SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 13, No. 2, June 2016, 157-174

UDC: 004.42+004.43]:621.391.822 **DOI: 10.2298/SJEE151118002M**

A Novel-Iterative Simulation Method for Performance Analysis of Non-Coherent FSK/ASK Systems Over Rice/Rayleigh Channels using the Wolfram Language

Vladimir Mladenović[1], Danijela Milošević[1]

Abstract: In this paper, a new approach in solving and analysing the performances of the digital telecommunication non-coherent FSK/ASK system in the presence of noise is derived, by using a computer algebra system. So far, most previous solutions cannot be obtained in closed form, which can be a problem for detailed analysis of complex communication systems. In this case, there is no insight into the influence of certain parameters on the performance of the system. The analysis, modelling and design can be time-consuming. One of the main reasons is that these solutions are obtained by utilising traditional numerical tools in the shape of closed-form expressions. Our results were obtained in closed-form solutions. They are resolved by the introduction of an iteration-based simulation method. The Wolfram language is used for describing applied symbolic tools, and SchematicSolver application package has been used for designing. In a new way, the probability density function and the impact of the newly introduced parameter of iteration are performed when errors are calculated. Analyses of the new method are applied to several scenarios: without fading, in the presence of Rayleigh fading, Rician fading, and in cases when the signals are correlated and uncorrelated.

Keywords: Wolfram language, Iteration-Based Simulation Method, Fading, Closed-form solution, Closed-form expression.

1 Introduction

Several challenges such as vehicular communications, mobile-to-mobile communications, vehicle-to-vehicle channels, fixed-to-vehicle channels, fixed-to-fixed channels, and moving scatterers, have become a major current research challenge in the growing mobile industry [1]. So far, calculating, designing and analysing modern mobile communication systems are based on mathematical models necessary for obtaining the desired results. These models have been applied more often by using numerical tools. Many computer-based numerical

[1]Faculty of Technical Sciences Čačak, University of Kragujevac, Svetog Save 65, 32000 Čačak, Serbia;
E-mails: vladimir.mladenovic@ftn.kg.ac.rs; danijela.milosevic@ftn.kg.ac.rs

algorithms have been developed to adapt to computer calculations, and the main task of numerical processing is the construction of algorithms. These tools give results that do not present closed-form solutions, performing their processes through numerical calculations and simulations [2, 3]. As a consequence of these approaches, the user might easily lose insight into the phenomenon being investigated, and numerical computation manipulates numerical values; therefore, very often it is important to take care in the final results: accuracy, execution time of the numerical algorithm and efficiency-based reasoning [4]. In particular, this is important with very complex calculations derived by hand.

On the other hand, workers have a problem in practice on how to encode mathematical formulae from scientific papers and monographs in software packages, as well as how to check whether their program gives correct results.

In recent decades, software tools providing the solution of complex problems through mathematical formulation in a closed form have been developed. One of the software packages that allows the symbolic calculation is *Mathematica*, which ensures that each system or process becomes a symbolic expression, and in such a form is further processed using the concept of *Wolfram language* [5]. The *Wolfram language* enables us to build, program and execute many processes, understand as everything-expressed structures. The term "everything-expressed structure" means that the use of a common expression is not cosmetic only, but it allows us to present everything as an expression [6].

In this paper, we derive performances of a digital telecommunication non-coherent FSK/ASK system, introducing a new method named the iteration-based simulation method. By applying defined mathematical formulations, we introduce a new parameter in the form of iteration, as a variable in this method. It helps that any form of closed-form expression leads to the final analytical form. In this way, we can follow all parameters in some system or process, and we can perform any kind of mathematical manipulation with them. Derivations are performed for three scenarios. First, we observe the system without the influence of fading. Secondly, the communication channel has Rayleigh fading; and thirdly, the communication channel has Rician fading. Also, all of these cases include correlated and uncorrelated noise.

The paper is structured as follows. Section 2 provides the state-of-art from the viewpoint of simulation type and subject of research. In Section 3, definitions of closed-form solutions and closed-form expressions are given. The *Wolfram language* is introduced through the method of simplification of closed-form expressions. Section 4 describes the reference model and analysis of the communication system. In Section 5, the iteration-based simulation method is explained and the *Wolfram language* is used for modelling. The mentioned method is used to derive closed-form solutions for three scenarios of propagation waves in wireless communications. According to the obtained results, the numerical simulations are performed as a special kind of symbolic computation and are presented in Section 6. Finally, Section 7 concludes this paper.

2 Related Work

Closed-form expressions of the classes of sum-of-cissoids processes are obtained for modelling and simulation of frequency-nonselective mobile Rayleigh fading channels, as presented in [7]. There are examples of the correct solutions of closed-form expressions that can be reached using the level-crossing rate and the average duration of fades of the channel capacity [8]. Often, these can be obtained where a solution exists through a special function, but it is known that the closed-form expression is in the background.

In general, sophisticated channel models are based on wideband scenarios that incorporate the frequency selectivity of the physical channel. Many approaches are used to develop mobile channel models by a finite sum of multipath propagation components [9, 10]. For these investigations, several approaches were proposed in [11, 12] to mitigate the drawbacks of this algorithm. All the mentioned models are derived using manipulation by hand and are simulated using by numerical tools.

The application of a computer algebra system has been presented in a number of papers [13 – 15], explaining that an automated symbolic solving problem in different areas can be used without manual execution and verification of results. Of course, such an approach has its limitations. It should always be borne in mind that the software programs that use symbolic tools can provide solutions only if knowledge is entered into the software tools and inasmuch real solutions exist [16 – 19].

3 Wolfram Language

For the initial formulation of the mathematical descriptions of the process and systems, let us define the key definitions.

Definition 1. An equation is said to be a closed-form solution if it solves a given problem in terms of functions and mathematical operations from a given generally accepted set [20].

Definition 2. A Closed-form expression represents an implicit solution that is contained in a mathematical expression. As an example, the integral with a non-final solution as the closed-form solution can be solved numerically or via special functions only.

In other words, a closed-form solution provides explicitly a solution of a process and systems, while a closed-form expression shows an implicit or insufficient solution.

In the engineering sciences, the first step for much problem solving is to understand various engineering fields, such as the processes and systems. The next step is the knowledge transfer into the corresponding software, which commonly uses procedures for solving the given problems. The best-known language to encapsulate expert knowledge in software is *Wolfram language* [21]. It allows some computations to be performed quite simply. It possesses many built-in functions that can be used for term rewriting. Also, it enables us to build, program and execute many processes, and to understand as everything-expressed structures. Using simplification rules of the *Wolfram language*, or by rearranging expressions, it is very easy to check that this automated derivation generates the same result [6]. Calculation of the previously presented expressions is often very arduous and complex; sometimes there is no output solution. In these cases, the special functions used to obtain a closer solution are applied, so that we can carry out the necessary assessments. The basic idea is to prepare examples for users who are used as an exemplary document or software for analysis, simulation or processing.

4 Reference Model and Analysis

In wireless communications, each wave interaction with objects that are in the propagation environment causes a large number of replica-sent signals that arrive at the input of the receiver with different attenuations, phase shifts and delays. The superposition of these replicas at reception causes strengthened or weakened signals, which is referred to in the literature as fading [10]. For modelling the signal transmission in multiple propagation environments, depending largely on the appearance of fading, the Rayleigh, Rice, Nakagami-m, Nakagami-q and Weibull distributions are commonly used. Application of a specific model is conditioned by the specifics of the propagation environment. Since the calculation of all performance indicators depends on the probability density function (PDF) of the receiving signal, it is important to determine the function parameters for a given propagation environment. The cumulative probability function (CDF) is calculated by integrating the PDF, which defines the probability of system failure.

The receiver system with a non-coherent detection of FSK/ASK signal is shown in Fig. 1 [6].

We have explained in detail the general statement of the observed system in [6]. Non-coherent detection is present on both branches of the receiver. The upper branch contains a matched filter with frequency ω_0, and the lower branch contains a matched filter with frequency ω_1. Each branch is composed of a phase detector, and further signal passes to a subtraction circuit; the difference of the signal goes into the sampler [22]. Information is sent by two symbols (named codewords) on frequencies ω_0 and ω_1, with equal probability, and interference is present in both branches [6]. In one case the noise is correlated, and the coefficient of correlation is marked by *R*. In a second case, the noise is uncorrelated (i.e. $R = 0$).

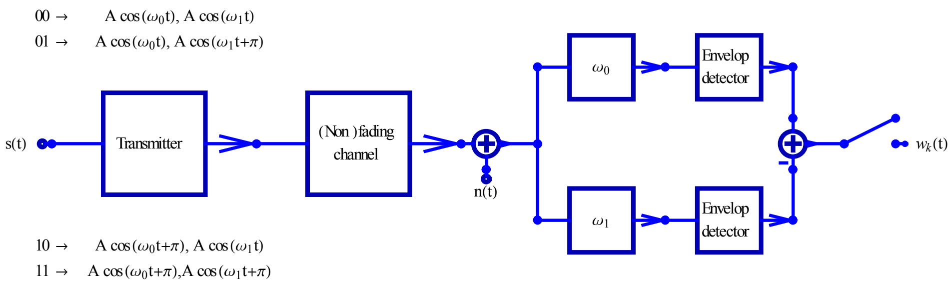

Fig. 1 – *Non-coherent FSK/ASK system.*

The transmitter sends codewords, where each one corresponds to a hypothesis. For example, codeword 00 corresponds to hypothesis H_0H_0, codeword 01 corresponds to hypothesis H_0H_1, etc.

The general form of the hypothesis on the upper branch is:

$$\begin{aligned} & H_k : w_{k1} = \\ & = (-1)^{k+1} A\cos\omega_0 t + x_k \cos\omega_0 t - y_k \sin\omega_0 t = \\ & = z_{Upk} \cos(\omega_0 t + \phi_{Upk}) \end{aligned} \tag{1}$$

In (1) $k = 0$, 1 and index "Up" represent the upper branch.

Similarly, the general form of the hypothesis on the lower branch is:

$$\begin{aligned} & H_k : w_{k1} = \\ & = (-1)^{k} A\cos\omega_1 t + x_k \cos\omega_1 t - y_k \sin\omega_1 t = \\ & = z_{Dwk} \cos(\omega_1 t + \phi_{Dwk}) \end{aligned} \tag{2}$$

In (2) $k = 0$, 1 and index "Dw" represent the lower branch.

By substituting

$$\begin{aligned} & (-1)^{k+1} A + x_k = z_{Upk} \cos\phi_{Upk} \\ & y_k = -z_{Upk} \sin\phi_{Upk} \end{aligned} \tag{3}$$

the z_{Upk} and φ_{Upk} ($k = 0$, 1) are envelopes and phases of input signal for the upper branch, it follows that:

$$z_{Upk} = \sqrt{((-1)^{k+1} A + x_k)^2 + {y_k}^2}$$
$$\phi_{Upk} = -\arctan \frac{y_k}{(-1)^{k+1} A + x_k} \tag{4}$$

The general form of the condition joint probability density function is:

$$p(x_0, x_1, y_0, y_1) = \frac{1}{4\pi^2\sigma^4\sqrt{1-R^2}} \exp\left\{ -\frac{x_0^2 + x_1^2 + y_0^2 + y_1^2 - 2R(x_0x_1 + y_0y_1)}{2\sigma^2(1-R^2)} \right\}, \tag{5}$$

where R is the coefficient of correlation and σ is variance.

For solving general settings, the Jacobian for the upper and lower branches must be found. For the upper branch:

$$|J|_{Up} = \frac{\partial(x_0, y_0, x_1, y_1)}{\partial(z_{Up1}, \phi_{Up1}, z_{Up2}, \phi_{Up2})} = \begin{vmatrix} \frac{\partial x_0}{\partial z_{Up1}} & \frac{\partial y_0}{\partial z_{Up1}} & \frac{\partial x_1}{\partial z_{Up1}} & \frac{\partial y_1}{\partial z_{Up1}} \\ \frac{\partial x_0}{\partial \phi_{Up1}} & \frac{\partial y_0}{\partial \phi_{Up1}} & \frac{\partial x_1}{\partial \phi_{Up1}} & \frac{\partial y_1}{\partial \phi_{Up1}} \\ \frac{\partial x_0}{\partial z_{Up2}} & \frac{\partial y_0}{\partial z_{Up2}} & \frac{\partial x_1}{\partial z_{Up2}} & \frac{\partial y_1}{\partial z_{Up2}} \\ \frac{\partial x_0}{\partial \phi_{Up2}} & \frac{\partial y_0}{\partial \phi_{Up2}} & \frac{\partial x_1}{\partial \phi_{Up2}} & \frac{\partial y_1}{\partial \phi_{Up2}} \end{vmatrix} = z_{Up1} z_{Up2} \tag{6}$$

Respectively, the Jacobian for the lower branch is $|J|_{Dw} = z_{Dw1} z_{Dw2}$.

The next step is determining the condition joint probability density function (JPDF). For the upper and lower branches, respectively, the condition joint probability density function is

$$p_{mn}(z_{Up1}, z_{Up2}, \phi_{Up1}, \phi_{Up2} / A) = |J|_{Up}\, p_{mn}(x_0, x_1, y_0, y_1); \quad m, n = 0, 1 \tag{7}$$

$$p_{mn}(z_{Dw1}, z_{Dw2}, \phi_{Dw1}, \phi_{Dw2} / A) = |J|_{Dw} \cdot p_{mn}(x_0, x_1, y_0, y_1); \quad m, n = 0, 1 \tag{8}$$

The joint probability density function (JPDF) is obtained by integrating condition joint probability density function for all amplitudes of A.

The following equation applies to the upper branch:

$$p_{mn}(z_{Up1}, z_{Up2}, \phi_{Up1}, \phi_{Up2}) = \int_0^\infty p_{mn}(z_{Up1}, z_{Up2}, \phi_{Up1}, \phi_{Up2} / A)\, p(A)\, \mathrm{d}A, \quad m, n = 0, 1, \tag{9}$$

and to the lower branch:

$$p_{mn}(z_{Dw1}, z_{Dw2}, \phi_{Dw1}, \phi_{Dw2}) = \int_0^\infty p_{mn}(z_{Dw1}, z_{Dw2}, \phi_{Dw1}, \phi_{Dw2} / A) p(A) \, \mathrm{d}A, \quad m, n = 0, 1. \tag{10}$$

Now, it is necessary to obtain the function of phase distribution. This is calculated by integrating JPDF for all envelopes z_{Up1}, z_{Up2} for the upper branch, and z_{Dw1}, z_{Dw2} for the lower branch, respectively.

Upper branch:

$$p_{mn}(\phi_{Up1}, \phi_{Up2}) = \int_0^\infty \int_0^\infty \mathrm{d}z_{Up1} \, \mathrm{d}z_{Up2} \, p_{mn}(z_{Up1}, z_{Up2}, \phi_{Up1}, \phi_{Up2}), \quad m, n = 0, 1. \tag{11}$$

Lower branch:

$$p_{mn}(\phi_{Dw1}, \phi_{Dw2}) = \int_0^\infty \int_0^\infty \mathrm{d}z_{Dw1} \, \mathrm{d}z_{Dw2} \, p_{mn}(z_{Dw1}, z_{Dw2}, \phi_{Dw1}, \phi_{Dw2}), \quad m, n = 0, 1. \tag{12}$$

Generally, 16 events appear at the input of the receiver. When hypothesis H_0H_0 is present at the output of the transmitter, codeword 00 is detected at the input of the receiver and the event D_0D_0 is applied; when hypothesis H_0H_1 is present at the output of the transmitter, codeword 01 is detected at the input of the receiver and the event D_0D_1 is applied, etc. At the input of the receiver, the following is detected:

$$\begin{aligned} p(D_m D_n / H_i H_j) = & \int_{\frac{(-1)^{m+1}}{2}\pi}^{\frac{(m+1)}{2}\pi} \mathrm{d}\phi_{Up1} \int_{\frac{(-1)^{n+1}}{2}\pi}^{\frac{(n+1)}{2}\pi} \mathrm{d}\phi_{Up2} \times \\ & \times \int_{\frac{(-1)^{m+1}}{2}\pi}^{\frac{(m+1)}{2}\pi} \mathrm{d}\phi_{Dw1} \int_{\frac{(-1)^{n+1}}{2}\pi}^{\frac{(n+1)}{2}\pi} \mathrm{d}\phi_{Dw2} \, p_{ij}(\phi_{Up1}, \phi_{Up2}) \, p_{ij}(\phi_{Dw1}, \phi_{Dw2}), \end{aligned} \quad m, n, i, j = 0, 1 \tag{13}$$

The total probability error is:

$$P_e = 1 - \sum_{i=0}^{1} \sum_{j=0}^{1} P(H_i H_j D_i D_j); \quad i, j = 0, 1 \tag{14}$$

$$P(H_i H_j D_i D_j) = p(D_i D_j / H_i H_j) P(H_i H_j); \quad i, j = 0, 1 \tag{15}$$

$$P(H_i H_j) = P(H_i) P(H_j) = \frac{1}{2}\frac{1}{2} = \frac{1}{4}; \quad i, j = 0, 1. \tag{16}$$

Such an approach shows a symbolic representation of resolving the general method for each designing, modelling and simulation. The numerical approach is a special case of symbolic calculation only.

5 Iteration-Based Simulation Method and Wolfram Language Modelling

The iteration-based simulation method is a simplification of complex algebraic expressions, expressed as a closed-form expression, in a manner that is acceptable for analysis, and can be reduced to a closed-form solution.

Since integrals exist in most of the analyses, we approach them by simply calculating when the integrals are presented using a Riemann sum. In this case, the number of iterations is a parameter that can be manipulated. The Riemann sum in the *Wolfram language* is defined as follows:

```
RiemannSumDouble[f_, {x_, a_, b_}, {y_, c_, d_}, q_] :=
 Sum[
   Evaluate[
    (f /. {x → (a + (b - a) k / q), y → (c + (d - c) k / q)})],
   {k, 0, q - 1}] (b - a)/q (d - c)/q

RiemannSumDouble[f, {x, a, b}, {y, c, d}, q]
(-a + b) (-c + d) f / q
```

Fig. 2 – *Riemann sum defined using Wolfram language.*

The definition of double integrals over the Riemann sum is undertaken by assigning the function name `RiemannSumDouble`. The form in Fig. 2 shows a double integral where `f` is marked as function, `x` and `y` are variables, `a`, `b`, `c`, `d` are below the upper limits of integral, and `q` is iteration. In this way, the defined integral is used to obtain closed-form solutions in performing complex mathematical formulae. The mark `:=` means assigning one variable or more variables to function with delayed values. The assigned variables are in an unevaluated form until the function that allocates concrete variables or variable values is called. Every time, when the function appears, it is replaced by variables, evaluated afresh each time.

We now define the scenarios that are considered.

5.1 No fading channel

The first analysis involves an illustration of the transfer of information through the channel without fading. Cases where the noise is correlated and uncorrelated are observed. No fading is in the channel and the PDF of amplitude is constant. So,

$$p_{uniformn} = \frac{1}{2\pi}. \tag{17}$$

A record of the PDF written in the *Wolfram language* is shown in Fig. 3.

```
pAUniformn := 1/(2 π)

pAUniformn

1/(2 π)
```

Fig. 3 – *Riemann sum defined using Wolfram language for no fading channel.*

The JPDF defined in (7) and (8) is typewritten in the *Wolfram language* in the following way:

```
p[k_, l_, z1_, z2_, s_, R_, A_, f1_, f2_] :=
  (z1 * z2)/(4 * π^2 * (s)^4 * Sqrt[1 - R^2]) *
   Exp[
    1/(2 (s)^2 (1 - R^2)) ((-1)^k * (-A + z1 * Cos[f1])^2 + ((-1)^l * A + z2 * Cos[f2])^2 +
      (-z1 * Sin[f1])^2 + (-z2 * Sin[f2])^2 -
      2 * R * (((-1)^k * A + z1 * Cos[f1]) ((-1)^l A + z2 * Cos[f2]) +
         (z1 * z2 * Sin[f1] * Sin[f2])))];
```

Fig. 4 – *Condition joint probability density function using Wolfram language.*

We calculate JPDF using (9) and (10), and define the formulation in Fig. 2, as shown in Fig. 5. It is assumed that the envelopes are equal $z_1 = z_2 = z$ and normalised $0 \le z \le 1$. All changes are performed by `/.` applying a rule or list of rules, in an attempt to transform each subpart of an expression. The list of rules is defined as:

```
change = {z1 → z1, z2 → z2, s → σ, f1 → ϕ1, f2 → ϕ2}
```

so, it follows that symbolic computation of joint probability density function using Wolfram language for no fading channel is as in Fig. 5.

The final expression defined by `PDensityFunUniformn`, shows the PDF for `m=n=0`. It is important to notice that the parameter `q` providing the PDF of p_{00} is in a closed-form solution. In the same way, we derive the remainder of the PDFs, but we do not describe them because of complexity.

Fig. 6 depicts the final closed-form solution of probability error for the no fading channel. The iteration `q` exists in closed-form solution as a parameter.

```
PDensityFunUniformn[R_, q_, A_] :=
 Table[RiemannSumDouble[
   1/4 * pAUniformn * p[k, l, z, z, s, R, A, f, f], {z, 0, 1},
   {f, (-1)^k π/2, (-1)^(k+1) π/2}, q], {k, 2}, {l, 2}] // Flatten

PDensityFunUniformn[R, q, A][[1]] /. change
```

$$\frac{e^{\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-2\,R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}}{32\,\pi^2\,q^3\,\sqrt{1-R^2}\;\sigma^4}$$

Fig. 5 – *Symbolic computation of joint probability density function using Wolfram language for no fading channel.*

$$\frac{e^{\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-2\,R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}}{32\,\pi^2\,q^3\,\sqrt{1-R^2}\;\sigma^4}+$$

$$\frac{e^{\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2+\left(A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2-2\,R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)\left(A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)\right)}{2\left(1-R^2\right)\sigma^2}}}{32\,\pi^2\,q^3\,\sqrt{1-R^2}\;\sigma^4}-$$

$$\frac{e^{\frac{\frac{8\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+2\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2-2\,R\left(\frac{4\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)\right)}{2\left(1-R^2\right)\sigma^2}}}{8\,\pi^2\,q^3\,\sqrt{1-R^2}\;\sigma^4}-$$

$$\frac{e^{\frac{\frac{8\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2+\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2-2\,R\left(\frac{4\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}}{8\,\pi^2\,q^3\,\sqrt{1-R^2}\;\sigma^4}$$

Fig. 6 – *Closed-form solution of probability error for no fading channel.*

5.2 Rayleigh fading with correlated and uncorrelated noise

The Rayleigh channel model is commonly used for modelling a fading signal that extends in environments where there is no line-of-sight (NLOS) between the transmitter and the receiver. This model is well suited to describe the statistics received signal in urban areas (and urban areas with tall buildings). The Rayleigh distribution describes the PDF [9, 10]:

$$p_{\text{Rayleigh}}(A) = \frac{A}{\sigma^2}\exp\left\{-\frac{A^2}{2\sigma^2}\right\},\ 0 \le A \le \infty . \tag{18}$$

Similarly, a preview subsection of the record of the PDF written in the *Wolfram language* is shown in Fig. 7.

```
pARayleght[A_, s_] := A/(s)^2 * Exp[-A^2/(2 (s)^2)]

pARayleght[A, s] /. change
```

$$\frac{A\, e^{-\frac{A^2}{2\sigma^2}}}{\sigma^2}$$

Fig. 7 – *Riemann sum defined using Wolfram language for Rayleigh fading.*

We calculate the JPDF using (9) and (10), and the *Wolfram language* formulation in Fig. 4 is as shown in Fig. 8.

```
PDensityFunRayleght[R_, n_, A_] :=
  Table[RiemannSumDouble[1/4 * pARayleght[A, s] * p[k, l, z, z, s, R, A, f, f],
    {z, 0, 1}, {f, (-1)^k π/2, (-1)^(k+1) π/2}, n], {k, 2}, {l, 2}] // Flatten

PDensityFunRayleght[R, q, A][[1]] /. change
```

$$\frac{A\, e^{-\frac{A^2}{2\sigma^2}+\frac{\frac{2\cos\left[\frac{\pi}{q}\right]^2}{q^2}-2R\left(\frac{\cos\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\sin\left[\frac{\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}}{16\pi q^3\sqrt{1-R^2}\,\sigma^6}$$

Fig. 8 – *Symbolic computation of joint probability density function using Wolfram language for Rayleigh fading.*

In the final expression defined by `PDensityFunRayleigh`, the PDF for `m=n=0` is shown. It is important to see that parameter `q` providing the PDF of p_{00} is in a closed-form solution.

Fig. 9 depicts the final closed-form solution of probability error for Rayleigh fading. Also, the iteration q exists in closed-form solution as a parameter.

$$\frac{A\, e^{-\frac{A^2}{2\sigma^2}+\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-2R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}}{16\,\pi\, q^3\sqrt{1-R^2}\;\sigma^6}+$$

$$\frac{A\, e^{-\frac{A^2}{2\sigma^2}+\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2+\left(A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2-2R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)\left(A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)\right)}{2\left(1-R^2\right)\sigma^2}}}{16\,\pi\, q^3\sqrt{1-R^2}\;\sigma^6}-$$

$$\frac{A\, e^{-\frac{A^2}{2\sigma^2}+\frac{\frac{8\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+2\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2-2R\left(\frac{4\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)\right)}{2\left(1-R^2\right)\sigma^2}}}{4\,\pi\, q^3\sqrt{1-R^2}\;\sigma^6}-$$

$$\frac{A\, e^{-\frac{A^2}{2\sigma^2}+\frac{\frac{8\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2+\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2-2R\left(\frac{4\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}}{4\,\pi\, q^3\sqrt{1-R^2}\;\sigma^6}$$

Fig. 9 – *Closed-form solution of probability error for Rayleigh fading.*

5.3 Rician fading with correlated and uncorrelated noise

The Rician model is used in intercity areas and suburban areas, which includes the line-of-sight (NLOS).

The Rician distribution describes the PDF [9, 10]:

$$p_{\mathrm{Rician}}(A)=\frac{A}{2\sigma^2}\exp\left\{-\frac{\mu^2+A^2}{2\sigma^2}\right\}I_0\left(\frac{\mu\cdot A}{2\sigma^2}\right),\quad 0\le A\le\infty\,. \tag{19}$$

$I_0(\cdot)$ is a modified Bessel function with zero order. The record of the PDF written in the *Wolfram language* is shown in Fig. 10.

We calculate the JPDF using (9) and (10), and the *Wolfram language* formulation as in Fig. 4 as shown in Fig. 11.

```
pARician[A_, s_] := A/(s)^2 * Exp[-(μ^2 + A^2)/(2 (s)^2)] * BesselI[0, μ * A/(s)^2]

pARician[A, s] /. change
```

$$\frac{A\, e^{-\frac{A^2+\mu^2}{2\sigma^2}}\ \mathrm{BesselI}\left[0, \frac{A\mu}{\sigma^2}\right]}{\sigma^2}$$

Fig. 10 – *Riemann sum defined using Wolfram language for Rician fading.*

```
PDensityFunRician[R_, n_, A_] :=
 Table[RiemannSumDouble[1/4 * pARician[A, s] * p[k, l, z, z, s, R, A, f, f],
    {z, 0, 1}, {f, (-1)^k π/2, (-1)^(k+1) π/2}, n], {k, 2}, {l, 2}] // Flatten

PDensityFunRician[R, q, A][[1]] /. change
```

$$\frac{A\, e^{-\frac{A^2+\mu^2}{2\sigma^2}+\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-2R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}\ \mathrm{BesselI}\left[0, \frac{A\mu}{\sigma^2}\right]}{16\pi q^3\sqrt{1-R^2}\ \sigma^6}$$

Fig. 11 – Symbolic computation of joint probability density function using Wolfram language for Rician fading.

$$\frac{A\, e^{-\frac{A^2+\mu^2}{2\sigma^2}+\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-2R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}\ \mathrm{BesselI}\left[0, \frac{A\mu}{\sigma^2}\right]}{16\pi q^3\sqrt{1-R^2}\ \sigma^6}+\frac{1}{16\pi q^3\sqrt{1-R^2}\ \sigma^6}$$

$$A\, e^{-\frac{A^2+\mu^2}{2\sigma^2}+\frac{\frac{2\,\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}-\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2+\left(A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)^2-2R\left(\frac{\mathrm{Cos}\left[\frac{\pi}{q}\right]^2}{q^2}+\left(-A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)\left(A+\frac{\mathrm{Sin}\left[\frac{\pi}{q}\right]}{q}\right)\right)}{2\left(1-R^2\right)\sigma^2}}$$

$$\mathrm{BesselI}\left[0, \frac{A\mu}{\sigma^2}\right]-\frac{1}{4\pi q^3\sqrt{1-R^2}\ \sigma^6}$$

$$A\, e^{-\frac{A^2+\mu^2}{2\sigma^2}+\frac{\frac{8\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+2\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2-2R\left(\frac{4\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)\right)}{2\left(1-R^2\right)\sigma^2}}$$

$$\mathrm{BesselI}\left[0, \frac{A\mu}{\sigma^2}\right]-\frac{1}{4\pi q^3\sqrt{1-R^2}\ \sigma^6}$$

$$A\, e^{-\frac{A^2+\mu^2}{2\sigma^2}+\frac{\frac{8\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(-A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2+\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2-2R\left(\frac{4\,\mathrm{Cos}\left[\frac{2\pi}{q}\right]^2}{q^2}+\left(A+\frac{2\,\mathrm{Sin}\left[\frac{2\pi}{q}\right]}{q}\right)^2\right)}{2\left(1-R^2\right)\sigma^2}}$$

$$\mathrm{BesselI}\left[0, \frac{A\mu}{\sigma^2}\right]$$

Fig. 12 – *Closed-form solution of probability error for Rician fading.*

In the final expression defined by `PDensityFunRician`, the PDF for m=n=0 is shown. It is important to see that the parameter `q` providing the PDF of p_{00} is in a closed-form solution.

Fig. 12 depicts the final closed-form solution of probability error for Rician fading. Also, the iteration `q` exists in closed-form solution as a parameter.

6 Graphics Interpretation using Closed-Form Solutions in Wolfram language

In the following explanation, the transmission characteristics of information for all three scenarios will be shown. The closed-form solutions using the *Wolfram language* are used for numerical simulation and are presented on Figs. 13 – 15. All simulations are performed for σ = 3 and μ = 1.

6.1 PDFs and Probability Errors

In Fig. 13, the PDF and probability errors are displayed for the no fading channel. The characteristics are obvious for different values of the number of iterations, as well as for the impact of correlated noise on the shapes of the characteristics.

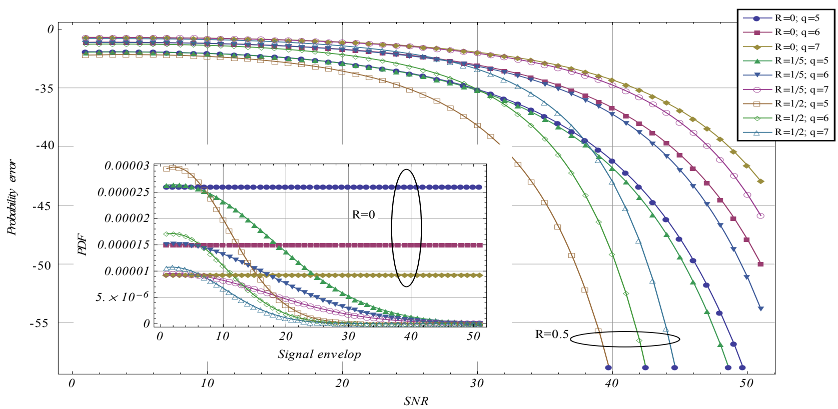

Fig. 13 – *Probability density function and probability errors for no fading channel.*

The characteristics of the PDF and probability errors are displayed for Rayleigh fading in Fig. 14.

The characteristics of the PDF and probability errors for Rician fading are displayed in Fig. 15. Also, the influence of correlated noise on the shapes of the characteristics, and characteristics for different values of the number of iterations, are shown.

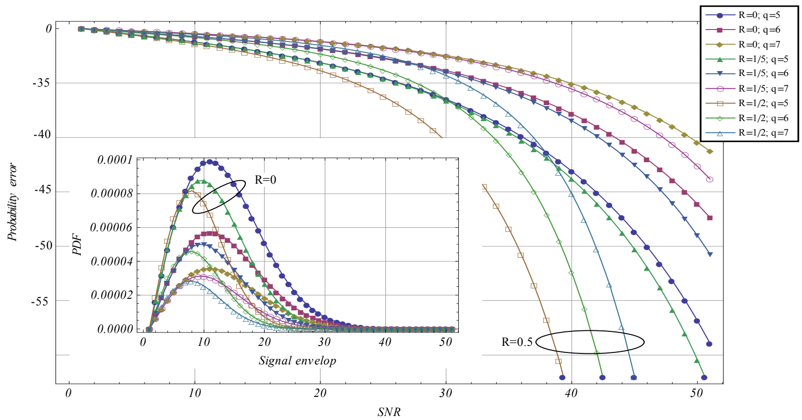

Fig. 14 – *Probability density function and probability errors for Rayleigh fading.*

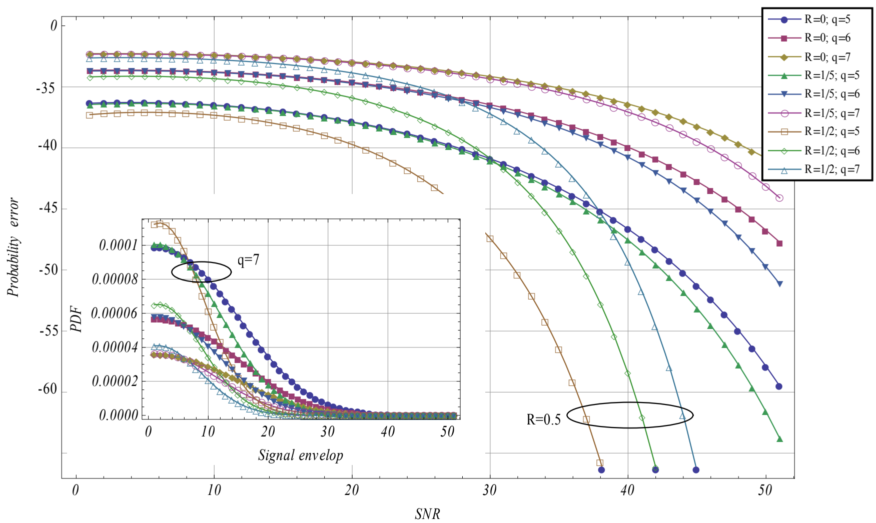

Fig. 15 – *Probability density function and probability errors for Rician fading.*

6.2 Analysis of Error Functions

This subsection analyses the error obtained from closed-form solutions for all three scenarios. To facilitate the description of phenomena that affect the mistake, the symbolic expressions obtained will be used for numerical simulation.

The errors as a function of iterations are shown in Fig. 16. They are obtained from the previously obtained probability errors. The arrows show the direction of reducing errors with regard to the correlation coefficient for all three scenarios.

On the other hand, the number of iterations q affects the simulation speed. It is significant to note that the order of magnitude of all iterations is 10^{-6}. The error falls by more than twice for Rayleigh and Rician fading when the noise is uncorrelated between the third and the fifth iterations. The error falls by around four times in the second iteration in Rayleigh and Rician fading, while a reduction in the errors is evident after the fifth iteration.

All three scenarios doubtlessly have an error calculation below 10^{-6} in the sixth iteration. Sometimes, there can be a large number of iterations with respect to the environment in which the analysis is performed. The error becomes almost constant after the eighth iteration.

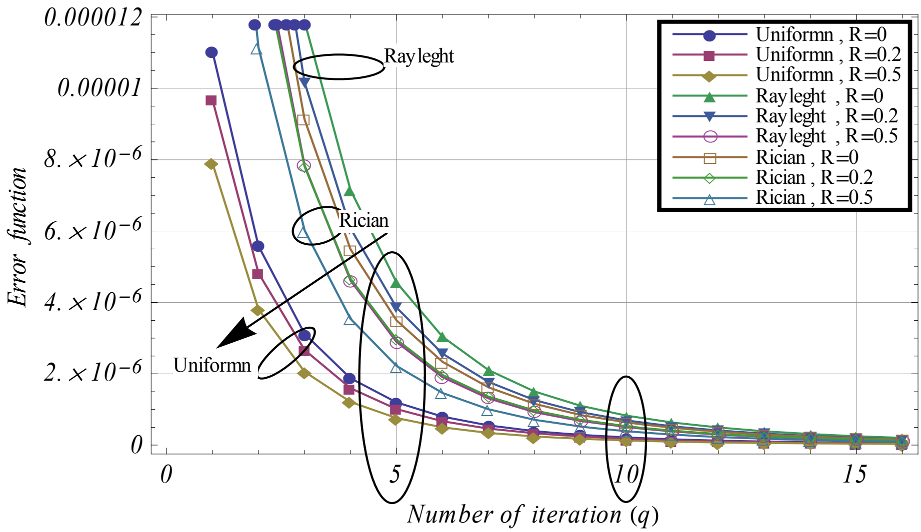

Fig. 16 – *Error function in terms of number of iterations.*

7 Conclusion

This paper has presented a new method to simulate the processes and systems where there is no opportunity to represent mathematical formulae in the final form. Often, many phenomena show that closed-form expressions and simulations are executed by numerical-based tools. In such cases, the users do not have insight into phenomena that affect the flow of processes, perhaps leading to incorrect assumptions and results. The presented iteration-based simulation method gives the possibility that the closed-form solutions could provide analytical forms that can be mathematically manipulated. The presented method has been performed on examples of wireless communications for three scenarios of no fading, and Rayleigh and Rician fading.

8 Acknowledgement

This work is partially funded by the Serbian Ministry of Education, Science and Technological Development within the project No. TR 32023, entitled: "Optimisation performance energy-efficient computing and communication systems".

9 References

[1] A. Borhani, M. Patzold: Modeling of Vehicle-to-Vehicle Channels in the Presence of Moving Scatterers, Vehicular Technology Conference, Quebec, Canada, 03-06 Sept. 2012, pp. 1 – 5.

[2] M. Patzold, U. Killat, Y. Li, F. Laue: Modeling, Analysis and Simulation of Nonfrequency-Selective Mobile Radio Channels with Asymmetrical Doppler Power Spectral Density Shapes, IEEE Transactions on Vehicular Technology, Vol. 46, No. 2, May 1997, pp. 494 – 507.

[3] N. Hajri, N. Youssef, M. Patzold: On the Statistical Properties of Phase Crossings and Random FM Noise in Double Rayleigh Fading Channels, IEEE Transactions on Vehicular Technology, Vol. 65, No. 4, April 2016, pp. 1859 – 1867.

[4] J.P. Barringer: Telecommunications: Applications, Modern Technologies and Economic Impact, Nova Science Publishers, New York, USA, 2014.

[5] M. Lutovac, D. Tošić: SchematicSolver Version 2.3, 2015. http://www.wolfram.com/products/applications/schematicsolver/.

[6] V. Mladenovic, M. Lutovac: Non-coherent FSK/ASK Signal Detection with Two Symbols Correlated Noise, 2nd International Conference on Electrical, Electronic and Computing Engineering (IcETRAN 2015), Silver Lake (Srebrno Jezero), Serbia, 08-11 June 2015, p. EKI 1.7.

[7] B.O. Hogstad, C.A. Gutierrez, M. Patzold, P.M. Crespo: Classes of Sum-of-cisoids Processes and Their Statistics for the Modeling and Simulation of Mobile Fading Channels, EURASIP Journal on Wireless Communications and Networking, Vol. 2013, Dec. 2013, p. 125.

[8] B.O. Hogstad, M. Patzold, N. Youssef, V. Kontorovitch: Exact Closed-Form Expressions for the Distribution, Level-Crossing Rate, and Average Duration of Fades of the Capacity of MIMO Channels, IEEE Transactions on Vehicular Technology, Vol. 58, No. 2, Feb. 2009, pp. 1011 – 1016.

[9] M. Patzold: Mobile Radio Channels, John Wiley and Sons, Chichester, UK, 2011.

[10] M. Patzold: Mobile Fading Channels, John Wiley and Sons, Chichester, UK, 2002.

[11] W. Li, Q. Ni: Joint Channel Parameter Estimation using Evolutionary Algorithm, IEEE International Conference on Communications, Cape Town, South Africa, 23-27 May 2010.

[12] A. Papoulis, S. Pillai: Probability, Random Variables and Stochastic Processes, McGraw-Hill, NY, USA, 2004.

[13] V. Mladenovic, M. Lutovac: Simulation of Symbol-Error Ratio and Energy Efficiency using Computer Algebra Tools: A Case Study of OFDM Receiver, International Scientific Conference of IT and Business-Related Research, Belgrade, Serbia, 16-17 April 2015, pp. 224 – 231. (In Serbian).

[14] M. Lutovac, V. Mladenovic: Contemporary Electronics with LTSpice and Mathematica, International Scientific Conference of IT and Business-Related Research, Belgrade, Serbia, 16-17 April 2015, pp. 134 – 138.

[15] M. Lutovac, V. Mladenovic: Teaching Contemporary Electronics using Wolfram Language, 2nd International Conference on Electrical, Electronic and Computing Engineering (IcETRAN 2015), Silver Lake, Serbia, 08-11 June 2015, p. EKI1.2.

[16] V. Mladenovic, M. Lutovac, D. Porrat: Symbolic Analysis as Universal Tool for Deriving Properties of Non-linear Algorithms – Case study of EM Algorithm, Acta Polytechnica Hungarica, Vol. 11, No. 2, 2014, pp. 117 – 136.

[17] V. Mladenovic, M. Lutovac M. Lutovac: Automated Proving Properties of Expectation-Maximization Algorithm using Symbolic Tools, Telfor Journal, Vol. 4, No. 1, 2012, pp. 54 – 59.

[18] V. Mladenovic, D. Porrat, M. Lutovac: The Direct Execution of the Expectation-Maximization Algorithm using Symbolic Processing, 10th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Services – TELSIKS, Nis, Serbia, 05-08 Oct. 2011, Vol. 4, pp. 265 – 268.

[19] M. Lutovac, V. Mladenovic, M. Lutovac: Development of Aeronautical Communication System for Air Traffic Control Using OFDM and Computer Algebra Systems, Studies in Informatics and Control, Vol. 22, No. 2, 2013, pp. 205 – 212.

[20] J.M. Borwein, R.E. Crandall: Closed Forms: What They Are and Why We Care, Notices of the American Mathematical Society, Vol. 60, No. 1, Jan. 2013, pp. 50 – 65.

[21] S. Wolfram: The Mathematica Book, Wolfram Media, Champaign, IL, USA, 2003.

[22] J. Proakis, M. Salehi: Digital Communication, McGraw-Hill, NY, USA, 2007.