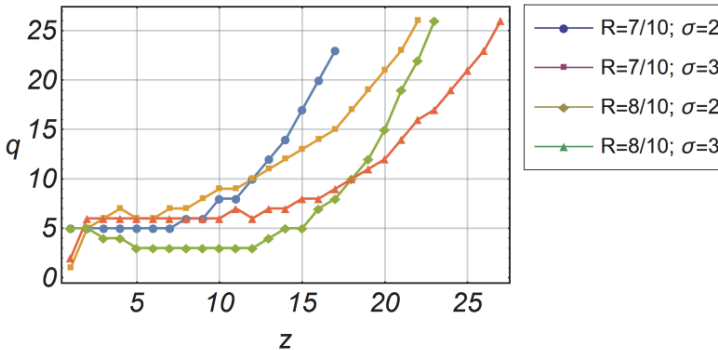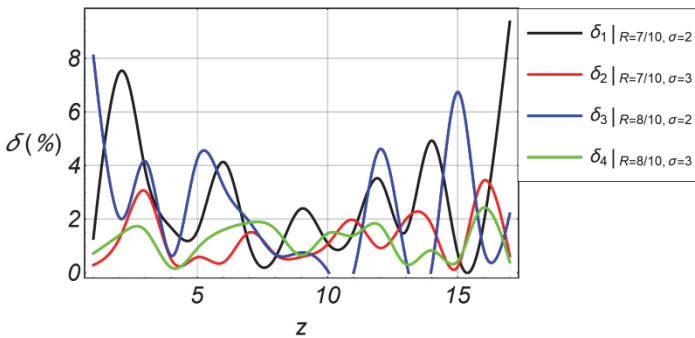

Fig. 9 – *The number of iterations in term of envelop z.*

Table 1
Reduced number of iterations.

z	q_1 $R = 7/10; \sigma = 2$	q_2 $R = 7/10; \sigma = 3$	q_3 $R = 8/10; \sigma = 2$	q_4 $R = 8/10; \sigma = 3$
1	5	1	5	2
2	5	5	5	6
3	5	6	4	6
4	5	7	4	6
5	5	6	3	6
6	5	6	3	6
7	5	7	3	6
8	6	7	3	6
9	6	8	3	6
10	8	9	3	6
11	8	9	3	7
12	10	10	3	6
13	12	11	4	7
14	14	12	5	7
15	17	13	5	8
16	20	14	7	8
17	23	15	8	9

Since the absolute error is not precisely characterized by accuracy, the relative error is used as

$$\delta = \frac{s - P_{outage}}{P_{outage}} \ . \tag{23}$$

Fig. 10 – *Relative error functions in term of the envelop z.*

Relative errors do not exceed more than 8% of the value.

In the Fig. 11, the comparative characteristics of P_{outage} and the characteristics obtained by accelerated calculation are presented. The matching point shows that we have successfully set the values of the mentioned characteristics to a great extent, which show that the algorithm has shown good results.
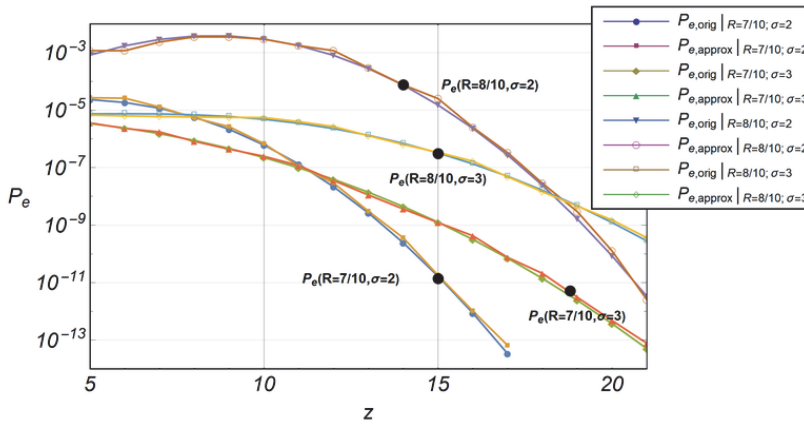

Fig. 11 – *Comparative characteristics of P_{outage} and accelerated outage probability s.*

And finally, the calculation of formula P_{outage} takes total calculation time 697.984s (11 min, 38 sec), so, the average time per iteration is 41.0579s. Using our algorithm total calculation time of accelerated formula is 1.42188s, so, the average time per iteration is 0.0836397s!

The speeding up is

$$Ratio = \frac{Original}{Accelerated} = \frac{697.984}{1.42188} \approx 490 \text{ times}. \tag{24}$$

6   Related Work

In the last decade, many new methods have been developed to improve and speed up calculating and solving of performances. They give results that include parameters that have not been taken in consideration, but can significantly affect the design of wireless systems as in [11].

The growth of urban areas with a marked density, especially in urban areas with tall buildings, as well as a sharp rise in users of mobile communications lead to efforts to find adequate methods and ways for proper exchange of data and information. For correct design rules of mobile systems, it starts from an environment in which the exchange of data will be used, as well as the parameters that will be calculated for the proper functioning of the system in all conditions as in [14 – 16]. The analyses, simulations and calculations of the

parameters of the system usually perform statistical calculations using numerical-based tools as in [20]. These tools can draw certain characteristics that lead us to solutions, but may represent a bottleneck in the case of very complex analyses of cases when it is necessary to have an analytical expression that describes the observed system.

So far, the performance analysis of wireless systems is investigated for differences scenarios. The most common form of simulations is based on traditional numerical tools as in [21, 22].

In this paper, we introduce a new approach for analyzing and solving the parameters of wireless systems. According to this, the closed form solutions based on closed form expressions of outage probability is obtained by introducing a new parameter – the step of iteration. The symbolic tools are used in software package Mathematica with new concept of the Wolfram language. We get final analytical forms of outage probability as the closed form solution, and easy analyze the signal detection and error function.

7   Conclusion

This paper presents a new method to speed up computation of the iteration-based simulation method for simulation the processes and systems where there is no opportunity to represent mathematical formulas in the final form. Often, many phenomena show that closed-form expressions and simulations are executed by the numerical based tools. In such cases, the users do not have insight into phenomena that affect the flow of processes, and it can lead to wrong assumptions and results.

The method shown accelerates the calculation significantly, provides better insight into processes and systems using symbolic processing that cannot be analyzed using numerical processing.

As a method for accelerating the calculation, Kummer's transformation was used. In this way, the calculations are accelerated approximately to 500 times, which is very important in the analysis and simulation of very complex systems and processes. The presented method has been performed on the examples of non-coherent ASK with shadowing when noise is correlated.

8   Acknowledgement

9   References

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