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Digital Filter-based 1D TLM Model of Dispersive Anisotropic Conductivity Panel

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Abstract: One-dimensional (1D) Transmission Line Matrix (TLM) method with Z-transforms, is applied in the paper to allow for an efficient time-domain simulation of thin anisotropic conductive panel of dispersive behavior. It uses a digital filter model to incorporate the scattering coefficients of the panel at two TLM cells interface in order to avoid a fine meshing of panel thickness. Model validation is done for a panel made of carbon-fibre material whose electric conductivity is anisotropic and assumed to be frequency dependent according to the Drude model. Fine TLM mesh results are used to verify the model accuracy.

Keywords: 1D TLM method, Bilinear Z–transform, Digital filter, Carbon-fibre, Anisotropic conductivity, Drude model.

1 Introduction

The transmission Line Matrix (TLM) method that uses Z-transformation techniques (so-called Z–TLM method) to transfer frequency dependent behaviour of electromagnetic (EM) characteristics of dispersive materials into the time-domain [1] has been successfully applied for modelling of a number of different media [2 − 6]. It is also suitable to apply digital filter (DF) techniques in order to create compact models for an efficient modelling of frequency-dependent external and/or internal boundaries describing thin slab of material [7, 8]. Instead of fine meshing of boundaries and panels across their thickness, which might be time-consuming due to small cell size and therefore small time-step or unnecessary in cases when it is not of interest to know what is EM field inside the interior of the panel/surface, these compact models can significantly speed up the simulation without sacrificing the accuracy.

In this paper, a DF–based compact model, initially developed for thin frequency independent isotropic conductive panel [8] and later generalized for panel exhibiting anisotropic electric conductivity [9], is extended to account for dispersive conductivity behaviour. The example of such material is carbon-fibre

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composite with a few millimeters thickness and anisotropic electric conductivity [1]. This conductivity may exhibit the dispersive behaviour as investigated in [10], so it is of interest to modify the compact model in [9] to include the variation of carbon-fibre anisotropic conductivity across the frequency range of interest for analysis. For simplicity it is assumed that this variation can be described by the Drude model [2], but other dispersive models described by a single pole or a pair of complex poles or even multiple poles (in the Pade form) can be easily adopted. One-dimensional (1D) Z−TLM method is used to incorporate the extended DF-based compact model whose accuracy and efficiency are verified through comparison with the fine TLM mesh results.

2 Compact Model

Thin dispersive anisotropic conductive panel is modelled by compact DF-based model as internal boundary condition positioned by the meshing at the interface of two TLM cells, Fig. 1. The scattering coefficients of the panel, obtained e.g. numerically by using the fine TLM mesh, are converted to the discrete time-domain by bilinear Z−transform so that can influence the connection procedure of incident/reflected pulses between two cells.

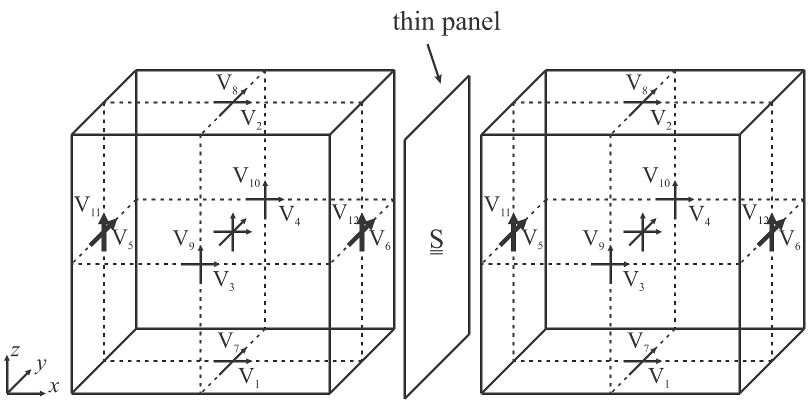

Fig. 1 – *Thin anisotropic conductive panel modelled as internal boundary condition, placed between appropriate TLM cells.*

For 1D case where there is propagation across the panel in the x-direction, the overall frequency-domain scattering matrix of the panel $\underline{\underline{S}}$, i.e. co-polarized and cross-polarized reflection and transmission coefficients connect pulses $V_{11}^{inc}, V_{11}^{ref}, V_{12}^{inc}, V_{12}^{ref}$ and $V_5^{inc}, V_5^{ref}, V_6^{inc}, V_6^{ref}$ associated with E_z and E_y field components, respectively using

$$\left[V_{12}^{inc} \; V_{11}^{inc} \; V_6^{inc} \; V_5^{inc} \right]^T = \underline{\underline{S}} \cdot \left[V_{12}^{ref} \; V_{11}^{ref} \; V_6^{ref} \; V_5^{ref} \right]^T . \tag{1}$$

In (1) the notation for incident and reflected waves is determined with the respect of two TLM cells centres separated by anisotropic panel. Each element of matrix $\underline{\underline{S}}$, after Vector Fitting (VF) method [11 – 13], can be presented as

$$F(\mathrm{s}) = \sum_{i=0}^{NP-1} \frac{C_i}{s - s_{pi}}, \tag{2}$$

where s_{pi} stands for set of complex pole frequencies, C_i are pole residues, where $i \in \{0,1,2,...,NP-1\}$, and NP is a number of poles.

Discrete-time model is developed by applying bilinear Z-transform [1] on (2)

$$s \xrightarrow{Z} \frac{2}{\Delta t}\left(\frac{1-z^{-1}}{1+z^{-1}}\right). \tag{3}$$

To make sure that precise calculations of bilinear Z–transform are performed, poles and zeros of NP degree rational function that corresponds to (2) are divided into *n* groups of m elements where *n***m* = *NP*. Based on this, main function can be described as

$$F(z) = \sum_{i=1}^{n} F^i(z), \tag{4}$$

$$F^i(z) = \frac{\sum_{k=0}^{m} B_k^i z^{-k}}{\sum_{k=0}^{m} A_k^i z^{-k}} = \frac{B_0^i + \sum_{k=1}^{m} B_k^i z^{-k}}{1 + \sum_{k=1}^{m} A_k^i z^{-k}} \xrightarrow{FPE} B_0^i + \frac{\sum_{k=1}^{m} B'^i_k z^{-k}}{1 + \sum_{k=1}^{m} A_k^i z^{-k}}, \tag{5}$$

where *A*′s and *B*′s coefficients can be easily found following approach in [7] and FPE stands for partial fraction expansion.

3 Numerical Results

Thin anisotropic panel considered here is assumed to be made of carbon-fibre composite material represented with three layers having an isotropic relative permittivity of $\varepsilon_{r\mathrm{I}} = \varepsilon_{r\mathrm{II}} = \varepsilon_{r\mathrm{III}} = 43$ and thickness $d_{\mathrm{I}} = d_{\mathrm{II}} = d_{\mathrm{III}} = 3.75$ mm, Fig. 2 [1, 9, 10].

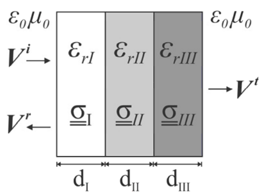

Fig. 2 – *Geometry of a carbon-fibre composite material.*

Electric conductivity of each layer is:

$$\underline{\underline{\sigma}}_{\mathrm{I}} = \begin{bmatrix} \sigma_e^1(\omega) & 0 \\ 0 & 0 \end{bmatrix}, \tag{6}$$

$$\underline{\underline{\sigma}}_{\mathrm{II}} = \begin{bmatrix} \sigma_e^2(\omega) & \sigma_e^2(\omega) \\ \sigma_e^2(\omega) & \sigma_e^2(\omega) \end{bmatrix}, \tag{7}$$

$$\underline{\underline{\sigma}}_{\mathrm{III}} = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_e^1(\omega) \end{bmatrix}. \tag{8}$$

Dispersive behaviour of electric conductivity is described by using the Drude model

$$\sigma_e^i(\omega) = \sigma_{e0}^i / (1 + j\omega\tau_e^i)\,, \quad i = 1,2\,. \tag{9}$$

Parameters of the Drude model, σ_{e0}^i and τ_e^i, are chosen so that electric conductivity of each layer varies with frequency in such way that at central frequency of considered frequency range real parts of conductivity have same values like in [1, 9] where conductivity is frequency independent. (Fig. 3)

$$\mathrm{Re}\left\{\sigma_e^1(\omega)\right\}_{|f=5\mathrm{GHz}} = 12\,, \quad \mathrm{Re}\left\{\sigma_e^2(\omega)\right\}_{|f=5\mathrm{GHz}} = 8.5\,. \tag{10}$$

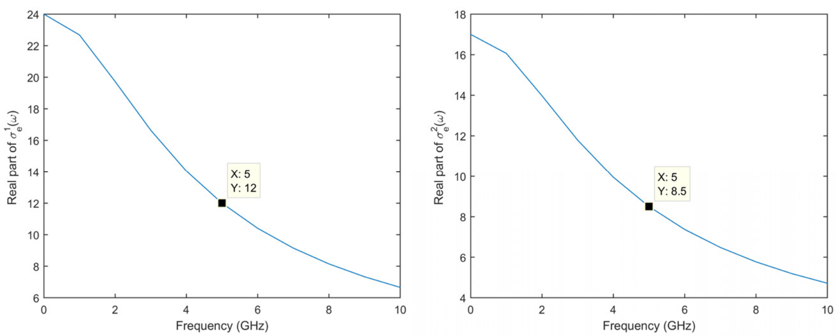

Fig. 3 – *Frequency dependence of conductivity matrix parameters* $\sigma_e^1(\omega)$ and $\sigma_e^2(\omega)$.

For assumed 1D propagation across the panel, surrounded with air at both ends, the overall frequency-domain co-polarized and cross-polarized reflection and transmission coefficients are already known as they are obtained in [10] using the Z–TLM method and applying a fine mesh with 40 nodes per each layer thickness in order to accurately represent EM field inside the layers. The real and imaginary parts of the frequency-domain reflection R_{ij} and transmission T_{ij} coefficients (the first index correspond to the component of the reflected and transmitted fields and the second index expresses the polarization of the incident wave), obtained by fine Z–TLM mesh, are marked in Figs. 4–7 with solid lines.

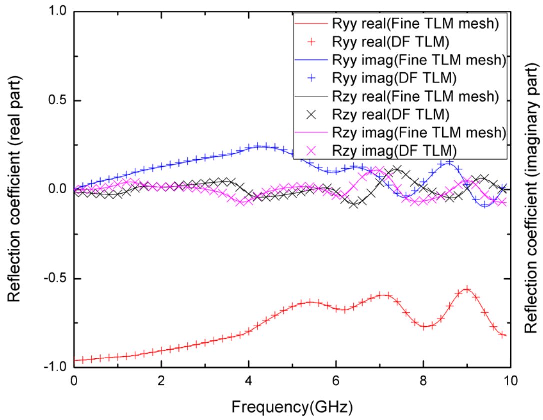

Fig. 4 – *Reflection coefficients Ryy and Rzy - real and imaginary parts.*

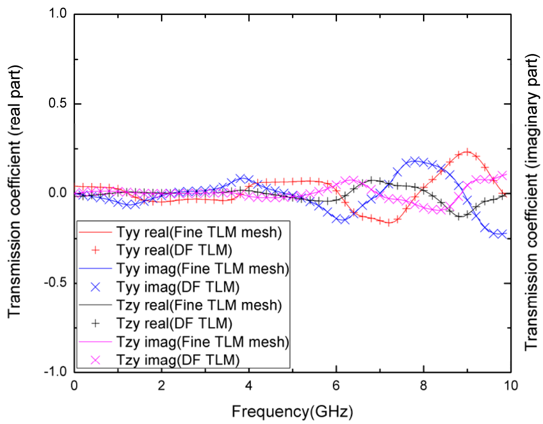

Fig. 5 – *Transmission coefficients Tyy and Tzy - real and imaginary parts.*

In order to develop the DF-based model for considered dispersive anisotropic carbon-fibre conductive panel, the rational approximations of order 24 (number of poles $NP = 24$) of overall reflection and transmission coefficients are obtained by using the VF method. Rational approximation of each reflection/transmission coefficient is then divided into n=6 groups of m=4 elements where each group consists out of 2 complex conjugate couples except group 1 where first two elements are real. In-house developed code has then been applied to efficiently represent boundary condition parameters. To do this, a compact model has been used, in which 120 nodes previously used in fine mesh to represent three layers of carbon fibre material are replaced with one internal boundary condition connecting two air layers, which reduced total number of nodes from 200 to 80. The real and imaginary parts of the frequency-domain reflection R_{ij} and transmission T_{ij} coefficients, obtained by Z–TLM mesh with incorporated DF-based compact model, are marked in Figs. 4–7 with cross and plus symbols. An excellent agreement between fine Z–TLM mesh results and results obtained by using DF Z–TLM can be observed.

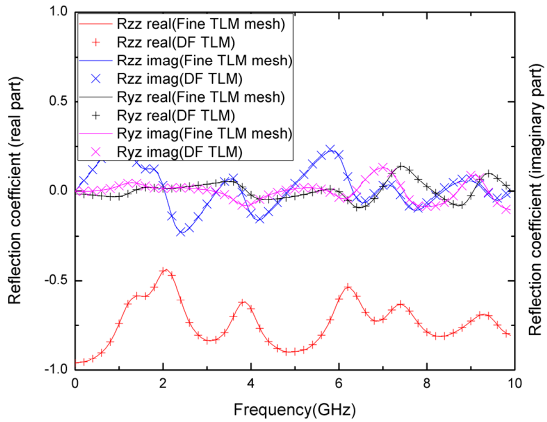

Fig. 6 – *Reflection coefficients Rzz and Ryz - real and imaginary parts.*

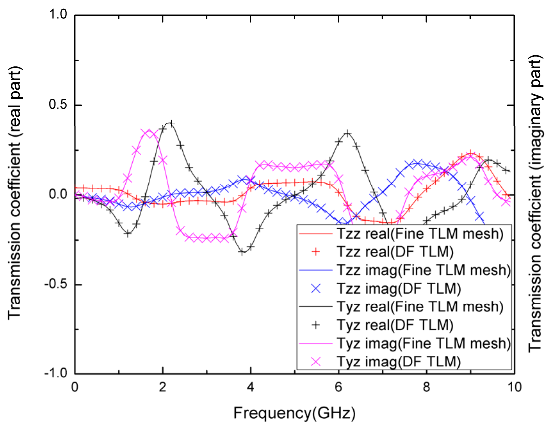

Fig. 7 – *Transmission coefficients Tzz and Tyz - real and imaginary parts.*

5 Conclusion

DF-based compact model of thin panel made of dispersive anisotropic carbon-fibre material, implemented into 1D Z–TLM method, is presented in this paper. In comparison with the fine meshing technique, significant reduction of required memory space and computation time is obtained while the accuracy is preserved. The presented approach will be extended to a graded TLM mesh and dispersive anisotropic metamaterials in future research.

6 Acknowledgement

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