SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 1, No. 2, June 2004, 175 - 186

Dielectric Body with Arbitrary Shaped and Positioned Cavity in Homogeneous Transversal Electric Field

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Abstract: This paper presents an application of Charge Simulation Method for calculation of the electric field strength and the potential in the arbitrary shaped and positioned cavity of the dielectric body. The dielectric body can also have an arbitrary, but known shape. Several examples of the dielectric body with the cavity are presented. The whole system is in the homogeneous transverse electrical field. The results obtained in this way have been compared to the results obtained by the program's packages: Bela 1.0 and Quickfield 4.2 (Student's Edition).

Keywords: Charge Simulation Method, Fictitious sources, Boundary conditions.

1 Introduction

One of the mostly used methods for numerical solving of the electrostatic problems is Charge Simulation Method (CSM) or Method of fictitious sources (MFS) [1]. Basic idea of this method is that the electrodes are replaced by a discrete set of inner charge distributions, whose position and type are predetermined, but whose magnitude is unknown [2]. The unknown intensities of fictitious sources (FS) are determined to satisfy boundary conditions on the electrode's surfaces [3]. In that way, system of the linear equations with FS as unknown values is formed. After solving this system by standard methods, the unknown charges of the FS can be determined. Using standard electrostatic formulas the potential and the electric field strength can be computed.

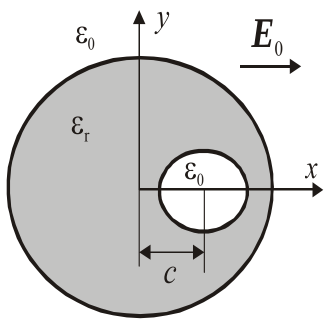

Fig. 1 - *Dielectric body with cavity.*

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The correct choice of the type and the form of the FS is very important [7]. Point charges (for three-dimensional systems), line charges with constant density per unit length (for plan-parallel systems) and linear circular loops (for systems with axial symmetry), can be used for FS. Also, a very different and complex FS can be used depending on the problem geometry and on the experience of the investigator.

In this paper the CSM is applied for calculation of the electric field strength in the arbitrary shaped cavity of the arbitrary shaped dielectric body in the homogeneous transverse steady electrical field.

2 Application of the method

The whole system is divided in three FS systems. The FS are placed with respect to the cylindrical or the spherical surfaces (depending on the problem geometry). The first FS system is presented on Fig. 2. For determination the potential outside the dielectric bodythe FS are placed inside the body.

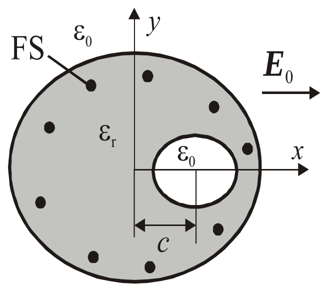


Fig. 2 - *The first system.*

The second FS system (for determination the potential inside the dielectric body) is on Fig. 3. The FS are placed outside the body and inside the cavity. For determination the potential inside the cavity, FS are placed inside the body (around the cavity) (Fig. 4).

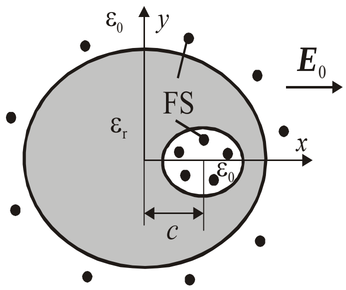


Fig. 3 - *The second system.*

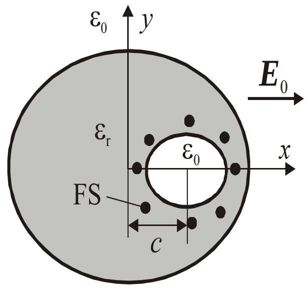


Fig. 4 - *The third system.*

The potential is given by

$$\varphi(r) = \sum_j p_j(r) q_j \ ,$$

where q_j represents the (unknown) magnitude of the charge on the j th distribution, and $p_j(r)$ is a coefficient that depends only on the type of the distribution and the position of the point r. The intensities of used FS can be determined after solving the linear equations satisfying the boundary conditions on the electrode's surface and on the surface, which separates layers (the dielectric layers and the cavity).

3 Numerical results

In order to demonstrate the application of the CSM several examples of the dielectric body with the eccentric cavity will be presented.

Example 1

It considers the dielectric circular cylinder with the eccentric circular cavity. The cylinder has radius *a*, eccentricity *c*, and radius of the cavity *b* (Fig. 5).

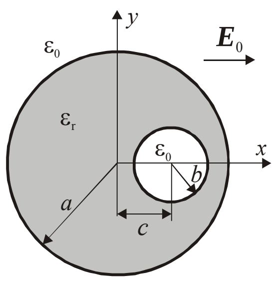


Fig. 5 - *Dielectric cylinder with circular cavity.*

Using the existing symmetry, N_1 fictitious sources are placed inside the body's volume, at points

$$x_{1j} = f_1 a \cos\theta_{1j} \text{ and } y_{1j} = f_1 a \sin\theta_{1j},\ 0 < f_1 < 1,$$

$$\theta_{1j} = \pi\frac{2j-1}{N_1},\ j = 1,\dots,N_1. \tag{1}$$

The potential outside dielectric cylinder can be expressed as

$$\varphi_{\rm I} = \varphi_0 - \sum_{j=1}^{N_1}\frac{q'_{1j}}{2\pi\varepsilon_0}\ln r_1 + C_1, \tag{2}$$

$$\text{where } r_1 = \sqrt{(x-x_{1j})^2+(y-y_{1j})^2} \text{ and } \varphi_0 = -E_0 x. \tag{3}$$

Also, for determination the potential inside the dielectric body and inside the cavity, we formed two systems. The potentials for the second system are:

$$\varphi_{\rm II} = -\sum_{\rm j=1}^{N_2}\frac{q'_{2\rm j}}{2\pi\varepsilon}\ln r_2 - \sum_{\rm j=1}^{N_3}\frac{q'_{3\rm j}}{2\pi\varepsilon}\ln r_3 + C_2, \tag{4}$$

where

$$r_2 = \sqrt{(x - x_{2j})^2 + (y - y_{2j})^2} \text{ and } r_3 = \sqrt{(x - x_{3j})^2 + (y - y_{3j})^2}\ , \tag{5}$$

and for the third system

$$\varphi_{\text{III}} = -\sum_{j=1}^{N_4} \frac{q'_{4j}}{2\pi\varepsilon_0} \ln r_4 + C_3\ ,$$

$$r_4 = \sqrt{(x - x_{4j})^2 + (y - y_{4j})^2}\ , \tag{6}$$

where (x_{ij}, y_{ij}) $(i = 1,2,3,4)$ are positions of the fictitious sources for each system. C_1, C_2 and C_3 are constant values. The unknown line charges can be determined by solving the system of linear equations that satisfies boundary conditions on the system surfaces. The total number of unknowns is $N = N_1 + N_2 + N_3 + N_4$, where N_i is the total number of the fictive charges per unit length in each system. Normalized values are used for the line charges

$$q'_{jn} = \frac{q'_j}{2\pi\varepsilon_0 E_0 a}\ .$$

By solving this system, the potential and the electric field strength are calculated.

Table I

Normalized values for potential $(\varphi / E_0 a)$.

x/a	$N_e = 30$	$N_e = 80$	Bela 1.0	QuickField
-1.0	0.53360	0.52374	0.52544	0.58628
-0.8	0.43028	0.42474	0.42564	0.48165
-0.6	0.33088	0.32670	0.32733	0.36932
-0.4	0.23275	0.22991	0.23023	0.25671
-0.2	0.13707	0.13552	0.13554	0.14657
0.0	0.04702	0.04668	0.04643	0.04326
0.2	-0.02573	-0.02500	-0.02596	-0.04206
0.4	-0.18200	-0.17991	-0.18064	-0.21504
0.6	-0.33952	-0.33622	-0.33700	-0.38624
0.8	-0.49880	-0.49466	-0.49513	-0.55089
1.0	-0.57590	-0.57077	-0.57241	-0.62412

Fictitious sources are placed on the fictitious cylindrical surfaces. Constants f_1, f_2, f_3 and f_4 determine positions of the fictitious sources. They should be neither close nor far from the cylindrical surfaces [6].

For the input values:

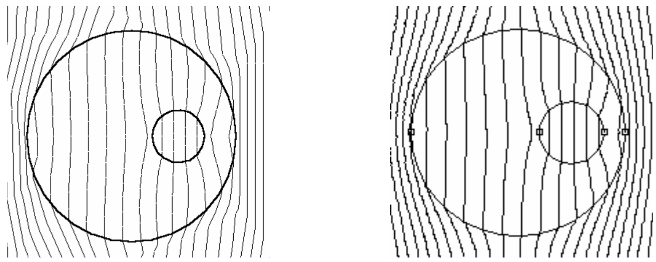
$$f_1 = 0.9\ ,\ f_2 = 1.2\ ,\ f_3 = 0.8\ ,\ f_4 = 1.2\ ,\ b/a = 0.3\ ,$$

$$c/a = 0.5\ ,\ \varepsilon_r = 3\ ,\ N_1 = N_2 = N_3 = N_4 = N_e\ ,$$

the results are shown in Table I and Table II.

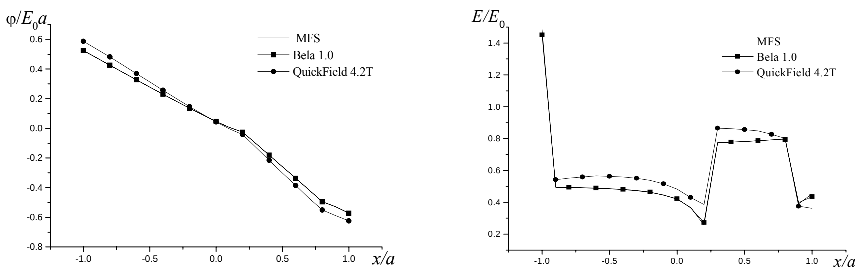
Table II
Normalized values for electric field strength (E/E_0).

x/a	$N_e = 30$	$N_e = 80$	Bela 1.0	QuickField
-1.0	1.43113	1.48412	1.45072	1.53345
-0.8	0.49910	0.49220	0.49363	0.55152
-0.6	0.49446	0.48768	0.48921	0.56499
-0.4	0.48597	0.47938	0.48056	0.55855
-0.2	0.46848	0.46218	0.46425	0.53712
0.0	0.42458	0.41877	0.42194	0.48207
0.2	0.26269	0.25727	0.27318	0.38544
0.4	0.78399	0.77762	0.77789	0.86538
0.6	0.79162	0.78612	0.78614	0.85604
0.8	0.79142	0.79926	0.79375	0.82677
1.0	0.45001	0.45260	0.43498	0.36252



(*a*) (*b*)

Fig. 6 - *Equipotential curves of dielectric cylinder using Quickfield 4.2* (a) *and using Bela1.0* (b).



(*a*) (*b*)

Fig. 7 - *Potential* (a) *and electric field strength* (b) *distribution in eccentric cavity of dielectric cylinder with circular cross-section.*

The results obtained using the Charge Simulation Method have been compared to the results obtained by the program's packages Quickfield 4.2 (Student Edition) [13] and Bela1. 0 [12] (Fig. 6 - 7).

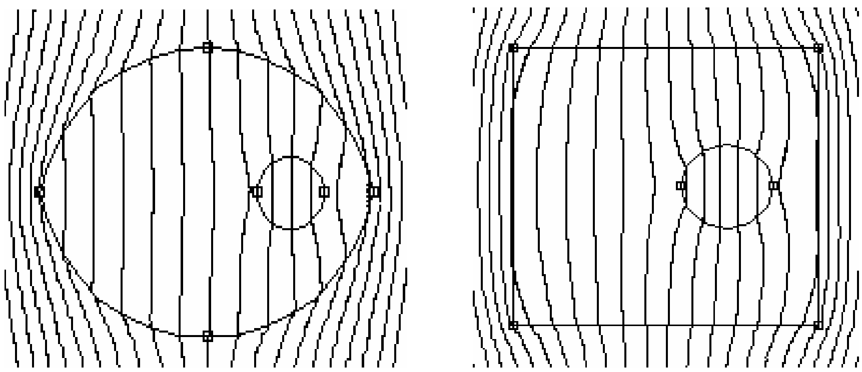
In Fig. 7 MFS - curves are identical to Bela1. 0-curves. That agreement can also be seen from Table I and Table II.

Example 2

Using the Charge Simulation Method it is possible to determine the electric field strength and the potential inside the circular cavity of the dielectric cylinder with arbitrary cross-section. The dielectric cylinder with elliptical cross-section including an eccentric circular cavity is described by the semi-axis *m* and *n*, eccentricity *c*, and radius of the cavity *b* [10].

The dielectric cylinder with the square cross-section and the eccentric circular cavity is presented on Fig. 8*b* and it is described by the side 2*a*, eccentricity *c* and radius of the cavity *b*. Like in the previous example, the whole system is derived in three systems. The line charges are used as fictitious sources. Using the boundary conditions we can form a system of linear equations. The unknown intensities of the FS can be determined by solving this system.

The program package Bela 1.0 gives the equipotential curves (Fig. 8).



(*a*) (*b*)

Fig. 8 - *Equipotential curves of dielectric cylinder with elliptical* (a) *and square* (b) *cross-section, obtained from Bela 1.0.*

The potential and the electric field strength distributions in the eccentric cavity of the dielectric cylinder with the elliptical and the square cross-section are shown in Fig.9 and Fig. 10, respectively.

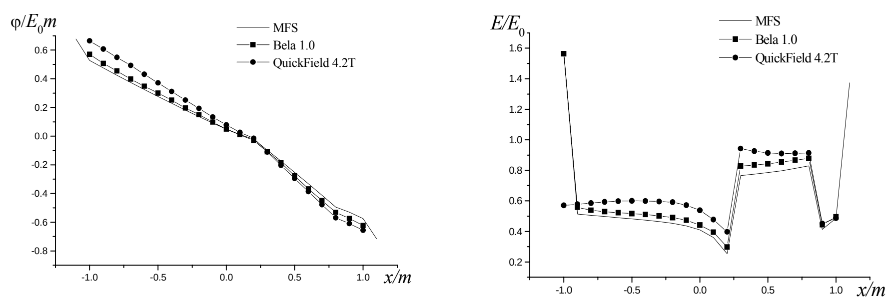
The input values are:

$$f_1 = 0.9,\ f_2 = 1.2,\ f_3 = 0.8,\ f_4 = 1.2,\ n/m = 0.8,\ b/m = 0.3, c/m = 0.5,\ \varepsilon_r = 3$$

(for the problem shown in Fig. 8*a*).

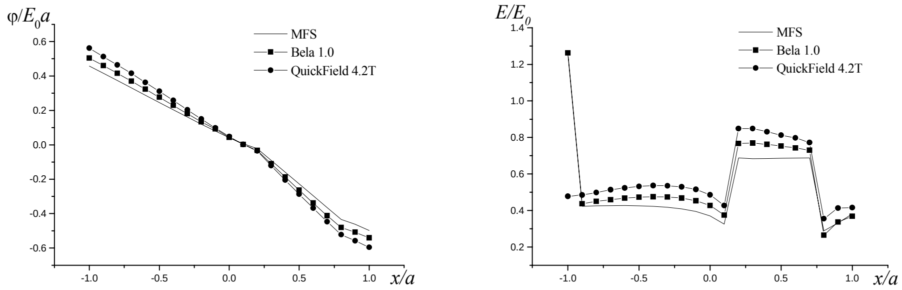
For the problem shown in the Fig. 8*b*, the input values are:

$$f_1 = 0.9,\ f_2 = 1.2, f_3 = 0.8,\ f_4 = 1.2,\ b/a = 0.3,\ c/a = 0.4\,.$$



(a) (b)

Fig. 9 - *Potential* (a) *and electric field strength* (b) *distribution in eccentric cavity of dielectric cylinder with elliptical cross-section.*



(a) (b)

Fig. 10 - *Potential* (a) *and electric field strength* (b) *distribution in eccentric cavity of dielectric cylinder with square cross-section.*

Example 3

The dielectric circular cylinder with the arbitrary shaped eccentric cavity is presented. Fig. 11*a* presents the dielectric circular cylinder including an eccentric cavity with the elliptical cross-section. The cylinder has radius *a*, eccentricity *c* and semi-axis of the cavity *m* and *n*.

The input values are;

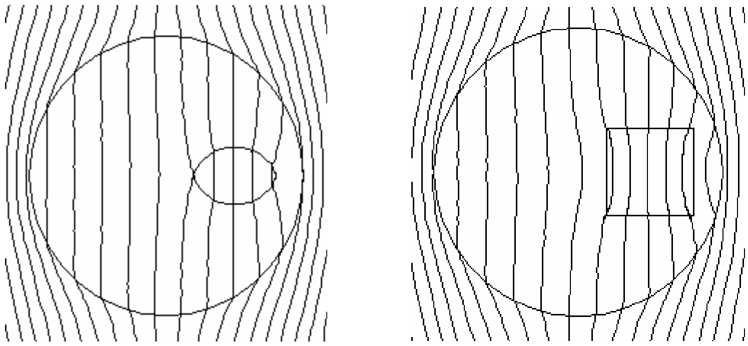
$f_1 = 0.9$, $f_2 = 1.2$, $f_3 = 0.8$, $f_4 = 1.2$, $m/a = 0.3$, $n/a = 0.2$, $c/a = 0.5$ and $\varepsilon_r = 3$.

The results are given in Fig. 12.

The dielectric circular cylinder with the eccentric cavity with square cross-section is presented on Fig. 11*b* and it is described by the radius of the cylinder *a*, eccentricity *c* and side of the cavity 2*b*. The input values are:

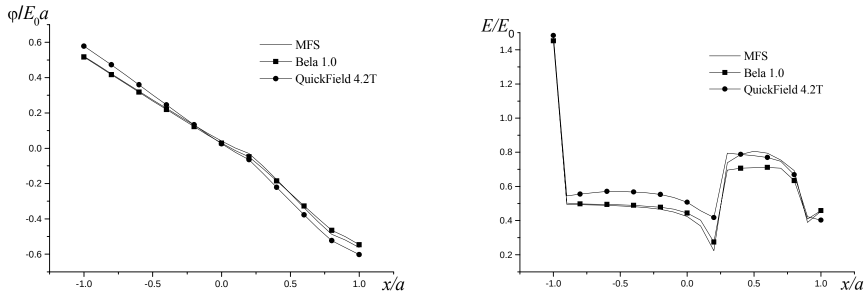
$$f_1 = 0.9,\ f_2 = 1.2,\ f_3 = 0.8,\ f_4 = 1.2,\ b/a = 0.3,\ c/a = 0.5 \text{ and } \varepsilon_r = 3.$$

The potential and the electric field strength distributions are shown in Fig. 13.



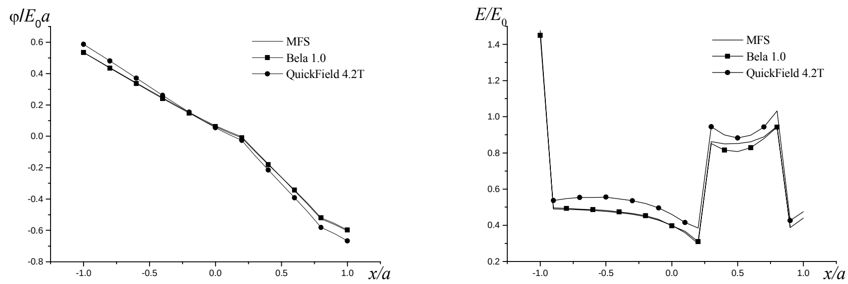
(a) (b)

Fig. 11 - *Equipotential curves of dielectric cylinder with cavity with elliptical* (a) *and square* (b) *cross-section using Bela 1.0.*



(a) (b)

Fig. 12 - *Potential* (a) *and electric field strength* (b) *distribution in eccentric cavity with elliptical cross-section of dielectric cylinder.*



(a) (b)

Fig. 13 - *Potential* (a) *and electric field strength* (b) *distribution in eccentric cavity with square cross-section of dielectric cylinder.*

Example 4

In the case of the dielectric sphere with the eccentric spherical cavity in the external steady homogeneous electric field using the same procedure we are used.

The point charges are used as the fictitious sources. We can form one mesh of the fictitious sources for each subsystem [9]. After solving the system of linear equations with FS as unknown values, the potential and the electric field strength can be calculated. The results are shown in Table III and Table IV.

Table III

Values for potential, φ *in* V.

x/a	$Ne=36$	$Ne=64$	$Ne=100$
-1.0	0.20530	0.22597	0.23548
-0.8	0.38758	0.17915	0.18829
-0.6	0.29452	0.13525	0.14185
-0.4	0.19985	0.09129	0.09554
-0.2	0.10544	0.04766	0.04967
0.0	0.01364	0.00548	0.00539
0.2	-0.06363	-0.02884	-0.03020
0.4	-0.17922	-0.08504	-0.09025
0.6	-0.29104	-0.14083	-0.15028
0.8	-0.38909	-0.18484	-0.19624
1.0	-0.46647	-0.22360	-0.23818

Table IV

Values for electric field strength, E in V/m.

x/a	$Ne=36$	$Ne=64$	$Ne=100$
-1.0	7.72280	7.45688	7.29376
-0.8	1.81188	2.18328	2.32068
-0.6	1.88712	2.19988	2.32068
-0.4	1.89532	2.19576	2.30892
-0.2	1.87476	2.16088	2.27020
0.0	1.77116	2.02464	2.12104
0.2	1.10088	1.06320	1.04232
0.4	2.26048	2.79660	3.00108
0.6	2.19072	2.77440	2.99968
0.8	1.60456	1.81692	1.89248
1.0	5.32576	5.46816	5.35348

The potential and the electric field strength distributions are shown in Fig. 15. Fig. 14 presents the equipotential curves obtained by program package Mathematica 3.0.

The input values are:

$$f_1 = 0.7\,,\ f_2 = 1.3\,,\ f_3 = 0.7\,,\ f_4 = 1.3\,,\ b/a = 0.25\,,\ c/a = 0.45 \text{ and } \varepsilon_r = 3\,.$$

Sphere is described by the radius of the sphere *a*, eccentricity *c* and radius of the spherical cavity *b*.

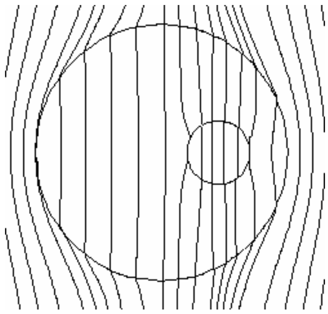
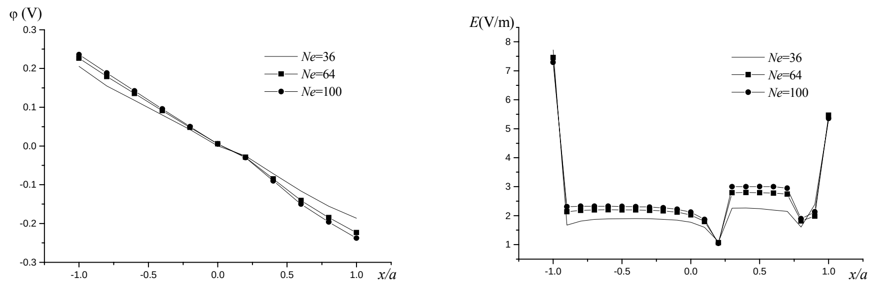
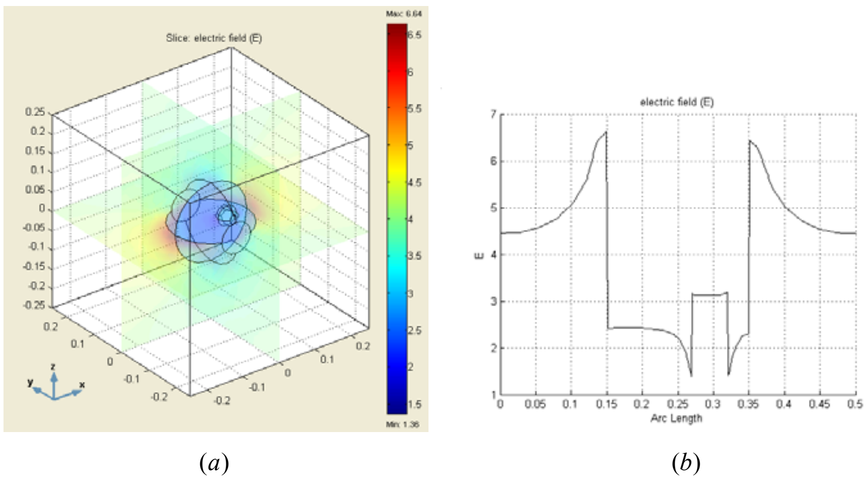


Fig. 14 - *Equipotential curves of dielectric sphere in xy plane using Mathematica* 3.0.



(*a*) (*b*)

Fig. 15 - *Potential* (*a*) *and electric field strength* (*b*) *distribution in eccentric spherical cavity of sphere.*



(*a*) (*b*)

Fig. 16 - *3D presentation* (*a*) *and distribution* (*b*) *of electric field strength across the x-axis in sphere.*

Milko Kuilekov, colleague from Technical University in Ilmenau, calculated values for potential and electric filed strenght inside the sphere using program package FEMLAB [14]. That results are presented in Fig. 16 and Fig. 17. Comparing the values from Table III and Table IV with values from Fig. 16 and Fig. 17 we have found very good agreement. For program package FEMLAB input values are:

$$E_0 = 2\text{V},\ b/a = 0.25,\ c/a = 0.45 \text{ and } \varepsilon_r = 3,$$

where E_0 is the value for the homogeneous transversal electric field.

4 Conclusion

In this paper, an application of the Charge Simulation Method for calculation of the electric field strength and the potential inside the arbitrary shaped cavity of the dielectric arbitrary shaped body in homogeneous transverse steady electrical field is presented. Theoretically, the precision of the solution depends on the number and position of the fictitious points, i.e. a higher precision can be realized by increasing the number of FS. If they are very close or too far from the cylindrical or spherical surfaces the obtained error is higher.

The results obtained in this way have been compared to the results obtained by the program packages Bela 1.0 and Quickfield 4.2. The program packages Quickfield and Bela 1.0 use the Finite Element Method.

Certain disagreement with the CSM results is product of the limited number of the nodes of the Student's Quickfield (max. 200 nodes).

We have found very good agreement with the results obtained by the program package Bela 1.0 (error rate is less then 2%).

Because the Department of Theoretical Electrical Engineering in Nis doesn't have any 3D program package, Milko Kuilekov, colleague from Technical University in Ilmenau, calculated values using program package FEMLAB. Comparing these results we have found very good agreement.

Using Method of fictitious sources we can calculate the electric field strength inside the arbitrary shaped cavity of the dielectric body for two- and three- dimensional problems.

5 Acknowledgment

The author of this paper is very grateful to colleague Milko Kuilekov from Technical University in Ilmenau. He has sent the values for the potential and the electric filed strenght inside the sphere using program package FEMLAB.

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