# Shielded Coupled Multilayered Microstrip Lines Analysis using HBEM

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**Abstract:** Shielded symmetrical coupled multilayered microstrip lines analysis have been done using the hybrid boundary element method (HBEM), which is developed a few years ago at the Faculty of Electronic Engineering in Niš. The quasi-TEM approximation is applied. Influences of different parameters as well as dimensions of such microstrip lines on characteristic parameters distribution are investigated. The results are presented in graphs and tables. In order to verify the obtained results, some comparative results are shown. The authors found them to be in very good agreement with the HBEM results.

**Keywords:** Characteristic parameters, Finite element method, Hybrid boundary element method, Multilayered structures, Shielded coupled microstrip lines.

# **1** Introduction

Scientists and researchers in the world, during the past six decades, try to develop methods that produce enough accurate and precise results for microstrip lines characteristic parameters. Different configurations are subject of many papers. Also, different methods have been developed and applied. List of commonly used methods for microstrip lines analysis is very wide, but here it can be mention some of them such as the variational method [1], the boundary element method (BEM) [2, 3], the finite element method (FEM) [4, 5], the finite difference method (FDM) [6], the equivalent electrodes method (EEM) [7], etc.

Most of those methods, in different manners, calculate the capacitance per unit length of the analysed microstrip line. After that, it is possible to obtain the effective permittivity,  $\varepsilon_r^{eff}$ , and characteristic impedance,  $Z_c$ , using well known expressions, given in (1) and (2), respectively:

$$\varepsilon_{\rm r}^{\rm eff} = \frac{C'}{C_0'},\tag{1}$$

$$Z_{\rm c} = \frac{1}{c\sqrt{C'C_0'}} = \frac{Z_{\rm c0}}{\sqrt{\epsilon_{\rm r}^{\rm eff}}},$$
 (2)

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where  $c = 3 \cdot 10^8$  m/s is the speed of light and C' is the capacitance per unit length of the microstrip line.  $C'_0$  and  $Z_{c0}$  are the capacitance per unit length and characteristic impedance of the same microstrip line when the microstrip is airfilled (the dielectric layers are replaced by air), respectively.

The hybrid boundary element method (HBEM), described in detail in [8] and [9], is suitable and effective as other numerical methods for microwave transmission lines analysis.

The HBEM is based on the BEM, on the EEM, on the point-matching method (PMM) for the potential of the perfect electric conductor (PEC) electrodes and for the normal component of the electric field at the boundary surface between any two dielectric layers. Open structures, covered and coupled microstrip lines were until now researched using the HBEM, [8 - 14]. In this paper, the method is applied for characteristic parameters determination of shielded coupled multilayered microstrip lines. The calculation is based on a quasi-static TEM approach. This approach involves an evaluation of microstrip lines as transmission lines of parallel plates, which support a quasi TEM mode of propagation.

As basic components for directional couplers design, band-pass filters, delay lines, semiconductor devices, etc. are used coupled microstrip lines. The conductor metallization thickness has an influence on the microstrip lines characteristic parameters. It should be mention that this effect is more noticeable for the coupled microstrip lines then for the single. The PMM, used in [15], analyses the shielded microstrip lines with the finite metallization thickness. In [16] the authors state that the PMM gives acceptable results only if parameter t/w is smaller than 0.01. An extended point-matching method is developed in [16] to analyse microstrip lines with thick electrodes.

The HBEM results obtained for the shielded coupled microstrip line will be compared with those from [16] and [17]. Also, influences of different parameters on characteristic impedance and effective dielectric permittivity will be shown. In order to compare some of the HBEM results, the analysed structure will be also modelled using FEMM software [18]. Both modes ("even" and "odd") are supported and analysed.

# 2 Theoretical Background

A shielded multilayered microwave transmission line with an arbitrary cross-section is shown in Fig. 1. Dielectric layers are assumed to be lossless, isotropic and homogeneous.

The HBEM is very suitable to analyse microstrip lines of different configurations, with arbitrary number of conductors and dielectric layers as well as infinitesimally thin or finite metallization thickness. It can be also applied to analyse single and coupled transmission lines. It can be applied if a conductor touches a dielectric interface, straddles a dielectric interface or is a totally within one dielectric media as shown in Fig. 1.

In order to form an equivalent HBEM model, each surface of the conductors as well as the boundary surface between any two dielectric layers is divided into a large number of segments [9]. Each of those segments is replaced by equivalent electrodes (EEs), placed at their centres. The corresponding HBEM model for the system from Fig. 1 is shown in Fig. 2.

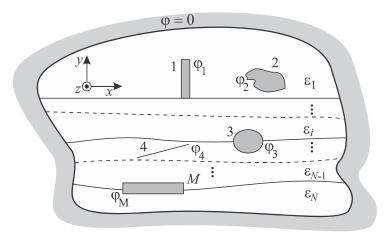


Fig. 1 – Shielded multilayered microwave transmission line.

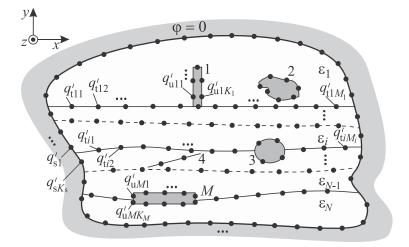


Fig. 2 – HBEM model.

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The segments at any boundary surfaces between two layers are replaced by discrete equivalent total charges placed in the air. In Fig. 2 those charges are denoted with  $q'_{tij}$ , where i = 1, ..., (N-1) and  $j = 1, ..., M_i$ . N is the number of dielectric layers with permittivity  $\varepsilon_n$  (n = 1, ..., N) and N-1 is the number of boundary surfaces between two dielectric layers. Index "t" denotes the total charges. Those charges are equal to the polarized charges, because the free charges at those surfaces do not exist. The free charges exit only at the PEC surfaces.

At the boundary surface between the PEC and the dielectric, the free and polarized charges exist. Their sum gives the total charges. But, the satisfying results are obtain using approximation that polarized charges at that boundary surface can be neglected and only free charges taken into account [8]. Those charges are placed in the corresponding dielectric layer and denoted in Fig. 2 with  $q'_{umk}$  and  $q'_{sl}$ , where m = 1, ..., M,  $k = 1, ..., K_m$  and  $l = 1, ..., K_s$ . M is the number of PECs. Indices "u" and "s" denote charges placed on the PECs and on the shield, respectively.

The type of equivalent electrodes depends on the problem geometry. When the system is plan parallel as those from Fig. 1, the EEs are thin cylindrical electrodes. In the case of 3D problems, spherical electrodes can be used as EEs. For solving 2D problems with axial symmetry, toroidal electrodes are used as EEs.

The potential of the equivalent electrodes placed at the PECs surface is the same as the potential of PECs themselves:

$$\phi_{mk} = \phi_m, \quad k = 1, ..., K_m, \quad m = 1, ..., M,$$
(3)

where  $\phi_{mk}$  is the potential of k -th EE at m -th PEC and  $\phi_m$  is the potential of m -th PEC, Fig. 2.

Using the Green's function, the electric scalar potential at any point of the system can be defined. The HBEM application has the aim to obtain the quadratic system of linear equations with unknown free charges of PECs, total charges per unit length at boundary surfaces between dielectric layers, and unknown additive constant  $\phi_0$  that depends on the chosen referent point for the electric scalar potential. As it is described in detail in [9], using the PMM for the potential of the inner and the outer conductors, the PMM for the normal component of the electric field, and the electrical neutrality condition, it is possible to determine unknown free charges per unit length on conductors, total charges per unit length on the boundary surfaces between layers, and the unknown constant  $\phi_0$ .

A computer code for solving that system and characteristic parameters calculation is developed by authors of this paper.

The HBEM results have been compared with those obtained by FEMM [18]. The results divergence is defined as:

$$\delta[\%] = \frac{\left|Z_{c}^{\text{HBEM}} - Z_{c}^{\text{FEMM}}\right|}{Z_{c}^{\text{FEMM}}} \cdot 100.$$
(4)

# **3** Numerical Results

The geometry of shielded symmetrical coupled microstrip line with finite metallization thickness *t* and three layers with permittivities  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$  is shown in Fig. 3.

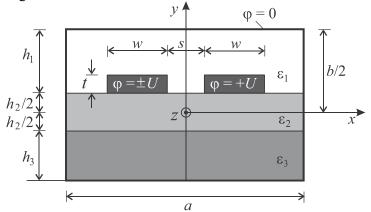


Fig. 3 – Shielded symmetrical coupled multilayered microstrip line.

With w is denoted the width of both inner conductors, s is the distance between the inner conductors,  $h_1$ ,  $h_2$  and  $h_3$  are heights of corresponding dielectric layers, a and b are the width and height of the shield, respectively.

Some of typical materials which can be used as dielectric layers are mentioned in [19]. Some of them are used in this paper for characteristic parameters calculation. Their permittivities are:  $\varepsilon_r = 3.78$ ,  $\varepsilon_r = 6.1$ ,  $\varepsilon_r = 9.35$  and  $\varepsilon_r = 11.0$ .

The convergence of microstrip line characteristic impedance for "even" and "odd" modes is shown in Fig. 4. Dimensions and layers permittivities of microstrip line are:

$$\varepsilon_{r1} = \varepsilon_{r3} = 1$$
,  $\varepsilon_{r2} = 9.35$ ,  $a/w = 4.0$ ,  $s/w = 1.0$ ,  
 $t/w = 0.1$ ,  $h_1/w = h_3/w = 0.8$ ,  $h_2/w = 0.4$ .

 $N_{\rm tot}$  is the total number of unknowns.

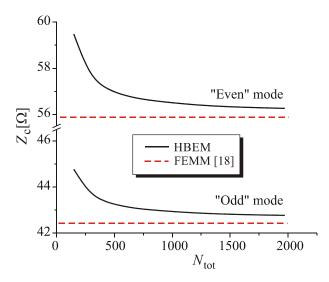


Fig. 4 – Characteristic impedance results convergence for "even" and "odd" modes.

A good convergence of the results is achieved for both modes. The values obtained using FEMM [18] are denoted with dashed lines.

In order to verify the accuracy of the HBEM values, the comparison of HBEM and FEMM results for the effective dielectric permittivity and the characteristic impedance versus  $h_3/w$  is given in **Tables 1** and **2**.

The microstrip dimensions and layers permittivities are:

$$\varepsilon_{r1} = 1$$
,  $\varepsilon_{r2} = 6.1$ ,  $\varepsilon_{r3} = 9.35$ ,  $a/w = 4.0$ ,  $b/w = 2.0$ ,  $s/w = 1.0$ ,  
 $t/w = 0.05$ ,  $h_2/w = 0.2$  and  $h_1/w = (b - h_2 - h_3)/w$ .

Table 1Verification of results for characteristic parameters of shieldedcoupled microstrip line versus  $h_3/w$  ("even" mode).

$h_3$	HBEM		FEMM		\$50/1
w	$\epsilon_{\rm r}^{\rm eff}$	$Z_{c}[\Omega]$	$\epsilon_r^{\rm eff}$	$Z_{c}[\Omega]$	δ[%]
0.2	4.8751	31.787	4.8971	31.730	0.18
0.4	4.6878	38.780	4.7156	38.662	0.30
0.6	4.4323	43.225	4.4634	43.062	0.38
0.8	4.1446	45.590	4.1779	45.394	0.43
1.0	3.8130	45.972	3.8490	45.745	0.50
1.2	3.4023	44.045	3.4410	43.797	0.57

h <sub>3</sub>	HBEM		FEMM		
$\frac{3}{W}$	$\epsilon_{\rm r}^{\rm eff}$	$Z_{c}[\Omega]$	$\epsilon_{\rm r}^{\rm eff}$	$Z_{c}[\Omega]$	δ[%]
0.2	4.5736	30.370	4.5955	30.307	0.20
0.4	4.3414	36.041	4.3702	35.913	0.35
0.6	4.1493	39.130	4.1819	38.961	0.43
0.8	3.9903	40.461	4.0251	40.269	0.48
1.0	2.7196	41.096	2.7574	40.821	0.68
1.2	2.5861	36.015	2.6249	35.772	0.68

Table 2Verification of results for characteristic parameters of shielded<br/>coupled microstrip line versus  $h_3/w$  ("odd" mode).

From those tables is evident that the results divergence is less than 0.7 %.

Increasing the height of the third layer, for "even" mode, the effective permittivity decreases and the characteristic impedance increases. In the other case (for "odd" mode) the characteristic impedance first increases than decreases. The effective permittivity decreases in that case.

It should be mentioned that, during the FEMM application, the simulation was done twice – one for the case when the shielded line is air-filled (the dielectric layers are replaced by air) and the second case when the dielectric layers are placed as it is shown in Fig. 3. In both cases the capacitance per unit length of the shielded microstrip line was calculated so the characteristic parameters can be determined.

The comparison of the results obtained applying the HBEM with the results reported in the literature is shown in Fig. 5. The characteristic impedance for "even" and "odd" modes as a function of electrode thickness, for different values of parameter s/w, is shown this figure. The results given in [16] and [17] are also presented in the figure as well as the FEMM results [18].

The microstrip parameters are:

 $\varepsilon_{r1} = \varepsilon_{r3} = 1$ ,  $\varepsilon_{r2} = 4$ , a/w = 10.0,  $h_1/w = h_2/w = 1.0$ ,  $h_3/w = 2.0$ .

Total number of unknowns is about 1500. As it can be seen from this figure, the results agreement is very good.

Increasing the electrode thickness, the characteristic impedance decreases for both modes, Fig. 5. Increasing the distance between conductors in the case when s/w > 1.0, the characteristic impedance decreases for "even" and increases for "odd" mode.

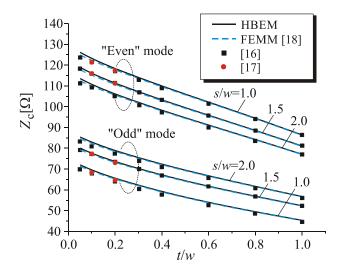


Fig. 5 – Characteristic impedance of shielded coupled microstrip line as a function of electrode thickness.

An influence of layer's permittivities is also investigated. The characteristic impedance distribution for "even" and "odd" modes for different values of parameter s/w and permittivities  $\varepsilon_{r2}$  and  $\varepsilon_{r3}$  are given in Figs. 6 and 7. The microstrip line dimensions and permittivity of the first layer are:  $\varepsilon_{r1} = 1$ , a/w = 4.0, t/w = 0.05,  $h_1/w = h_3/w = 0.8$  and  $h_2/w = 0.4$ .

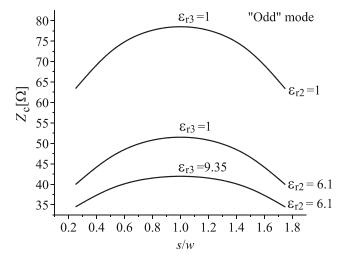
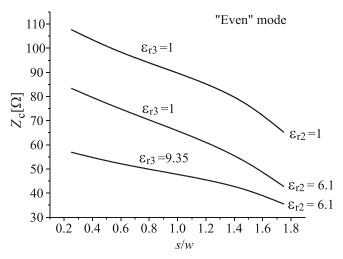


Fig. 6 – Characteristic impedance of shielded coupled microstrip line as a function of distance between conductors and layers permittivities ("odd" mode).



**Fig.** 7 – Characteristic impedance of shielded coupled microstrip line as a function of distance between conductors and layers permittivities ("even" mode).

The influence of relative permittivities  $\varepsilon_{r2}$  and  $\varepsilon_{r3}$  on the characteristic impedance distribution is evident. The characteristic impedance is greatest when permittivities are equal to 1. Increasing the value of third layer permittivity, when second layer permittivity has constant value, the characteristic impedance decreases for both modes. However, increasing the distance between conductors, the characteristic impedance for "odd" mode first increases than decreases. Such conclusion could not be performed from the Fig. 5. That figure presents results obtain when parameter s/w is 1.0, 1.5 and 2.0. These values are chosen in order to compare the HBEM results with those found in the literature [16, 17]. The characteristic impedance decreases for "even" mode when the distance between the conductors increases.

The second layer height has also the influence on potential distribution. That is shown in Figs. 8 and 9 for "even" and "odd" modes. The potential distribution is given along *x*-axis. Increasing the second layer height, the conductors distance from the *x*-axis increases to, so the potential decreases.

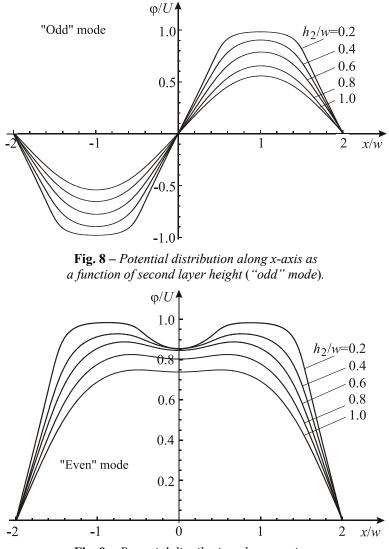
The microstrip parameters are:

$$\varepsilon_{r1} = \varepsilon_{r3} = 1$$
,  $\varepsilon_{r2} = 9.35$ ,  $s/w = 1.0$ ,  $t/w = 0.05$ ,  $h_1/w = h_3/w = (b - h_2)/w$ .

#### **3.1** Computation time

The computation time for different number of unknowns is shown in Fig. 10. The microstrip parameters are:

$$\varepsilon_{r1} = \varepsilon_{r3} = 1$$
,  $\varepsilon_{r2} = 9.35$ ,  $a/w = 4.0$ ,  $s/w = 1.0$ ,  
 $t/w = 0.1$ ,  $h_1/w = h_3/w = 0.8$ ,  $h_2/w = 0.4$ .



**Fig. 9** – Potential distribution along x-axis as a function of second layer height ("even" mode).

Increasing the number of unknowns, the computation time increases too. All calculations were performed on computer with dual core INTEL processor 2.8 GHz and 4 GB of RAM.

The term "computation time" describes the time spent for determining the number of unknowns using an initial number, positioning of the equivalent electrodes, forming the matrix elements, solving the system of equations and the characteristic parameters determination. A computation time distribution is shown in pie chart given in Fig. 11 for  $N_{tot} = 1500$ . It is evident that the most of the computation time goes to the matrix fill (89% of total computation time).

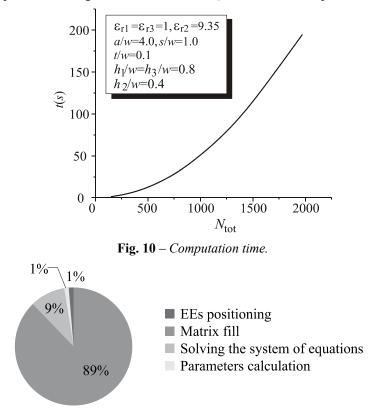


Fig. 11 – Distribution of computation time.

# 4 Conclusion

An analysis of shielded coupled multilayered microstrip lines has been done using the hybrid boundary element method. The calculation is based on a quasi-static TEM approach. The obtained results show influences of different parameters on microstrip lines characteristic parameters. Those results are verified with the FEMM results and data available in the literature obtained using other methods. Very good results agreement can be noticed.

Overall analysis showed that the HBEM is a simple, fast and sufficiently accurate procedure which can be applied to various structures with arbitrary number of dielectric layers and conductors. The finite metallization thickness is also taken into account so the influence of that parameter on characteristic impedance distribution was noticed. The computation time is very short. M.T. Perić, S.S. Ilić, S.R. Aleksić

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