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Proximity Effect in a Shielded Symmetrical Three-Phase Line

Dragan Filipović[1], Tatijana Dlabač[2]

Abstract: In this paper we present an approximate analysis of the proximity effect in a shielded symmetrical three-phase line with conductors of circular cross section. The system of two integral equations for current densities is solved approximately by assuming them in the form of two finite series with properly chosen basic functions. The unknown coefficients in these series are found by applying the point matching procedure. Numerical results are given for the AC to DC resistance ratio of the line conductors and for the power loss in the shield.

Keywords: Symmetrical three-phase line, Shield, Current density distribution, Proximity effect, AC to DC resistance ratio, Power loss.

1 Introduction

The phenomenon referred to as the proximity effect describes how conductors with time varying currents, when close to each other, mutually affect current distribution. This effect causes an increase of conductor resistance, i.e. additional Joule losses.

A powerful method for determining the current distribution in a system of several conductors with sinusoidal currents, proposed by Manneback [1], is the method of integral equations, where the current densities are described by a system of integral equations. Manneback used this method to solve in a closed form the problem of the current distribution in a solid round conductor influenced by a filament, and also derived a system of two integral equations for the determination of the current distribution in two parallel cylindrical conductors. Dwight [2] used these equations in considering some systems of two or three conductors of various cross sections and gave some exact and approximate solutions for current distribution. Both Manneback and Dwight used the method of successive approximations for solving integral equations.

Instead of using the method of successive approximations, an alternative method for solving a system of integral equations is frequently used. Namely, a solution is sought in the form of finite or infinite linear combinations of properly chosen basic functions with unknown coefficients. These coefficients

[1]Faculty of Electrical Engineering, Cetinjski put bb, 81000 Podgorica, Montenegro; E-mail: draganf@ac.me
[2]Faculty of Maritime Studies, Dobrota 36, 85330 Kotor, Montenegro; E-mail: tanjav@ac.me

are determined by the point matching method which consists in imposing that integral equations be satisfied at a sufficient number of distinct points of conductor cross sections. In this way the system of integral equations is converted into a system of linear algebraic equations from which the unknown coefficients that determine current distribution are readily found. This method was successfully used in [3 – 8].

In this paper we investigate the proximity effect in a shielded symmetrical three-phase line with conductors of circular cross section. The current densities are assumed in the form of two finite series with properly chosen basic functions and then the point matching method is applied for determining the unknown coefficients. Once the approximate current distribution is known one readily finds the AC to DC resistance ratio and the power loss in the shield. A simpler case of a shielded symmetrical three-phase line whose conductors are filaments is analyzed using the same method in [8], where an exact solution for current distribution in the shield is obtained.

Some other methods for solving the problem of current distribution include boundary integral equation formulation [9 – 10], modal functions [11], differential equation approach in terms of the magnetic vector potential [12 – 15], etc.

Finally, two main advantages of the integral equation method over the above mentioned methods should be pointed out – the absence of any boundary condition and a low order of the system of algebraic equations that a system of integral equations is reduced to.

2 Integral Equation for Current Densities in the Line Conductors and in the Shield

The cross section of a shielded symmetrical three – phase line is shown in Fig. 1. The line conductors have circular cross section of radius *a*, the distance between the conductor axes is *D*, the mean radius and the thickness of the shield are *R* and d respectively ($d \ll R$). The conductor currents are $I_1 = I$, $I_2 = I\,\mathrm{e}^{-\mathrm{j}2\pi/3}$ and $I_3 = I\,\mathrm{e}^{+\mathrm{j}2\pi/3}$.

Due to 2π/3 symmetry, the current densities in the conductors will differ only by a phase shift of 2π/3, i.e. we may assume

$$J_1(x,y) = J(x,y),$$

$$J_2(x,y) = J(x,y)\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}},$$

$$J_3(x,y) = J(x,y)\mathrm{e}^{+\mathrm{j}\frac{2\pi}{3}}.$$

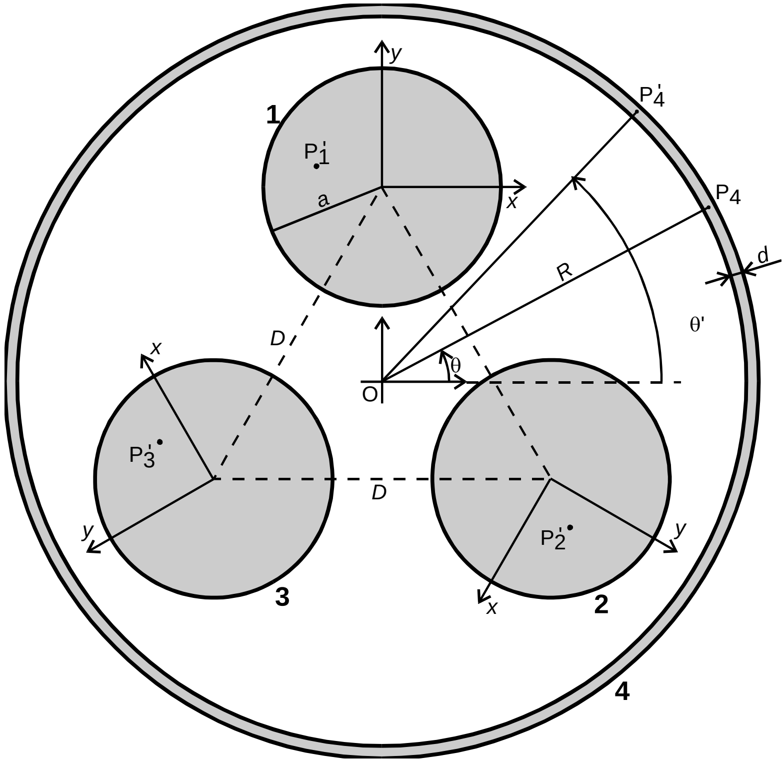

Fig. 1 – *Shielded symmetrical three-phase line with identical conductors of circular cross section.*

Consequently we have two unknown current densities: $J(x, y)$ (current density in conductor 1) and $J_{sh}(\theta)$ (current density in the shield). The integral equations for these two current densities are [6]:

$$J(x,y)=\frac{k^2a^2}{4\pi}\left[\int\limits_{S_1} J(x',y')\ln\left(\frac{\overline{P_1P_1'}}{H}\right)^2 \mathrm{d}x'\,\mathrm{d}y' + \right.$$
$$+\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}\int\limits_{S_2} J(x',y')\ln\left(\frac{\overline{P_1P_2'}}{H}\right)^2 \mathrm{d}x'\,\mathrm{d}y' +$$
$$\left. +\mathrm{e}^{+\mathrm{j}\frac{2\pi}{3}}\int\limits_{S_3} J(x',y')\ln\left(\frac{\overline{P_1P_3'}}{H}\right)^2 \mathrm{d}x'\,\mathrm{d}y' \right] + \tag{1}$$
$$+\frac{k^2Rd}{4\pi}\int\limits_0^{2\pi} J_{sh}(\theta')\ln\left(\frac{\overline{P_1P_4'}}{H}\right)^2 \mathrm{d}\theta' + K_1,$$

$$
\begin{aligned}
J_{sh}(\theta) = \frac{k^2 a^2}{4\pi} \Bigg[& \int_{S_1} J(x',y') \ln\left(\frac{\overline{P_4 P_1'}}{H}\right)^2 \mathrm{d}x' \mathrm{d}y' + \\
& + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}} \int_{S_2} J(x',y') \ln\left(\frac{\overline{P_4 P_2'}}{H}\right)^2 \mathrm{d}x' \mathrm{d}y' + \\
& + \mathrm{e}^{+\mathrm{j}\frac{2\pi}{3}} \int_{S_3} J(x',y') \ln\left(\frac{\overline{P_4 P_3'}}{H}\right)^2 \mathrm{d}x' \mathrm{d}y' \Bigg] + \\
& + \frac{k^2 R d}{4\pi} \int_0^{2\pi} J_{sh}(\theta') \ln\left(\frac{\overline{P_4 P_4'}}{H}\right)^2 \mathrm{d}\theta' + K_2,
\end{aligned}
\tag{2}
$$

where $k^2 = \mathrm{j}\omega\mu_0\sigma$ and K_i, i = 1, 2 are some unknown constants.

The distances $\overline{P_1 P_4'}, k = 1,2,3$ in (1) are

$$
\left(\overline{P_1 P_4'}\right)^2 = (x - x')^2 + (y - y')^2; \quad x, y \in S_1; \quad x', y' \in S_k, \quad k = 1,2,3, \tag{3}
$$

$$
\left(\overline{P_1 P_4'}\right)^2 = (x - R\cos\theta')^2 + \left(\frac{D}{\sqrt{3}} + y - R\sin\theta'\right)^2, \quad x, y \in S_1; \; x', y' \in S_4 . \tag{4}
$$

The coordinates $x',y' \in S_k$ (k = 1,2,3) are taken in the coordinate system related to conductor 1. Also, the integrals over S_k (k = 1,2,3) should be evaluated with respect to the same coordinate system.

The distances $\overline{P_4 P_k'}$ in (2) are

$$
\begin{gathered}
\left(\overline{P_4 P_1'}\right)^2 = (R\cos\theta - x')^2 + \left(R\sin\theta - y' - \frac{D\sqrt{3}}{6}\right)^2, \\
0 < \theta \le 2\pi, \; x', y' \in S_1,
\end{gathered}
\tag{5}
$$

$$
\begin{aligned}
\left(\overline{P_4 P_2'}\right)^2 = & \left(R\cos\theta + \frac{x'}{2} - \frac{y'\sqrt{3}}{2} - \frac{D}{2}\right)^2 + \\
& + \left(R\sin\theta + \frac{x'\sqrt{3}}{2} + \frac{y'}{2} + \frac{D}{2\sqrt{3}}\right)^2, \\
& 0 < \theta \le 2\pi, \; x', y' \in S_2,
\end{aligned}
\tag{6}
$$

$$\left(\overline{P_4P_3'}\right)^2 = \left(R\cos\theta + \frac{x'}{2} + \frac{y'\sqrt{3}}{2} + \frac{D}{2}\right)^2 + \\ + \left(R\sin\theta - \frac{x'\sqrt{3}}{2} + \frac{y'}{2} + \frac{D}{2\sqrt{3}}\right)^2, \quad 0 < \theta \le 2\pi,\ x', y' \in S_3, \tag{7}$$

$$\left(\overline{P_4P_4'}\right)^2 = 4R^2\sin^2\frac{\theta-\theta'}{2}, \quad \theta, \theta' \in [0, 2\pi]. \tag{8}$$

The coordinates $x', y' \in S_k$ (k = 1,2,3) are taken with respect to the local coordinate systems related to conductors 1, 2 and 3 respectively. The integrals over S_k (k = 1,2,3) are evaluated with respect to the coordinate system related to the shield (i.e. its center).

Finally, the distance H in (1) – (2) can be taken arbitrarily; we choose $H = a$.

3 Approximate Solution of the System of Integral Equations for the Current Densities

We seek for an approximate solution of integral (1) – (2) in the form

$$J(x,y) = \sum_{m=0}^{M_1}\sum_{n=0}^{N_1} a_{mn} x^m y^n, \tag{9}$$

$$J_{Sh}(\theta) = b_0 + \sum_{n=1}^{N_2}\left(b_n \cos n\theta + c_n \sin n\theta\right), \tag{10}$$

where a_{mn}, b_0, b_n and c_n are unknown coefficients.

By substituting (9) and (10) into (1) and (2) we obtain:

$$\sum_{m=0}^{M_1}\sum_{n=0}^{N_1} a_{mn}\left(F_{mn}^{(1,1)}(x,y) + F_{mn}^{(1,2)}(x,y) + F_{mn}^{(1,3)}(x,y)\right) + \\ + \sum_{n=1}^{N_2} b_n P_n^{(1,1)}(x,y) + \sum_{n=1}^{N_2} C_n P_n^{(1,2)}(x,y) = K_1, \tag{11}$$

$$\sum_{m=0}^{M_1}\sum_{n=0}^{N_1} a_{mn}\left(F_{mn}^{(2,1)}(\theta) + F_{mn}^{(2,2)}(\theta) + F_{mn}^{(2,3)}(\theta)\right) + \\ + \sum_{n=1}^{N_2} b_n P_n^{(2,1)}(\theta) + \sum_{n=1}^{N_2} C_n P_n^{(2,2)}(\theta) = K_2, \tag{12}$$

where

$$F_{mn}^{(1,1)}\left(x,y\right)=x^{m}y^{n}-\frac{k^{2}a^{2}}{4\pi}\int\limits_{S_1}x'^{m}y'^{n}\ln\left[\left(x-x'\right)^{2}+\left(y-y'\right)^{2}\right]\mathrm{d}\,x'\,\mathrm{d}\,y', \quad (13)$$

$$F_{mn}^{(1,2)}\left(x,y\right)=-\frac{k^{2}a^{2}}{4\pi}\mathrm{e}^{-\mathrm{j}\frac{2\pi}{3}}\int\limits_{S_2}x'^{m}y'^{n}\ln\left[\left(x-x'\right)^{2}+\left(y-y'\right)^{2}\right]\mathrm{d}\,x'\,\mathrm{d}\,y', \quad (14)$$

$$F_{mn}^{(1,3)}\left(x,y\right)=-\frac{k^{2}a^{2}}{4\pi}\mathrm{e}^{+\mathrm{j}\frac{2\pi}{3}}\int\limits_{S_2}x'^{m}y'^{n}\ln\left[\left(x-x'\right)^{2}+\left(y-y'\right)^{2}\right]\mathrm{d}\,x'\,\mathrm{d}\,y', \quad (15)$$

$$P_{n}^{(1,1)}\left(x,y\right)=-\frac{k^{2}Rd}{4\pi}\int\limits_{0}^{2\pi}\cos\theta'\ln\left[\left(x-R\cos\theta'\right)^{2}+\left(\frac{D}{\sqrt{3}}-y-R\sin\theta'\right)^{2}\right]\mathrm{d}\,\theta', \quad (16)$$

$$P_{n}^{(1,2)}\left(x,y\right)=-\frac{k^{2}Rd}{4\pi}\int\limits_{0}^{2\pi}\sin\theta'\ln\left[\left(x-R\cos\theta'\right)^{2}+\left(\frac{D}{\sqrt{3}}-y-R\sin\theta'\right)^{2}\right]\mathrm{d}\,\theta', \quad (17)$$

$$F_{mn}^{(2,1)}\left(\theta\right)=-\frac{k^{2}a^{2}}{4\pi}\int\limits_{S_1}x'^{m}y'^{n}\ln\left[\left(R\cos\theta-x'\right)^{2}+\left(R\sin\theta-y'-\frac{D}{\sqrt{3}}\right)^{2}\right]\mathrm{d}\,x'\,\mathrm{d}\,y', \quad (18)$$

$$\begin{aligned}F_{mn}^{(2,2)}\left(\theta\right)=-\frac{k^{2}a^{2}}{4\pi}\mathrm{e}^{-\frac{2\pi}{3}}\int\limits_{S_1}x'^{m}y'^{n}\ln\Bigg[&\left(R\cos\theta+\frac{x'}{2}-y'\frac{\sqrt{3}}{2}-\frac{D}{2}\right)^{2}+\\&+\left(R\sin\theta+x'\frac{\sqrt{3}}{2}+\frac{y'}{2}+\frac{D}{2\sqrt{3}}\right)^{2}\Bigg]\mathrm{d}\,x'\,\mathrm{d}\,y',\end{aligned} \quad (19)$$

$$\begin{aligned}F_{mn}^{(2,3)}\left(\theta\right)=-\frac{k^{2}a^{2}}{4\pi}\mathrm{e}^{+\frac{2\pi}{3}}\int\limits_{S_1}x'^{m}y'^{n}\ln\Bigg[&\left(R\cos\theta+\frac{x'}{2}+y'\frac{\sqrt{3}}{2}+\frac{D}{2}\right)^{2}+\\&+\left(R\sin\theta-x'\frac{\sqrt{3}}{2}+\frac{y'}{2}+\frac{D}{2\sqrt{3}}\right)^{2}\Bigg]\mathrm{d}\,x'\,\mathrm{d}\,y',\end{aligned} \quad (20)$$

$$P_{n}^{(2,1)}\left(\theta\right)=\cos n\theta-\frac{k^{2}Rd}{4\pi}\int\limits_{0}^{2\pi}\cos\theta'\ln\left[4R^{2}\sin^{2}\frac{\theta-\theta'}{2}\right]\mathrm{d}\,\theta', \quad (21)$$

$$P_{n}^{(2,2)}\left(\theta\right)=\sin n\theta-\frac{k^{2}Rd}{4\pi}\int\limits_{0}^{2\pi}\sin\theta'\ln\left[4R^{2}\sin^{2}\frac{\theta-\theta'}{2}\right]\mathrm{d}\,\theta'. \quad (22)$$

To find the unknown coefficients a_{mn} ($m = 0, 1, 2, ..., M_1$; $n = 0, 1, 2, ..., N_1$), b_n ($n = 0, 1, 2, ..., M_2$), c_n ($n = 1, 2, ..., M_2$) and two constants K_i ($i = 1, 2$) we apply the standard point matching procedure. We stipulate that (11) be satisfied at $(M_1 + 1)(N_1 + 1)$ distinct points of conductor 1, and that (12) be satisfied at $2N_2+1$ distinct points of the shield. Two additional equations are obtained from the known current I in conductor 1

$$I = \int_{S_1} J(x,y)\,\mathrm{d}y = \sum_{m=0}^{M_1}\sum_{n=0}^{N_1} a_{mn} \int_{S_1} x^m y^n \,\mathrm{d}x\,\mathrm{d}y = \\ = a^2\pi \sum_{m=0}^{M_1}\sum_{n=0}^{N_1} a_{mn} \frac{m!\left[2(n+1)\right]!}{2^{m+2n+1}(2n+1)\left(\frac{m}{2}\right)!(n+1)!\left(\frac{m}{2}+n+1\right)!}, \tag{23}$$

where the summation in m is performed only over odd values, and from the fact that the total induced current I_{Sh} the shield is equal to zero

$$I_{Sh} = \int_0^{2\pi} J_{Sh} R d \,\mathrm{d}\theta = b_0 R d\, 2\pi = 0\,,$$

from which it follows that

$$b_0 = 0\,. \tag{24}$$

In this way we obtain from (11), (12), (23) and (24) all necessary $(M_1 + 1)(N_1 + 1)+2N_2+2$ equations for the unknown coefficients a_{mn}, b_n, c_n and two constants K_1 and K_2. Once the coefficients a_{mn}, b_n and c_n are found, the current densities $J(x,y)$ and $J_{Sh}(x,y)$ are determined by (9) – (10).

4 AC to DC Resistance Ratio

One important consequence of the proximity effect is an increase of resistance, i.e. increase of power loss. This increase is measured through the AC to DC resistance ratio of the line conductors

$$\frac{R'_{a.c.}}{R'_{d.c.}} = a^2\pi \frac{\int_S \left|J(x,y)\right|^2 \mathrm{d}x\,\mathrm{d}y}{\left|\int_S J(x,y)\,\mathrm{d}x\,\mathrm{d}y\right|^2}\,, \tag{25}$$

where $J(x,y)$ is given by (9).

5 Power Loss in the Shield

Induced currents in the shield give rise to an additional power loss which is found from Joule's law

$$P'_{JSh} = \frac{1}{\sigma}\int_0^{2\pi} \left|J_{Sh}(\theta)\right|^2 Rd\,\mathrm{d}\theta = \frac{Rd\pi}{\sigma}\sum_{p=1}^{N_4}\left(\left|b_p\right|^2 + \left|c_p\right|^2\right). \quad (26)$$

6 Numerical Results

Fig. 2. shows the normalized current distribution, i.e. the lines of equal magnitudes of the current densities for conductors from Fig. 1. It was taken: $a = 8{,}8$ mm, $R = 3$ cm, $D = 2{,}5\,a$, $d = 0.5$ mm, $\sigma = 5.7\cdot 10^7$ S/m and $f = 60$ Hz. The same distribution obtained by implementing the programming package FEMM 4.0 (*Finite Element Method Magnetics*) is depicted in Fig. 3.

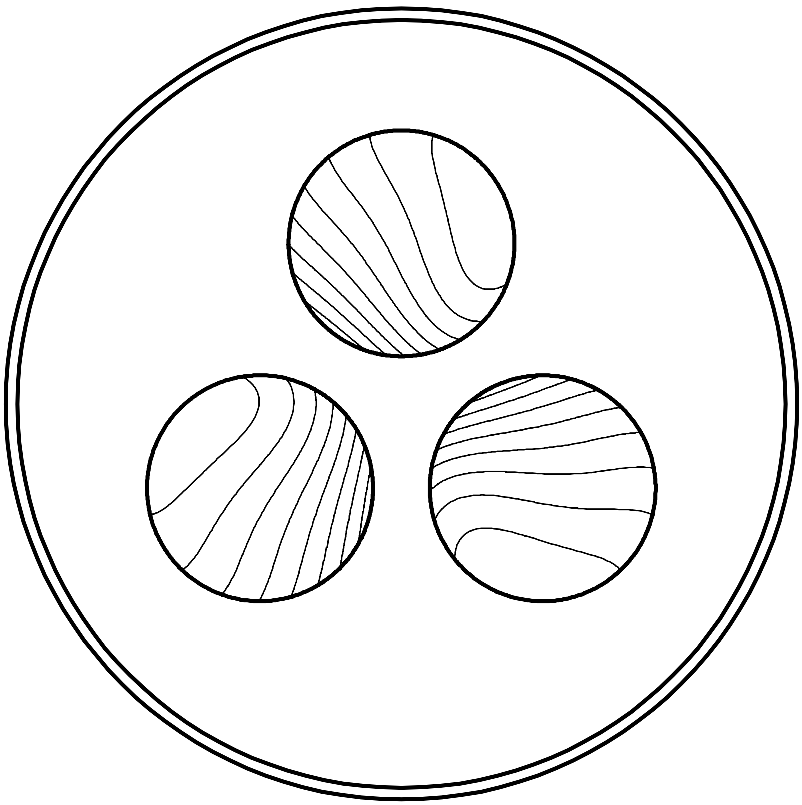

Fig. 2 – *Current distribution in the conductors from* Fig. 1.

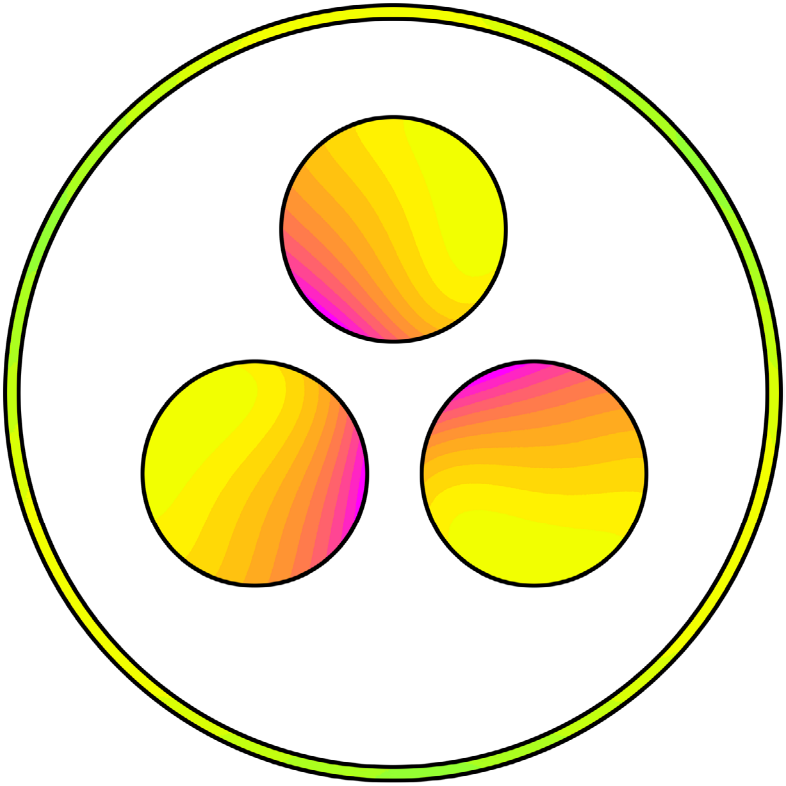

Fig. 3 – *Current distribution in the conductors from* Fig. 1.
(*obtained by implementing the* FEMM 4.0 *software package*).

Fig. 4. shows the AC to DC resistance ratio for the line conductors from Fig. 1 versus frequency f. It was taken: a = 8,8 mm, R = 3 cm, D = 2,2 cm, d = 0.5 mm and $\sigma = 5.7 \cdot 10^7$ S/m.

Fig 5. shows the normalized current density in the shield of the symmetrical three-phase line from Fig. 1. It was taken: a = 8,8 mm, R=3 cm, d = 0.5mm, D =2,5, $\sigma = 5.7 \cdot 10^7$ S/m and f = 60 Hz.

The dependence of power loss in the shield of the symmetrical three-phase line from Fig. 1 on frequency is shown by solid line in Fig. 6. It was taken: a = 8,8 mm, R = 3 cm, d = 0.5 mm, D =2,5 a, $\sigma = 5.7 \cdot 10^7$ S/m and I = 50 A. In the same figure the power loss in the shield, calculated by the exact formula from [8], which treats the conductors as filaments, is shown by dashed line.

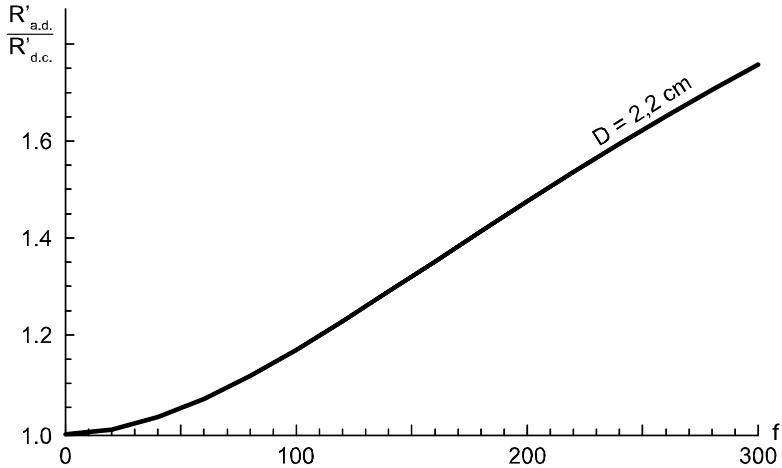

Fig. 4 – *Resistance ratio R_{AC}/R_{DC} for the line conductors from* Fig. 1. *versus frequency for D* = 2.2 cm.

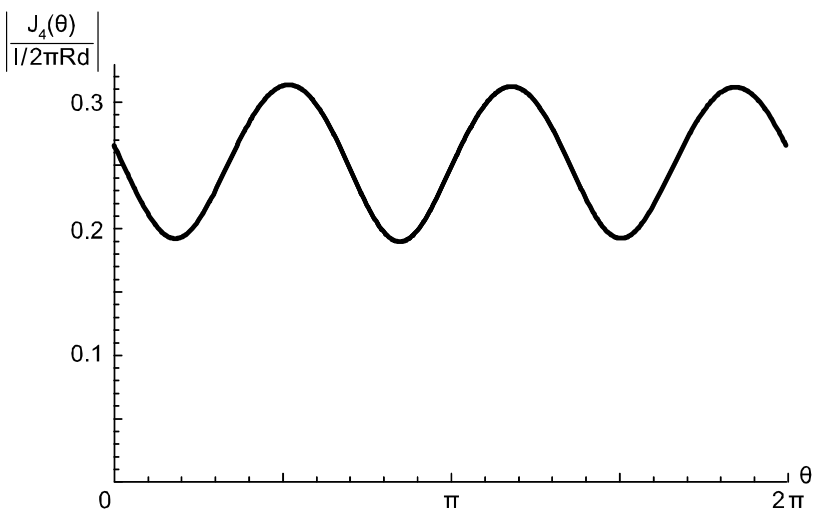

Fig. 5 – *Normalized current density in the shield of the symmetrical three-phase line from* Fig. 1.

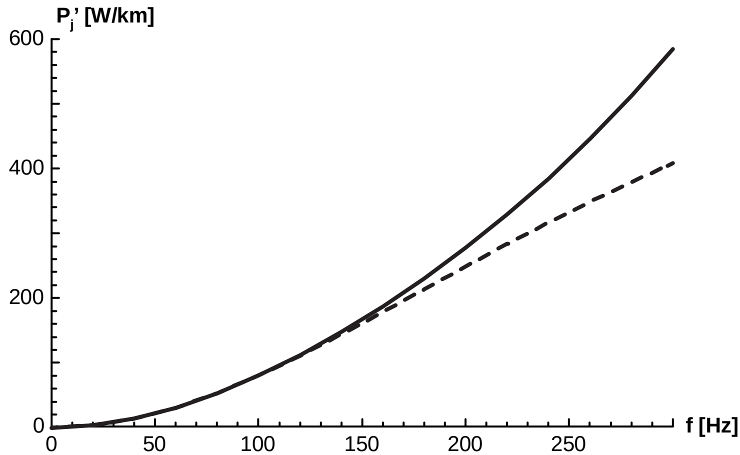

Fig. 6 – *Dependence of power loss in the shield of the symmetrical three-phase line from* Fig. 1.

7 Conclusion

This paper presents an approximate analysis of the proximity effect in a shielded symmetrical three-phase line with conductors of circular cross section. The system of two integral equations for the current densities in one of the line conductors and in the shield are approximately solved by assuming the current densities in the form of two finite series with properly chosen basic functions with unknown coefficients. These coefficients are determined by the point matching method. Also, the AC to DC resistance ratio for line conductors and the power loss in the shield is calculated.

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