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Measurement of Definite Integral of Sinusoidal Signal Absolute Value Third Power Using Digital Stochastic Method

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Abstract: In this paper a special case of digital stochastic measurement of the third power of definite integral of sinusoidal signal's absolute value, using 2-bit AD converters is presented. This case of digital stochastic method had emerged from the need to measure power and energy of the wind. Power and energy are proportional to the third power of wind speed. Anemometer output signal is sinusoidal. Therefore an integral of the third power of sinusoidal signal is zero. Two approaches are proposed for the third power calculation of the wind speed signal. One approach is to use absolute value of sinusoidal signal (before AD conversion) for which there is no need of multiplier hardware change. The second approach requires small multiplier hardware change, but input signal remains unchanged. For the second approach proposed minimal hardware change was made to calculate absolute value of the result after AD conversion. Simulations have confirmed theoretical analysis. Expected precision of wind energy measurement of proposed device is better than 0,00051% of full scale.

Keywords: AD conversion, Digital measurements, Electrical measurements, Probability, Stochastic processes.

1 Introduction

Wind is a vector quantity – to describe it, it is necessary to conduct detailed measurements of the wind speed (intensity), its line and direction. Typically, the measurement results are shown as a wind rose – Fig. 1.

For the accurate assessment of a location, for the construction of wind farms, it is necessary to perform measurements. The most important factor to consider before deciding on location of wind energy plant is the wind speed. The measurements are performed for a minimum period of one year in order to cover all seasons and get relevant information. In order for a location to be considered, the lowest annual wind speed should be higher than 5 m/s. Wind

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farm is a large investment, therefore it is of great importance to perform comprehensive measurements and choose the optimum location. Wrong choice of location is a mistake that cannot be corrected later. For example, price of wind turbines with installed power of 1 MW amounts to over one million euros. To obtain a complete picture of the wind resources at a desired micro-location we must set up monitoring devices for wind characteristics tracking.

Most widely used measuring instrument in meteorology for measuring the wind speed is an anemometer. The cup anemometers generate sinusoidal signal at their output, where amplitude and frequency are proportional to the wind speed. It is well known [1] that wind power (1) and energy are proportional to the third power of wind speed:

$$P = \frac{1}{2}\rho A v^3 . \tag{1}$$

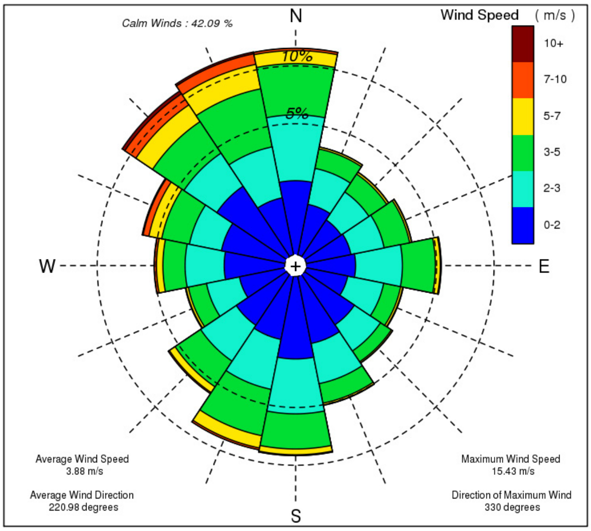

Fig. 1 – *Wind rose for Jan. 1, 2011 to Dec. 31, 2015.*

Where: P is power (W), ρ is density of air (kg/m^3), A is a cross-section area perpendicular to the wind that wind is passing through the windmill (m^2), and v is wind speed (m/s). German physicist Albert Betz concluded in 1919 that no wind turbine can convert more than 16/27 (theoretical maximum power efficiency $C_{P_{\max}}$ of any design of wind turbine is 59.3%) of kinetic energy of wind into mechanical energy turning a rotor. To this day, this is known as the Betz Limit or Betz's Law. Wind turbines cannot operate at this maximum limit. The real world limit is well below the Betz Limit with values of 35–45% common even in the best designed wind turbines. By the time we take into account other losses in a complete wind turbine system - e.g. gearbox, bearings, generator and so on - only 10–30% of the power of wind is ever actually converted into usable electricity. Hence, the power coefficient C_P needs to be factored in (1) and the extractable power from the wind is:

$$P = \frac{1}{2}\rho A v^3 C_p . \tag{2}$$

Fig. 2 – *Block diagram of application of a uniform random dither h to the measured signal y.*

2 Principle of Digital Stochastic Measurement Using 2-bit Flash AD Converter

Sampling measurement method has been the backbone of measurement evolution and is a standard in metrology, control, telecommunications, etc. Time-continuous signals are sampled (at discrete time instants) and converted into discrete digital values in AD converters. In the conversion process, accuracy and speed are opposing requirements.

Accurate measurements of low, noisy and distorted signals has been a challenging problem in theory and practice of measurement science and technology. Since 1956 [2], the possibility of reliable operation of instruments with inherent random error has been researched. It has been shown that adding a random uniform dither to the input signal prior to quantization decouples quantization error from the input signal [3 – 5]. Fig. 2 shows the principle of adding a uniform random dither signal h to the measured signal y.

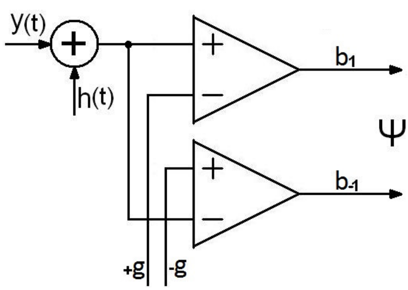

Fig. 3 – *Two-bit flash ADC with the minimal hardware structure.*

High speed of all electronic circuits implies the viability of application of the Central limit theorem for practical measurements. Digital stochastic measurement, formulated in [6], enables very simple hardware to be used and easy parallel processing (practically without additional delays in signal processing).

Let us assume that set of the following conditions are met:

$$|y| \le R,\ R = Za,\ |h| \le \frac{a}{2},\ |y+h| \le R + \frac{a}{2}, \tag{3}$$

where R is the range of ADC, Z is a number of quantum levels, and a is a quantum of ADC uniform quantizer. For $Z = 3$, 2-bit flash ADC is extremely simple, with minimal hardware structure shown in Fig. 3. Practically, ADC consists of two comparators that have decision levels
(threshold voltages) $-g$ and $+g$.

A probability density function (PDF) of uniform random dither signal h is $p(h) = \frac{1}{a}$ for $|h| \le \frac{a}{2}$. Voltage ranges and decision thresholds associated with mean input signal measuring process using uniform quantizer are presented in Fig. 4.

Ideal uniform quantizer characteristic of this 2-bit flash ADC is shown in Fig. 5. For this kind of 2-bit flash ADC, when $a = 2g$, (3) becomes (4):

$$|y| \le 2g\ \ R = 2g,\ |h| \le g,\ |y+h| \le 3g, \tag{4}$$

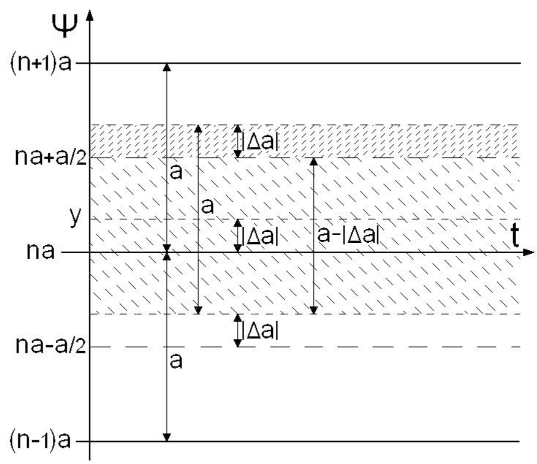

Fig. 4 – *Principle of stochastic measurement of DC signal y.*

Possible values of Ψ are $\Psi = \{-2g, 0, +2g\}$, and analytical expression for Ψ is:

$$\Psi = 2g(b_1 - b_{-1}), \tag{5}$$

where $b_1, b_{-1} \in \{0,1\}$ and $b_1 b_{-1} = 0$. It is not possible for b_1 and b_{-1} to be 1 simultaneously - it would mean that $y < 0$ and $y > 0$ are simultaneously.

Device from Fig. 6 have two 2-bit flash ADC from Fig. 3. If, during one measurement interval, *N* conversions of ADC are performed by each ADC, then accumulator from Fig. 6 accumulates the sum of *N* subsequent multiplier outputs: $\sum_{i=0}^{N} \Psi_1(i)\Psi_2(i)$. This accumulation can be used for simple calculation of the mean of a multiplier output $\overline{\Psi}$ over the measurement interval as:

$$\overline{\Psi} = \frac{1}{N}\sum_{i=1}^{N}\left[\Psi_1(i)\Psi_2(i)\right]. \tag{6}$$

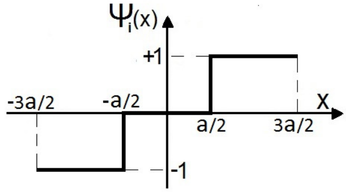

Fig. 5 – *Ideal quantizer characteristic of 2-bit flash ADC implemented using two comparators.*

The mean of the multiplier output $\overline{\Psi}$ (when sampling frequency is infinite) is mean of products of y_1 and y_2:

$$\begin{aligned}\overline{\Psi} &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \mathrm{d}t \int_{-2g}^{+2g} \delta[y_1 - f_1(t)] y_1 \,\mathrm{d}y_1 \int_{-2g}^{+2g} \delta[y_2 - f_2(t)] y_2 \,\mathrm{d}y_2 = \\ &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f_1(t) f_2(t)\,\mathrm{d}t = \overline{y_1 y_2}.\end{aligned} \tag{7}$$

Variance of the measurement error *e* for product of signals y_1 and y_2 is:

$$\sigma_e^2 = \frac{(2g)^2}{t_2 - t_1}\int_{t_1}^{t_2} \left| f_1(t)\, f_2(t) \right| \mathrm{d}t - \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f_1^2(t) f_2^2(t)\,\mathrm{d}t, \tag{8}$$

which enables us to see how the error varies when the power of signal is measured using a device from Fig. 6.

The necessary condition for validity of the Central limit theorem is that any third moment, including the central third moment, is limited [7]:

$$M_3 \leq 8(2g)^6 . \tag{9}$$

As a consequence of (9) and the Central limit theorem, statistical sampling theory can be applied to the error e. Therefore, following estimation can be made:

$$\sigma_{\bar{e}}^2 = \frac{\sigma_e^2}{N} , \tag{10}$$

where N is a finite number of samples within the time interval $T = (t_2 - t_1)$. Estimation of (10) is correct if discrete sets of samples $\Psi_1 \in \{\Psi_1(1), \Psi_1(2)....\Psi_1(N)\}$ can represent function $y_1 = f_1(t)$ and $\Psi_2 \in \{\Psi_2(1), \Psi_2(2)....\Psi_2(N)\}$ can represent function $y_2 = f_2(t)$, which means that the Nyquist sampling criterion, regarding a uniform sampling of the signals y_1 and y_2, is satisfied. In [7] it is not only shown that standard measurement uncertainty of type A can be defined as $\sigma_{\bar{e}}$, but it is also shown how this parameter can be determined on-line.

Expressions (6), (8) and (10) can be generalized for a product of S input signals. Then a device from Fig. 6 can be improved to accommodate S inputs $(y_i = f_i(t),\ i = 1, 2, ..., S)$, and 2-input multiplier can be replaced with S-input multiplier, thus being adjusted for measurement of the mean of multiplier output as follows:

$$\overline{\Psi} = \frac{1}{N} \sum_{i=1}^{N} \left[\Psi_1(i) \Psi_2(i) \dots \Psi_S(i) \right] . \tag{11}$$

In that case, the mean of the multiplier's output $\overline{\Psi}$ is given as:

$$\overline{\Psi} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_1(t) f_2(t) \cdots f_S(t) \, \mathrm{d}t , \tag{12}$$

and measurement error variance is:

$$\sigma_e^2 = \frac{(2g)^S}{t_2 - t_1} \int_{t_1}^{t_2} \left| f_1(t) f_2(t) \cdots f_S(t) \right| \mathrm{d}t - \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_1^2(t) f_2^2(t) \cdots f_S^{\,2}(t) \, \mathrm{d}t . \tag{13}$$

Of course, a standard measurement uncertainty of type A can also be calculated using (10).

In articles [8 – 12] theoretical aspect of error was considered, at the same time the theory was proven using simulations. Devices that were developed and their practical use has proven the efficiency of this method. We speak about measurements on the grid where the most common measurements are of RMS values of voltage and current, as well as measurements of power and energy.

3 Application of Digital Stochastic Measurement in Wind Speed Measurement

To measure wind power and energy, input signal (of the wind speed) must be raised to the third power. For this purpose, to device shown in Fig. 6, one more AD converter together with uncorrelated dither signal associated with – Fig. 7 was added. Outputs of all three AD converters are connected to the multiplier input.

Direct application of the device from Fig. 7, where $y_1(t) = y_2(t) = y_3(t) = f(t)$, (we connected the same input signal to all three AD converter inputs) calculates the third power of the input signal. Since $f(t)$ is a sinusoidal signal, the device at its output (as a result) generates value close to 0 (sinusoidal signal third power mean value), which is a direct application of (12) for $S = 3$. This result is independent of the wind speed. To be able to use this device, for measuring sinusoidal signal third power, it is necessary to change the input signal or to modify device itself. Both of these solutions are proposed.

The first solution is to use sinusoidal signal absolute value as an input - namely to modify it into full-wave rectified sinusoidal signal. In this case absolute value is calculated before AD conversion.

The second solution would be to leave the input signal unchanged, but requires a small change of device's hardware itself. This change can be done by modifying multiplier output to convert result -1 into 1. It is easily done by adding single OR gate on multiplier output. This way accumulator is not up-down counter any more, but becomes up counter.

In both cases mean signal absolute value is calculated and it depends on the wind speed.

Theoretical mean absolute value of the third power of sinusoidal signal is:

$$\frac{1}{T}\int_0^T [|U_m \sin(\omega t)|^3 \, \mathrm{d}t] = \ldots = \frac{4}{3\pi} U_m^3 . \tag{14}$$

Because of the fact that wind speed is extremely slow changing quantity (compared to electronics), we can significantly simplify device from Fig. 7. This simplification is shown in Fig. 8.

Anemometer A generates voltage u at its output. That voltage u is proportional to the wind speed v.

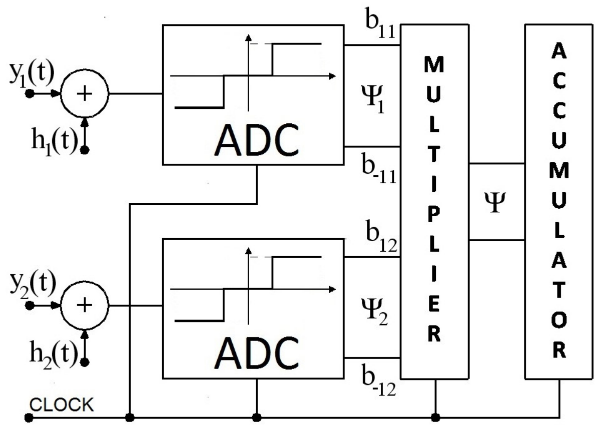

Fig. 6 – *Structure of a device, based on two 2-bit flash ADC, for measurement of definite integral product of two signals.*

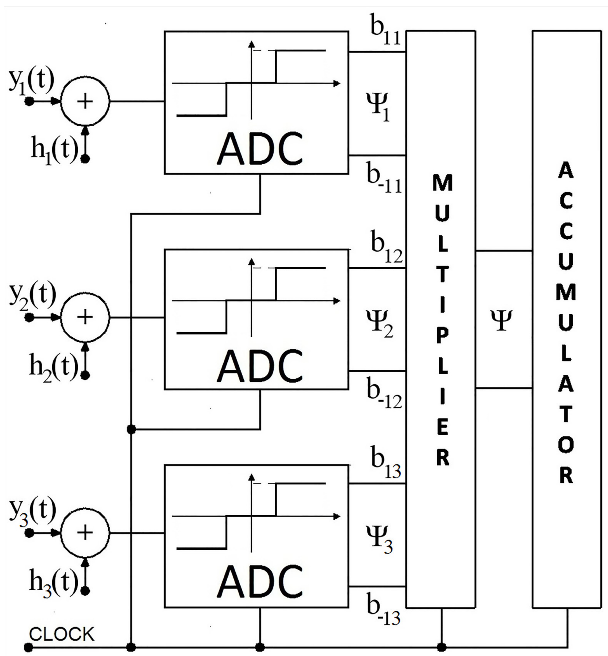

Fig. 7 – *Structure of a device, based on three 2-bit flash ADC, for the measurement of product of three signals definite integral.*

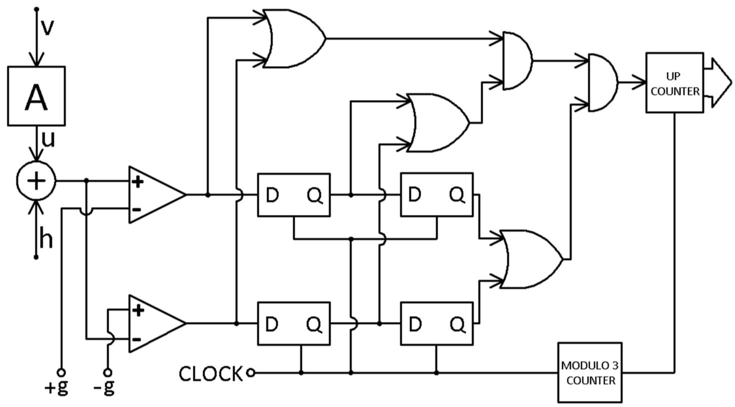

Fig. 8 – *Block diagram of a device for the measuring wind power and energy.*

To use only one dither signal h for calculating the third power, we must prove that there's no correlation between factors. This is achieved using three consecutive samples in every product. This is possible due to oversampling, because between 3 consecutive signal samples there's practically no difference (signals from anemometer are slow changing so we can assume that samples have equal values). Results of multiplying obtained this way (wind speed third power) are sent to the accumulator (up counter).

Two D flip-flops are used for each bit to remember previous two samples. They allow us to multiply current sample and two previous ones to form every product, i. e. third power of the signal current value.

Instead of multipliers used in a device in Fig. 7, Fig. 8 shows a simplified device that uses logic AND gates. If anyone of three samples is zero (both bits are logic FALSE), it means that the result of the corresponding AD conversion was zero. In this case there is no increase in the value of up counter.

Only in the case when we have 3 consecutive samples with AD conversion value other than 0 (-1 or 1), up counter will increase its value. That means that this combination of logic gates and D flip-flops form absolute value of the product for 3 consecutive dithered samples from the input signal.

4 Comparison of Theoretical Error and Error Obtained by Simulations

Theoretical value of the measurement error is calculated according to the (13), considering $S = 3$ and (10). Device from Fig. 8 measures the third power of the input signal absolute value. If $u(t) = U_m \sin \omega t$ then theoretical measurement error variance is:

$$\sigma_{e_T}^2 = \frac{2}{N}\left[(2g)^3 \int_0^{\frac{T}{2}} \left| U_m \sin \omega t \right|^3 \mathrm{d}t - \int_0^{\frac{T}{2}} \left| U_m \sin \omega t \right|^6 \mathrm{d}t \right]. \quad (15)$$

Even though there's no practical application, due to theoretical importance, simulation was carried out with sinusoidal signal which were connected to the inputs of all three AD converters. AD converter outputs (–1, 0, 1) were fed to the multiplier input, whose output was connected to accumulator. Multiplier can also have three values, accumulator can decrease by 1 or increase by 1 its value, or remain the same. Simulations were performed for input signals of amplitude from 0.2 V to 5.0 V with step of 0.2 V, sampling frequency was 100 kHz and duration of each measurement was 1 s. Each measurement was repeated 300 times. The results, of simulations, were close to zero, and they were statistically processed.

For practical application more important is second group of simulations. This group of simulations were performed for the same range of input signal amplitudes, with the same sampling frequency and duration of each individual measurement. Each measurement was repeated 300 times. Unlike the first group of simulations where multiplier's output has three (–1, 0, 1) possible values it was necessary for a small hardware change to have only two values (0, 1) at its output. This effect was achieved by adding single OR gate at multiplier output. This way the result of product with value –1 is now 1. That means that accumulator can only increase its value by 1 or remain the same.

Table 1
Measurement Error Standard Deviation.

% FS	TRUE VALUE	$\sigma_{\bar{e}_T}$	$\sigma_{\bar{e}_S}$	$\sigma_{\bar{e}_S}$-$\sigma_{\bar{e}_T}$	$\lvert\sigma_{\bar{e}_S}$-$\sigma_{\bar{e}_T}\rvert$	$\Delta\sigma_{\bar{e}}$
10	0.05	0.0081	0.0085	0.0003	0.0003	0.0009
20	0.42	0.0230	0.0233	0.0003	0.0003	0.0027
30	1.43	0.0419	0.0436	0.0017	0.0017	0.0048
40	3.40	0.0636	0.0659	0.0023	0.0023	0.0073
50	6.63	0.0868	0.0879	0.0011	0.0011	0.0100
60	11.46	0.1098	0.1022	-0.0076	0.0076	0.0127
70	18.20	0.1304	0.1301	-0.0003	0.0003	0.0151
80	27.16	0.1454	0.1426	-0.0028	0.0028	0.0168
90	38.67	0.1496	0.1492	-0.0005	0.0005	0.0173
100	53.05	0.1322	0.1382	0.0059	0.0059	0.0153

Duration of measurement was 1 s and sampling frequency was $f_c = 100\,\text{kHz}$. These results are presented in **Table 1**. In the same table, we also presented the value of error variance of wind power, for both simulations, of 300 repeated measurements. As we can see we have good match between the theoretical and (fist and second) simulation results of standard deviation.

Fig. 9 shows dependence of measurement error standard deviation in relation to percentage of full scale (FS):

- For the measurement error theoretical value of definite integral third power sinusoidal signal,
- For the measurement error simulation of definite integral third power sinusoidal signal and
- For the measurement error simulation of definite integral third power sinusoidal signal with calculated absolute value after AD conversion.

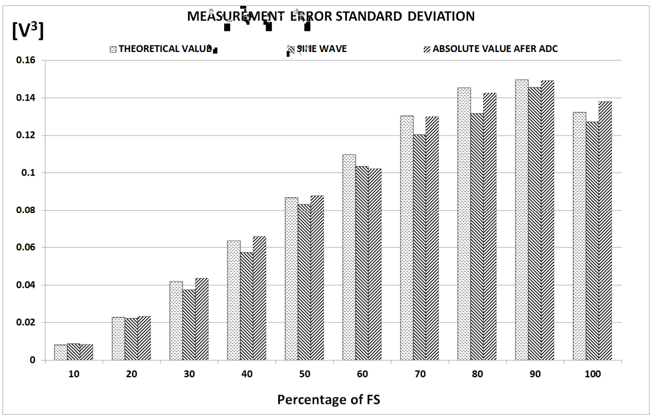

Fig. 9 – *Measurement error standard deviation in relation to FS percentage for the sinusoidal signal third power.*

We observed standard deviation theoretical value and standard deviation obtained by both groups of simulation. The results were consistent with expectations. Deviations from ideal (theoretical) curve for simulation results were minimal because simulation used built-in function to generate random number and threshold levels were constant.

If we were to conduct the experiment, higher deviations of experimentally obtained results, from ideal (theoretical) curve, should be expected due to non-ideal random number generator, voltage instability of threshold levels associated with practical device and other influences associated with real word measurements.

Let us now see how we can calculate the energy of the wind. If we assume that:

$$E = \overline{P}T \tag{16}$$

where E stands for energy, $\overline{P}$ for average power of measurements in time interval T, we can calculate the differential of energy as:

$$\mathrm{d}E = \mathrm{d}\overline{P}T + \overline{P}\mathrm{d}T, \text{ i.e.: } \frac{\mathrm{d}E}{E} = \frac{\mathrm{d}\overline{P}}{\overline{P}} + \frac{\mathrm{d}T}{T}, \tag{17}$$

where $\mathrm{d}T/T$ can be neglected because even at time interval of 1s relative precision of time measurement drops below $1 \cdot 10^{-6}$.

Therefore it can be concluded that relative precision (in this case, also accuracy, we assumed ideal components of the device) is the same for energy and average power.

By increasing the measurement interval from 1 s to one day, i.e. 86400 s (when measuring the wind power and energy, it is necessary to measure for at least a year), accuracy increases approximately 294 times $\left(\sqrt{86400\,\mathrm{s}/1\,\mathrm{s}}\right)$ and amounts to 0.00051% FS i.e. $(0.1492/294)\%\,\mathrm{FS}$.

By increasing the sampling frequency from 100 kHz to 12 MHz, accuracy increases about 11 times $\left(\sqrt{12\ \mathrm{MHz}/100\ \mathrm{kHz}}\right)$ and amounts to 0.000046% FS or below one ppm of FS.

5 Discussion

During analysis we were guided by the fact that theoretical error is known value defined by (10). We conducted simulation with 300 measurements, each lasting 1 s. By checking if theoretical results and results of simulation for wind power measuring are within acceptable limits, we came to conclusion that simulation error deviation from its theoretical value inside acceptable limits (**Table 1**). If we observe how wind energy is measured and how its relative precision behaves we came to conclusion that for long enough measurement period (longer period, greater precision) the precision of measuring wind energy is equal to the precision of measuring mean wind power. Therefore, with device from Fig. 8 we can measure wind power as well as its energy. Measuring device is very simple, as it can be seen in Fig. 8, so it has very small number of sources of systematic errors, which can easily be kept under control. On the other hand, its modern high-speed electric circuits make it very precise – in time interval of one day it can reach the accuracy of approximately 5 ppm of FS.

The only quantity that changes is the wind speed. It is measured and its third power integral calculated. It is shown that measurement precision is proportional to the square root of sampling frequency. The accurate values of wind power and wind energy are obtained by multiplying with constants. This is a task for the simplest supporting microprocessor. It needs to support commands for instrument operation, communication with other devices and display.

6 Conclusion

This paper arose from the need to measure wind power and energy which are proportional to the third power of wind speed. This paper elaborates developed theory of accurate and precise measurements of definite integral of sinusoidal signal absolute value third power using 2-bit flash ADC. Proposed solutions are in domain of hypothesis and have yet to prove themselves. In developed theory, no assumptions were made regarding waveform of the measured signals. Signal waveform directly affects both the measured quantity and standard deviation of measurement error. Consequently, if the waveform of measured signal is unknown, standard deviation of its measurement error cannot be calculated, but can be measured on-line. However, if the conditions of the sampling theorem are satisfied, then dithered 2-bit samples will contain sufficient information of the waveform of measured signal and it will be possible to accurately determine measurement uncertainty.

In discussed case waveform of the signal is known (sinusoidal) so theoretical measurement uncertainty can be calculated. Comparison between the theoretical and simulation results show good basis of developed theory in both considered cases. Therefore, it is realistic to expect, for measuring interval of one day, for above mentioned device, to achieve precision of 0.0005% FS, i.e., 5ppm FS.

7 Acknowledgement

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