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The Robustness of the Differential Quantizer in the Case of the Variable Signal to Noise Ratio

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Abstract: In this paper a short theoretical overview of differential quantizer and its implementations is given. Afterward, the effect of the order of prediction in differential quantizer and the effect of the difference in order of predictor in the input and output of differential quantizer is analyzed. Then it was proceeded with the examination of the robustness of the differential quantizer in the case in which a noise signal is brought to the input of the differential quantizer, instead of the clean speech signal. The analysis was conducted with a uniform distribution, as well as the noise with the gaussian distribution, and the obtained results were adequately commented on. Also, experimentally a limit was set which refers to the intensity of the noise and still enable results which are better that a regular uniform quantizer. The whole analysis is done by using the fixed number of bits in quantization, i.e. 12-bit quantizer is used in all the implementations of differential quantizer. In the conclusion of this paper there is a discussion about the possibility of implementing a differential quantizer which will be able to recognize which noise attacks the system, and in addition to that, in what form it adapts its coefficients so that it at any moment acquires the optimal signal to noise ratio.

Keywords: Differential quantizer, Digital speech processing, Predictor order, quantization, Signal to noise ratio.

1 Introduction

SPEECH is a signal which holds specific information and its main purpose is communication. Speech is the most natural way of communication among people and its importance cannot be downsized even nowadays when communication and information is available in other forms [1]. In systems dealing with speech communication voice is transferred, stored and processed in different ways. Digital representation of the signal is desirable, because digital signal is more convenient for transmission, processing and storage. One of branches of analog-digital conversion is quantization. This is a process of conversion of signal with the infinite number of levels into a signal with finite

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number of quantization levels [2]. This process leads to the inevitable distortion, namely one part of the information is lost due to the error caused by the finite number of quantization levels. Therefore, choice of the quantization procedure is extremely important in order to ensure satisfying quality of the signal. This topic is well covered in the literature including books [3 – 5] which describe the procedure and implementation of quantizers and offer a detailed overview of the quantization theory. Reference [6] describes the design of different quantization methods including the differential quantizers. However, it does not provide the analysis of quantizer quality depending on different parameters and the effects of the distortion to the system, which is common in practice. Therefore, the idea of this paper is to examine the robustness of the projected differential quantizer in the case where the signal noise ratio (SNR), which is the basic measure of quality of any quantizer, is not constant but the subject of variation. The idea of the author is to perform a systematic display of the influence of the distortion (noise) to a clean speech signal, and furthermore to show that the use of the differential quantizer in these situations is desirable.

Differential quantizer is widely used in digital speech processing. This paper analyses the performance of differential quantizer based on several parameters: the order of the predictor (first, second and third), it has been shown in which way the signal to noise ratio changes as the line of predictors multiply, as well as the change of the signal noise ratio when a pure speech signal is not brought to the input of the differential quantizer, but the superposition of the speech signal and noise. One of the main properties of the differential quantizer is that its input is not the value of the speech signal, but its difference from the estimated value. As a consequence the input to the quantizer is a signal with smaller amplitude which provides better results with the same number of bits or maintains the quality of quantization signal with less bits, both of which are a great advantage. However, this improvement comes with a cost of higher complexity, since the structure of differential quantizer is much more complex than the regular, uniform quantizer.

The paper is structured as follows: Section 2 provides the theoretical overview of the differential quantizer as well as the system of equations necessary for its implementation. Section 3 presents experimental results with a detailed analysis of the quantizer performance depending on two parameters: the order of the predictor and the different noises in which the speech signal becomes added for the purpose of simulating one kind of distortion.

2 Differential Quantizer

Calculation of the autocorrelation function of the speech signal suggests high correlation between adjacent samples. This indicates slow signal changes in the statistical sense and lesser variance of the differences of the adjacent samples than the variance of the signal itself. This is the primary motivation for the differential quantizer design. The differential quantizer structure is shown in Fig. 1.

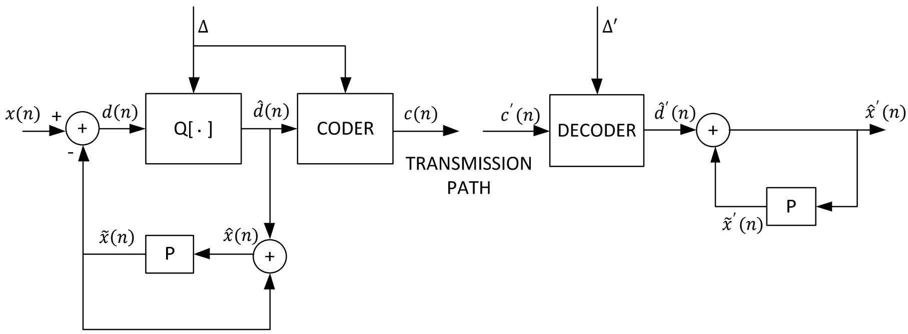


Fig. 1 – *Structure of the differential quantizer.*

The quantizer input signal in Fig. 1 is:

$$d(n) = x(n) - \tilde{x}(n), \tag{1}$$

which is the difference between the non-quantized speech signal $x(n)$ and the estimation (prediction) of that signal, denoted by $\tilde{x}(n)$. This prediction is the value of the predictor P output, whose input is the quantized value of speech signal, $\hat{x}(n)$. The difference signal can be also called the prediction error signal since that value represents the value of the error in the prediction of predictor compared to $x(n)$. The quantizer can be with fixed or adaptive structure, uniform or non-uniform. Regardless of that, its parameters should be set according to the variance of the signal $d(n)$. Quantized difference signal can be represented as:

$$\hat{d}(n) = d(n) + e(n), \tag{2}$$

where $e(n)$ is the quantization error. As depicted in Fig. 1 the quantized difference signal is $\hat{d}(n)$ is added to the estimation of the input signal $\tilde{x}(n)$, resulting in the output of the predictor P:

$$\hat{x}(n) = \tilde{x}(n) + \hat{d}(n). \tag{3}$$

Substituting (1) and (2) in (3) provides a following expression of the predictor P input as a function of the original signal $x(n)$ and the prediction error $e(n)$:

$$\hat{x}(n) = x(n) + e(n). \tag{4}$$

The obtained expression is the same as for the uniform quantizer, except it is derived by the quantization of the signal of difference $d(n)$. Therefore, if the achieved prediction is satisfactory, the variance of the signal $d(n)$ will be significantly lesser than the variance of the signal $x(n)$, leading to a greater SNR compared to the regular quantization procedure on the entire input signal. Further on, the quantized signal of difference is coded and ready for transmission and storage. The system for the reconstruction of the coded signal consists of the decoder and the predictor which can be the same as the predictor used before the encoder, but does not have to be the same in the general sense. If $\hat{c}(n) = c(n)$, than there were no errors in the transmission channel and $\hat{x}'(n)$ will be equal to $\hat{x}(n)$. By definition, the signal noise ratio is equal to:

$$SNR = \frac{E\{x^2(n)\}}{E\{e^2(n)\}} = \frac{\sigma_x^2}{\sigma_e^2}, \tag{5}$$

which can be written as:

$$SNR = \frac{\sigma_x^2}{\sigma_d^2}\frac{\sigma_d^2}{\sigma_e^2} = G_p SNR_Q, \tag{6}$$

where $SNR_Q(\text{dB}) = 10\log(\sigma_x^2/\sigma_d^2) = 6B + 4.77 - 20\log(d_{\max}/\sigma_d)$ is the signal noise ratio for the uniform quantizer, while the $G_p = \sigma_x^2/\sigma_d^2$ is the correction factor of the differential quantizer. SNR_Q depends on the quantizer which is used. If the value of G_p is greater than 1, the differential quantizer structure presented in Fig. 1 introduces an improvement in the quantization procedure. Clearly, the goal is to maximize the value of G_p using the appropriate predictor P. For the given signal, σ_x^2 is constant and the value of G_p can be maximized by the minimization of the denominator, or the minimization of σ_d^2.

The next step is to determine the predictor P and its nature. Commonly used predictor is the linear predictor where $\tilde{x}(n)$ is the linear combination of the delayed samples of $\hat{x}(n)$:

$$\tilde{x}(n)=\sum_{k=1}^{p}\alpha_k \hat{x}(n-k). \tag{7}$$

The variance of the prediction error from Fig. 1 is:

$$\begin{aligned}\sigma_d^2 &= E\left[d^2(n)\right]=E\left[\left(x(n)-\tilde{x}(n)\right)^2\right]\\ &= E\left[\left(x(n)-\sum_{k=1}^{p}\alpha_k \hat{x}(n-k)^2\right)\right].\end{aligned} \tag{8}$$

Therefore, let us consider the minimization of the criterion $J=\sigma_d^2$, by calculating the partial derivatives over the coefficients α_j:

$$\frac{\partial \sigma_d^2}{\partial \alpha_j}=0, \quad 1\le j\le p. \tag{9}$$

Using the relations $E\left[x(n)x(n-j)\right]=\Phi(j)$, where $\Phi(j)$ is the autocorrelation function of signal $x(n)$ and $E\left[e(n-j)e(n-k)\right]=\sigma_e^2\delta(j-k)$ and the assumption that the quantization error $e(n)$ and signal $x(n)$ are not correlated, autocorrelation function can be rewritten in the following form:

$$\Phi(j)=\sum_{k=1}^{p}\alpha_k\Phi(j-k)+\sum_{k=1}^{p}\alpha_k\sigma_e^2\delta_{kj}, \quad 1\le j\le p. \tag{10}$$

Dividing of equation (10) with σ_x^2 and defining the correlation coefficient as:

$$\rho(k)=\frac{\Phi(k)}{\sigma_x^2}, \tag{11}$$

the following matrix form of the system of p equations can be derived:

$$\begin{bmatrix}\rho(1)\\ \rho(2)\\ \vdots\\ \rho(n)\end{bmatrix}=\begin{bmatrix}1+\frac{1}{SNR} & \rho(1) & \cdots & \rho(p-1)\\ \rho(1) & 1+\frac{1}{SNR} & \cdots & \rho(p-2)\\ \vdots & \vdots & \ddots & \vdots\\ \rho(p-1) & \rho(p-2) & \cdots & 1+\frac{1}{SNR}\end{bmatrix}\cdot\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_p\end{bmatrix}. \tag{12}$$

The solution for the coefficients α_i, $1\le i\le p$, can be obtained from the matrix equation:

$$\alpha = C^{-1}\rho. \tag{13}$$

On the other hand, σ_d^2 can be derived as:

$$\begin{aligned}
\sigma_d^2 &= \mathrm{var}\left[d(n)\right] = E\left[d^2(n)\right] = E\left[\left(x(n) - \tilde{x}(n)\right)\right] \\
&= E\left[\left(x(n) - \tilde{x}(n)\right)x(n)\right] - E\left[\tilde{x}(n)\left(x(n) - \tilde{x}(n)\right)\right] \\
&= E\left[x^2(n)\right] - E\left[\sum_{k=1}^{p} \alpha_k \hat{x}(n-k)x(n)\right] \\
&= \sigma_x^2 - E\left[\sum_{k=1}^{p} \alpha_k \left(x(n-k) + e(n-k)\right)x(n)\right] \\
&= \sigma_x^2 - \sum_{k=1}^{p} \alpha_k \Phi(k) - \sum_{k=1}^{p} \alpha_k E\left[x(n)e(n-k)\right] \\
&= \sigma_d^2 = \sigma_x^2 \left(1 - \sum_{k=1}^{p} \alpha_k \rho(k)\right).
\end{aligned} \tag{14}$$

Next, we can write:

$$G_p = \frac{\sigma_x^2}{\sigma_d^2} = \frac{1}{1 - \sum_{k=1}^{p} \alpha_k \rho(k)}. \tag{15}$$

The signal noise ratio is:

$$SNR = \frac{SNR_Q}{1 - \sum_{k=1}^{p} \alpha_k \rho(k)}. \tag{16}$$

Therefore, there are $p+1$ equations and $p+1$ unknown parameters given by (13) and (16). For example, for the first order predictor ($p=1$) the problem is reduced to:

$$SNR = \frac{SNR_Q}{1 - \alpha_1 \rho(1)}, \tag{17}$$

$$\rho(1) = \left(1 + \frac{1}{SNR}\right)\alpha(1), \tag{18}$$

and the solution leads to the parameter $\alpha(1)$.

The aforementioned facts lead to a conclusion that the differential quantizer increases the SNR in comparison to the regular quantizer where the whole signal is introduced directly to the quantizer. The improvement depends on the correlation of the speech signal. Here we come to the essence and importance of this paper. The question is what will happen if instead of the signal $x(n)$ we bring the noisy speech signal $x_n(n)$? The answer lies in the fact that the aforementioned equations will mostly remain the same, as one of the coefficients of the predictors will be counted through the noisy speech signal, as well as the signal errors, as whereas the amplitude G_p will be counted by a formula for the noiseless speech signal, so we will inevitably come to a certain loss of quality in dependence of the intensity of the sound in which it affects the speech signal, but also depending on the nature of the sound itself and its distribution. In Fig. 2 the structure of the differential quantizer is shown. Finally, linear predictor cannot be the best solution for all speech signals. Therefore, there are many approaches to the adaptive differential quantizers which provide better results, but that is beyond the scope of this paper and will not be considered here. The idea for further analysis and results may be the following: to build a system which will be able to recognize the nature of the noise and in accordance to it adjust the predictor coefficients.

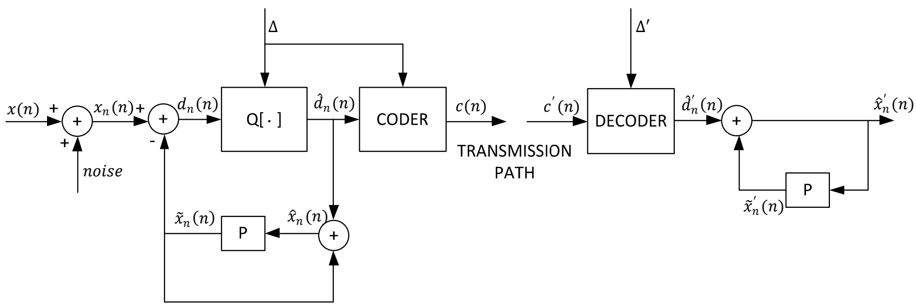


Fig. 2 – *Structure of the differential quantizer used in this paper.*

3 Experimental Results

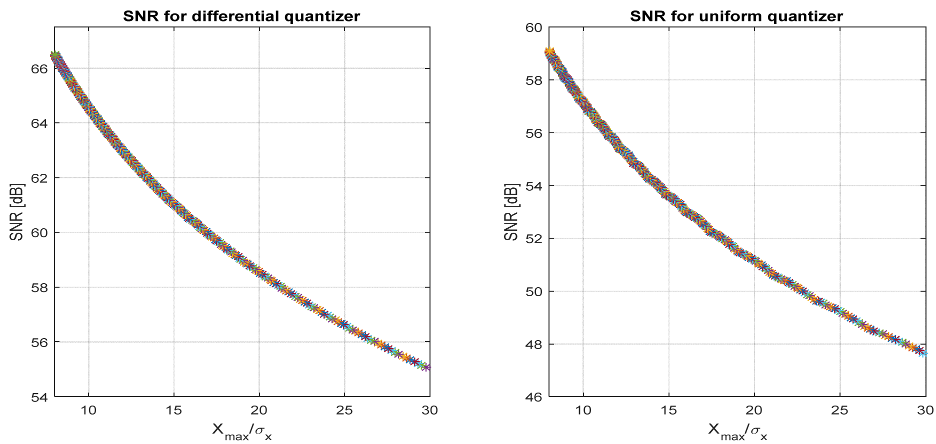
The design and analysis of the differential quantizer was performed in the programming software MATLAB 8.6.0 (R2015b).

The first phase, before the design of the differential quantizer itself, a sequence was recorded with sampling period $f_s = 10000\,\text{Hz}$ and duration of one minute, so that sequence had 600000 samples of speech signal, which contained a male and female voice. The sequence was recorded in Serbian language. The design was carried out with a 12-bit uniform quantizer in all steps of the analysis. The analysis included predictors of the first, second and third order.

The results concerning the signal noise ratio and the corrective factor are provided in dB.

Fig. 3 shows the comparison of signal noise ratio for a differential 12-bit quantizer and a uniform 12-bit quantizer depending on the parameter which refers to the volume of the speech signal. In this picture we only show signal noise ratio for the predictor of the first order for transparency reasons, and for the predictors of the second and third order there is a increase in numbers; a 6dB in regards to every lower row. The results indicate clear improvement of SNR in the case of differential quantizer. This confirms theoretical assumptions and results, described in the previous section. **Table 1** shows the signal noise ratio in decibels for a differential quantizer with the first, second and third order predictor, using a 12-bit uniform quantizer, and using pure speech signal.

What is left to determine is to what extent does the distortion on the system (in this case the noise) affects the quality of the speech signal, as well as the extent in which it diminishes the quality of the transfer based on the uniform distribution or the gaussian distribution of noise. Both of the analyses are shown in two tables, so that there is a systematic way of how the two different distributions of noise affect the quality of quantization, as well as the differential quantizer, and also the intensity of the noise which affects the factor of gain G_p of differential quantizer.



(a) (b)

Fig. 3 – (a) *SNR of the differential quantizer with the first order predictor and a 12-bit uniform quantizer;* (b) *SNR of 12-bit uniform quantizer.*

For the purpose of illustrating the effect of the measurable additive noise to the quality of performance of the differential quantizer, a large number of experiments have been performed and the received results have been shown in the Fig. 4 and 5. In Fig. 4 the way of the diminishing corrective factor G_p is presented with a increasing variance of the additive noise, and in the case that this noise is with a uniform distribution with zero expectation. The results given in Fig. 5 are gained in a similar way, with the difference that the additive noise is with a normal distribution. The results obtained display a common result that the corrective factor substantially diminishes with the variance noise increase, however, there are several effects which are worth mentioning. The first effect is that the predictors of the first order show much weaker results than predictors of the second and third order for small or large values of noise variance, however, for some middle values of the noise variance this difference becomes substantially smaller. On the other hand, the difference between the predictors of the second and third order is mostly unnoticeable unless the variance of noise becomes so strong and then the effect of the order clearly shows. Also, it is important to mention that the working quality of the differential quantizer is more insensible in the case of the uniform distribution of noise measure. From the diagrams shown we can tell that in the case of the uniform noise distribution, the corrective factor vanishes, or becomes closer to zero noise variance of value round 0.5, while in the case of the normal distribution the corrective factor becomes unnoticeable at a variance close to the value of 0.15.

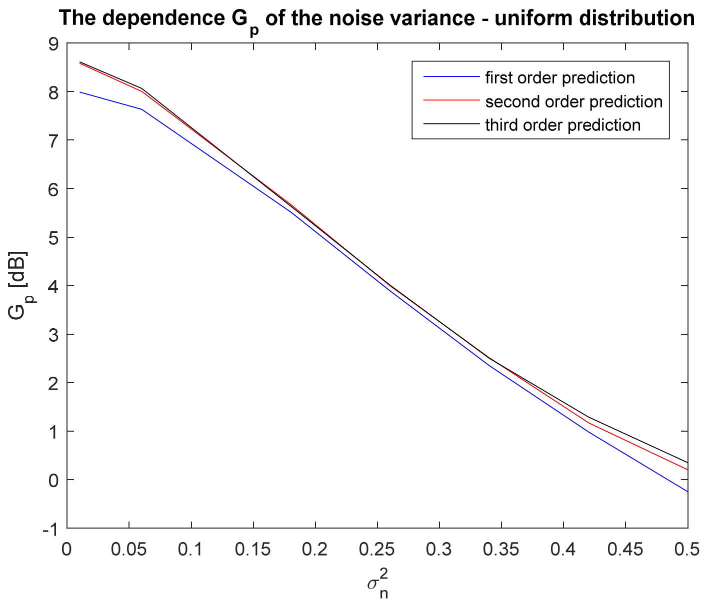


Fig. 4 – *The dependence correlation factor of the noise variance with uniform distribution, with the first, second and third order predictor and a 12-bit uniform quantizer.*

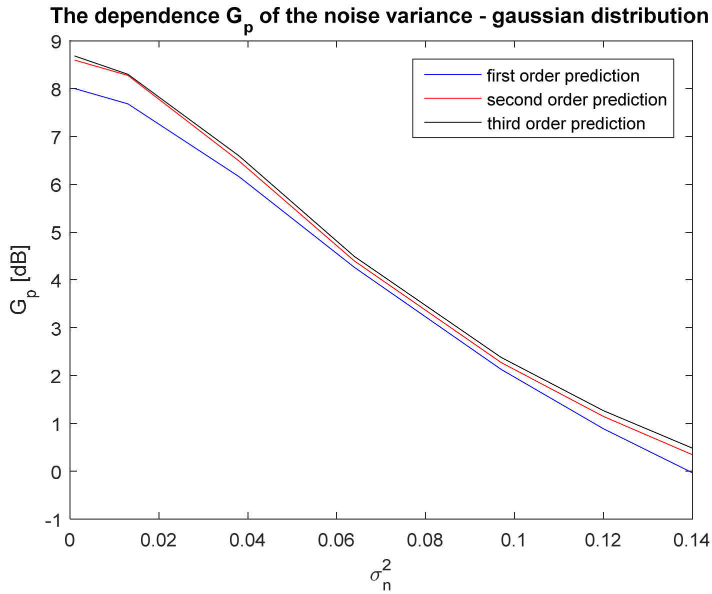


Fig. 5 – *The dependence correlation factor of the noise variance with gaussian distribution, with the first, second and third order predictor and a 12-bit uniform quantizer.*

Table 1

SNR in decibels for the differential quantizer with predictors of different order, 12-bit uniform quantizer.

Predictor order	$p=1$	$p=2$	$p=3$
Uniform quantizer	64.0496	64.0496	64.0496
Differential quantizer	72.5475	73.6288	73.6561
Correction factor G_p	8.4979	9.5792	9.6065

Table 2

SNR in decibels for the differential quantizer with predictors of different order, 12-bit uniform quantizer, uniform distribution of noise with zero expectation and variance 0.281.

Predictor order	$p=1$	$p=2$	$p=3$
Uniform quantizer	64.9059	64.8910	64.8931
Differential quantizer	68.3761	68.4048	68.6142
Correction factor G_p	3.4702	3.5138	3.7211

Table 3
SNR in decibels for the differential quantizer with predictors of different order, 12-bit uniform quantizer, uniform distribution of noise with zero expectation and variance 0.481.

Predictor order	$p = 1$	$p = 2$	$p = 3$
Uniform quantizer	64.9641	64.9648	64.9563
Differential quantizer	65.0068	65.3643	65.4886
Correction factor G_p	0.0426	0.3996	0.5323

Table 4
SNR in decibels for the differential quantizer with predictors of different order, 12-bit uniform quantizer, gaussian distribution of noise with variance 0.079.

Predictor order	$p = 1$	$p = 2$	$p = 3$
Uniform quantizer	64.9138	64.8923	64.8889
Differential quantizer	68.0609	68.2228	68.3326
Correction factor G_p	3.1471	3.3305	3.4437

Table 5
SNR in decibels for the differential quantizer with predictors of different order, 12-bit uniform quantizer, gaussian distribution of noise with variance 0.139.

Predictor order	$p = 1$	$p = 2$	$p = 3$
Uniform quantizer	64.9632	64.9650	64.9667
Differential quantizer	64.9902	65.3524	65.4968
Correction factor G_p	0.0270	0.3874	0.5301

Table 2 shows the signal noise ratio in decibels for a differential quantizer with the first, second and third order predictor, using a 12-bit uniform quantizer, whereas the distortion on the system has a uniform distribution with variance 0.281. In **Table 3** the same parameters are shown, only the intensity of the noise is much larger (almost two times – 0.481). Similarly, **Table 4** provides results obtained from the same quantizer, only the distortion has a gaussian distribution with variance 0.079. **Table 5** contains the same parameters as **Table 4**, only the noise intensity is much larger (almost two times – 0.139).

4 Conclusion

The presented analysis can lead to the following conclusions: 1) differential quantizer does indeed increase the signal noise ratio compared to the uniform quantizer which can be seen in Fig. 3; 2) the correction factor for each sequence was greater than 6 dB, which can be interpreted as one additional bit compared to the uniform quantizer or constant SNR with one bit less compared to the uniform quantizer. However, the cost of improved quantization quality is the increase in complexity of the system and quantization procedure. Therefore, the correct choice of parameters is conditioned by a compromise between the quality and complexity. Additionally, **Tables 2**, **3**, **4** and **5** show in which way the insertion of distortion to the system for speech regulation affects the diminishing signal noise ratio and what is the limit of intensity and strength of noise when the use of the differential quantizer is no longer the preferred action. On the other hand, because of the nature of the speech signal, which is a stochastic process, we cannot claim that the limits are always the way they are presented in this paper, but what is important is the result that shows the declining quality quantization trend depending on the type of distortion that is attacking the system. In theory, the results have been confirmed that the noise with the gauss distribution diminishes the signal to noise ratio more than the noise with the uniform distribution; for the reason that it directly affects the choice of the parameters of predictors that are acquired based on the correlation coefficients.

And there will be an additional examination on the influence of the noise on the quantization, e.g. a larger number of speech sequences. Furthermore, what needs to be done is to make an algorithm which will be able to recognize the noise that attacks and based on that information it will adjust its coefficients of predictors, which will contribute that at any moment the factor of repair G_p is at a high level.

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