SERBIAN JOURNAL OF ELECTRICAL ENGINEERING
Vol. 11, No. 4 (special issue), December 2014, 551-563

UDC: 621.375.826:546.666]:621.376.22 **DOI: 10.2298/SJEE1404551R**

Erbium – Doped Fiber Laser Systems: Routes to Chaos

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Abstract: Erbium-doped fiber laser systems exhibit a large variety of complex dynamical behaviors, bifurcations and attractors. In this paper, the chaotic behavior which can be achieved under certain conditions in a laser system with erbium-doped fiber, is discussed. The chaos in this system occurs through several standard scenarios. In this paper, the simulation sequence of quasi-periodic, intermittent and period-doubling scenario transitions to chaos is shown. Quasiperiodic and intermittent transitions to chaos are shown on the example system with a single ring. The electro-optical modulator was applied to the system for modulating the loss in the cavity. We used the sinusoidal and rectangular signals for modulation. Generation of chaos is achieved by changing the parameters of signal for modulation. Period-doubling transition to chaos is illustrated in a system with two rings. Simulation results are shown in the time domain and phase space.

Keywords: Erbium-doped fiber ring laser (EDFRL), Chaos.

1 Introduction

Erbium-doped fiber ring laser (EDFRL) is considered in many experimental and theoretical works as an interesting optical device for generating optical chaos [1 – 7]. EDFRL is a system with three energy levels and belongs to a class B laser in which the laser field and the population inversion build the dynamic equations [8, 9]. In order to be manifested as a chaotic system, as belonging to class B, another perturbation must be added to his equations [10, 11]. Several approaches to generate chaotic dynamic in EDFRL were discussed in literature. A group of papers is based on the introduction of an ion-pair model [1], and the other group is based on the introduction of modulation of certain parameters [3 – 6].

For a fixed set of parameters, equations that describe EDFRL define a dynamic system that manifest a particular behavior. For example, the trajectory can converge to an equilibrium point, a limit cycle or a strange attractor. The equilibrium point can be stable or unstable. The variation in the qualitative

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behavior of the system or bifurcations when parameters change the value are especially interesting. In this review paper, some simulations of chaotic phenomena and bifurcations, will be presented [10, 11]:

1. Quasiperiodic (torus breakdown) transition
2. Intermittency transition
3. Period–doubling transition

For better visualization of chaos and bifurcation phenomena we use:

1. Phase portraits of trajectory
2. Time waveforms of variables

The chaotic oscillations which can be generated in EDFR allow an attractive way to exploit the wide bandwidth available in optical communication systems. The discovery of Pecora and Carroll [12] that the synchronization of chaotic transmitters and receivers is possible has opened the way for the use of chaos in secure communications in electrical and optical domain [13]. The techniques for the modulation of the information in the broadband chaotic oscillations and signal transmission along the optical cable and demodulation of information in the receiver have been developed. Well described techniques in secure chaotic communications are message masking, message modulation and chaos shift keying [13 – 18].

The above scenario of chaos generation will be shown in the example of EDFRL that plays the role of the transmitter in one chaotic masking communication scheme. As well, the range of bifurcation parameter for which are obtained chaotic solutions and chaotic outputs, will be determined. The generation of chaos in dual ring EDFL which is also the part of transmitter, is then shown.

This paper is organized as follows: After the introduction, in the second chapter, we will show the basis of nonlinear optical phenomena. The third chapter describes EDFRL chaotic system and contains simulation sequences for quasiperiodic and intermittent scenario of transition to chaos. Then, in the fourth section, the EDFL dual ring system is described, and the simulated period-doubling route to chaos which is observed in it, is presented. At the end, conclusions were made.

2 Nonlinear Optical Effects

The nonlinear optical phenomena originate due to the optical properties of the material depending on the strength of the electromagnetic field of light that spreads through the material. They do not apply the principle of superposition, which means that different optical waves do not spread independently through non-linear optical materials, but it must be taken into account their mutual interference and generation of light at the other frequencies.

The non-linearity which is modeled by the nonlinear dependence of the material polarization vector $\vec{P}$ and electric field of light $\vec{E}$ is described in a general form as [19]:

$$\vec{P} = \varepsilon_0(\chi^{(1)} \cdot \vec{E} + \chi^{(2)} \cdot \vec{E} \cdot \vec{E} + \chi^{(3)} \cdot \vec{E} \cdot \vec{E} \cdot \vec{E} + \cdots), \tag{1}$$

where ε_0 is the permittivity in vacuum, and the $\chi^{(i)}$ susceptibility of the i-th order. Susceptibilities are generally tensor quantities. In the linearized approximation the susceptibility of the first order is only taken into account. The susceptibilities of higher order gives different nonlinear effects. One of the most interesting is the emergence of chaos in optical systems [20].

3 Model of Erbium-Doped Fiber Laser Systems

Erbium-doped fiber laser is considered as a three-level energy system that has a basic state, one metastable state and one state excited by the pump. The rate of the decay of polarization variables is of the order $10^{12}\,\mathrm{s}^{-1}$. Thus, the polarization variables can be adiabatically eliminated and the laser can be considered as laser class B. The population of pump excited state is negligible due to the fast non-radiant transition to the metastable state. After certain simplifying from standard laser equations in the theory of lasers [8, 9] the following equation for the lasing field E_L is obtained, as well as the population inversion D of the laser with the erbium doped fiber.

$$\begin{aligned} \dot{E}_L &= -kE_L + gE_L D + \xi, \\ \dot{D} &= -\frac{1}{\tau}\left[(1 + I_P + E_L^2)D - I_P + 1\right], \end{aligned} \tag{2}$$

where "." denotes the derivative, E_L lasing field, D the density of population inversion, τ a time of decay of meta-stable states of erbium. k and g are, respectively, coefficient of loss and gain coefficient of lasing fields. ξ represents the spontaneous-emission process small of amplitude random fluctuations. I_P is the power of the laser pump.

As we can see from the equation above, EDFRL is nonlinear system of second order. The dynamics of the system is described by two equations; one is for the field E_L, the other is for the population inversion D. Laser equations with two degrees of freedom require additional perturbation in the form of optical feedback with delay [21], the external signal injection [22] or feedback with phase conjugation [23] to generate chaos, because according to the

Poincaré-Bendixon theorem [10, 11] at least three degrees of freedom are required in order that the chaos in the dynamic system is to be expressed.

EDFRL is a dissipative system and the losses of his model are responsible for the absorption of energy during lasing action. In this paper we used the modulation loss in the resonator to produce a chaotic output from the system. This is achieved by inserting electro-optic modulators in the fiber loop. In this case, the coefficient of loss of the field is expressed as:

$$k = k_0(1 + m\sin\omega t), \tag{3}$$

where m is the modulation index and ω is the angular modulation frequency.

The laser with erbium-doped fiber and modulated losses described by equations (2) and (3) is the basis for obtaining a transmitter in communication schemes with chaotic signals.

The transmitter in the message-masking scheme, which is shown in Fig. 1. is described here. Main part of the transmitter is the fiber ring laser formed of coupler a_2, and a segment of the erbium-doped fiber. An electro-optical modulator is inserted in the ring that forms the fiber. Another coupler a_3 was inserted into the receiver to get the chaotic field E_L. Message signal S_{in} is mixed with E_L through couplers a_1 and a_4. Also, it affects the signal within the ring through couplers a_1 and a_2. The combined signal for transmission by a suitable choice of coupling coefficients $a_1 - a_4$, can be written in the form $\propto |E_L + S_{in}|^2$ [6]. This signal is transmitted toward the receiver.

The behavior of the transmitter is described, therefore, with the following equations:

$$\begin{gathered}
\dot{E}_L = -k\left(E_L - cS_{in}\right) + gE_L D + \xi, \\
\dot{D} = -\frac{1}{\tau}\left[(1 + I_P + E_L^2)D - I_P + 1\right], \\
k = k_0(1 + m\sin\omega t), \\
S_{in} = S_0(1 - \sin\omega_s t),
\end{gathered} \tag{4}$$

where S_0 is the amplitude and ω_s is the frequency of the message. c is a part of the signal introduced through the couplers.

The cavity parameters are been optimized to obtain chaos. The resulting field E is a series of positive chaotic impulses in fixed intervals. The field in between equals zero.

The population inversion D increases when the field E is equals zero because it arises due to the energy of the pump that enters the cavity continuously. D decreases rapidly during the formation of pulses because the stimulated emission of photons is accompanied by the transition of electrons from a higher to a lower energy level.

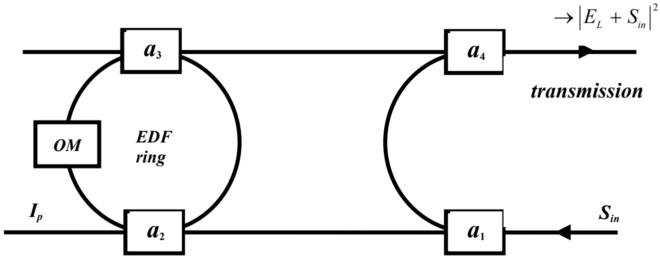

Fig. 1 – *Transmitter with EDFRL included.*

3.1 Simulation results – Quasiperiodic scenario of transition to chaos

Of all known transition to chaos quasiperiodic or Rulle-Takens-Newhouse's is earliest observed in [10, 11]. In the basis of the transition is Hopf 's bifurcation. Hopf's bifurcations occur in a wide variety of nature of physical systems – from the fluid flow to the optical resonator.

In our simulations, the modulation index m is changed within the range from 0 to 1, while other parameters values are shown in the third column of **Table 1** [4].

The existence of chaos is qualitatively verified by direct observational method, which is based on the phase portraits in the phase space [10, 11]. In the case of periodic motion in phase space the closed trajectory is observed. In the event of chaos in phase space strange attractors can be seen with non-closed trajectory.

The modulation index m was increased from 0 to 1 and outputs of the transmitter as shown in Fig. 2. For $m = 0$ the output is sinusoidal, as can be seen in Fig. 2a with a frequency that equals the frequency of a message ω_s. For $m = 0.01$ the output becomes quasiperiodic, as shown in Fig. 2b. By increasing m to 0.02 and more, outputs will still transit from quasiperiodic to chaotic. Fig. 2c shows the chaotic signal obtained for $m = 0.04$. Further increasing the modulation index to the value of 1 (Fig. 2d), the signal remains chaotic and the amplitude of chaotic impulses are increased 30 times. Thus, the modulation index m is the control parameter whose growth system moves from the periodic through quasiperiodic to a chaotic state. This is a quasiperiodic scenario of transition to chaos. It has also been noted that the dynamic range of the chaotic impulses increases with the increase of the modulation index. However, this generates more pulses with an amplitude of zero and increases the distance between the chaotic pulses, which can have a negative impact on the masking signal.

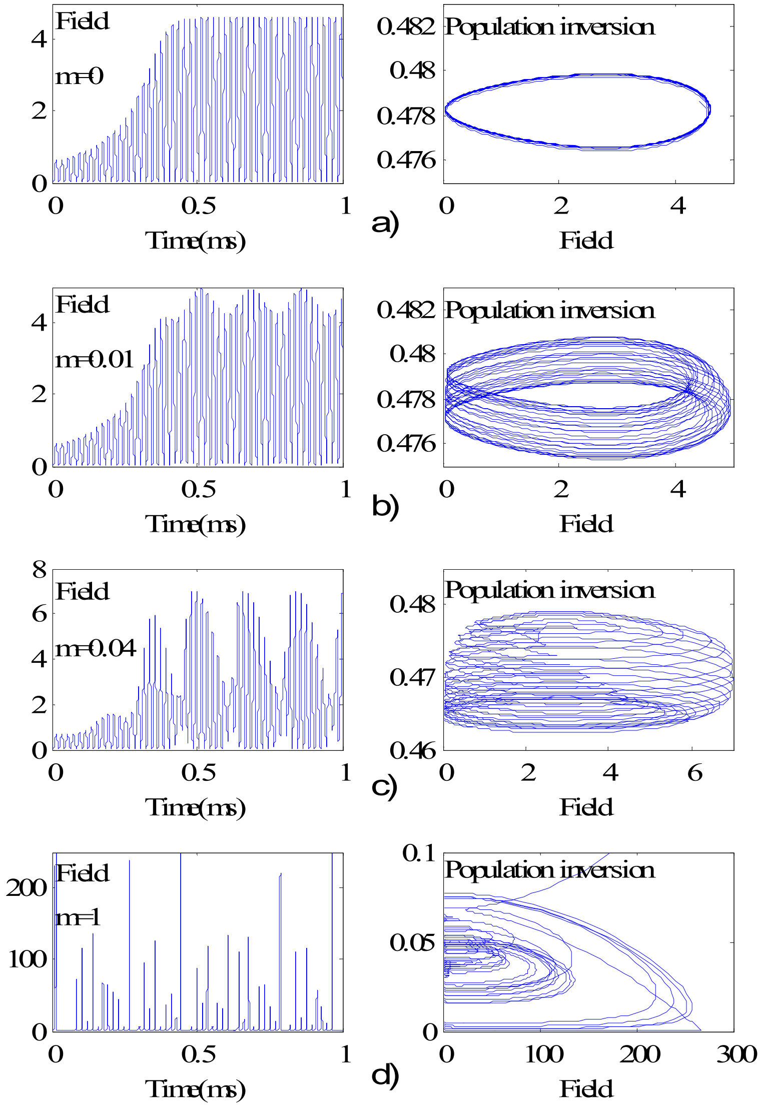

Fig. 2 – *Generating the chaos by increasing the modulation index m – quasiperiodic scenario of transition to chaos*: (a) m = 0 (*periodic limit cycle*), (b) m = 0.01 (*torus attractor*), (c) m = 0.04 (*torus attractor*), (d) m = 1 (*chaotic attractor*).

Table 1
Parameter values in EDFRL.

Parameter	Symbol	Sine wave modulation	Square wave modulation
Decay time of metastable state	τ	10 ms	10 ms
Spontaneous-emission factor	ξ	10^{-4}	10^{-4}
Pump power	I_P	10 mW	70 mW
Modulation index	m	0-1	0.3
Decay rate	k_0	$3.3 \cdot 10^7$	$3.3 \cdot 10^7$
Gain	g	$2k_0$	$5 \cdot 10^7$
Message amplitude	S_0	1	1
Modulating frequency	ω	$3.5 \cdot 10^5$	$5 \cdot 10^5$
Frequency of message	ω_s	$3.14 \cdot 10^5$	$3.14 \cdot 10^5$
Part of the signal introduced through the couplers	c	0 – 1	0 – 1

3.2 Simulation results – Intermittent scenario of transitions to chaos

Modulation of losses in the fiber can be achieved by a signal which is not purely sinusoidal. In our simulations, we used the rectangular periodic signal for loss modulation

$$k = k_0 \left(1 + \omega square\left(\omega t + \phi\right)\right). \tag{5}$$

In this way two additional parameters are obtained, and they can be changed in order to obtain a more complex chaotic signal and a higher degree of security communication [5]. These parameters are the duty cycle and the phase of rectangular periodic signal. Using this form of modulation and the duty cycle as a parameter that changes the intermittent scenario, we achieve transitions to chaos. The intermittency are alternating periodic-chaotic phases in the dynamics of systems that exist for the same value of the control parameter. They are characteristic for intermittent or Pomeau-Manneville's transition to chaos [10, 11].

As the system approaches the crossing point, the intervals of regular motion become shorter, and the turbulence hit becomes longer and longer, until there is only chaos.

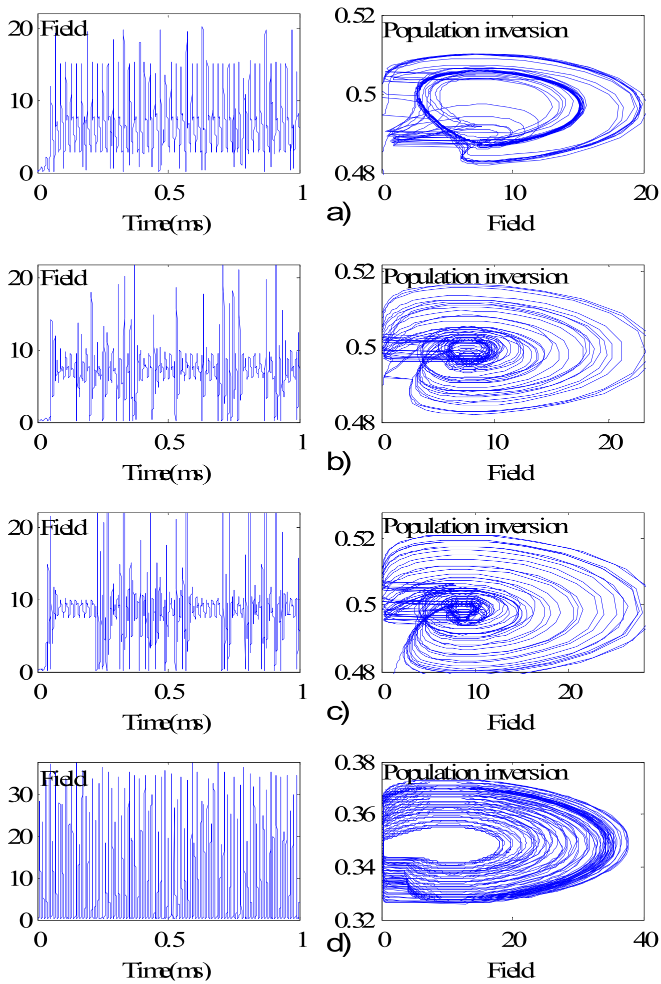

Fig. 3 – Simulation results for the intermittent scenario for transition to chaos: (a) *Duty cycle* = 0.05% (intermittency starts), (b) *Duty cycle* = 0.1% (intermittency), (c) *Duty cycle* = 0.2% (intermittency), (d) *Duty cycle* = 10% (chaos).

In our simulations, the bifurcation parameter is the duty cycle. The value of duty cycle is changed in the range from 0.05% to 90%, while the other parameters have the values as in the fourth column of **Table 1**. For $\mathit{Duty\ cycle} = 0.05\%$ in the Fig. 3a on the left is shown the time shape of the field which corresponds to the trajectory in Fig. 3a right. The time shape consisting of long regular parts and short peaks. This is the typical characteristic of intermittency. If we increase the *duty cycle* further, regular part is shortened, and the peaks occur more frequently, it can be seen in Figs. 3b and 3c. For the critical value of the control parameter $Duty\ cycle = 0.3\%$ it's reached the chaos. On Fig. 3d left ($Duty\ cycle = 10\%$) waveform looks completely chaotic. The corresponding chaotic attractor is shown in Fig. 3d right. Further increasing the value of the control parameter provides different chaotic attractors, whose complexity can be quantified by calculating the Lyapunov exponent spectrum.

4 Chaos in Erbium–Doped Fiber Dual-Ring Laser System

In the previous examples, the chaotic behavior is achieved by a different modulation of loss coefficient. Otherwise, the modulation of some of the parameters is the most common way to generate chaos in the laser class B. To illustrate the period-doubling route to chaos, we chose the generation of chaos in erbium-doped fiber dual-ring laser system [3]. In this system chaos is generated by changing the values of coefficients of gain, without modulation of some parameter of the system.

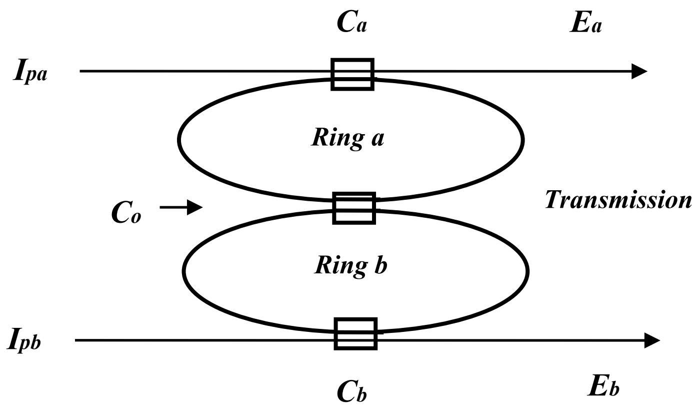

Fig. 4 – *Erbium-doped fiber dual-ring laser system.*

The fiber dual-ring laser system is shown in Fig. 4. The basic system contains two rings of erbium-doped fiber. They are coupled by coupler C_0 with small coupling coefficient. Both ring resonators are also coupled to a separate standard fiber through couplers C_a and C_b. When the lasing fields in the two rings are frequency locked through the coupler C_0 with phase change $\pi/2$ from one ring to the other, the equations for the fundamental system are as follows:

$$\begin{aligned}\dot{E}_a &= -k_a\left(E_a + C_0 E_b\right) + g_a E_a D_a,\\ \dot{E}_b &= -k_b\left(E_a - C_0 E_a\right) + g_b E_b D_b,\\ \dot{D}_a &= -\left(1 + I_{pa} + E_a^2\right) D_a + I_{pa} - 1,\\ \dot{D}_b &= -\left(1 + I_{pb} + E_b^2\right) D_b + I_{pb} - 1,\end{aligned} \tag{6}$$

where E_a and E_b are the lasing fields and D_a and D_b are the population inversions in ring a and ring b, respectively. The parameters k_a, k_b, g_a and g_b are respectively the decay rates and the gain coefficients of the lasing intensity in ring a and ring b. I_{pa} and I_{pb} represent the pump intensity in the respective fiber ring.

4.1 Simulation results - Period-doubling route to chaos

By using computer simulations we studied the dynamic behavior of the system. In this system chaos can be generated through period-doubling scenario [10, 11]. These are the results from the interaction of the two lasing fields through the coupler C_0. To illustrate the period-doubling route to chaos, erbium-doped fiber dual-ring system will be observed with the following parameter values: $k_a = 1000$, $k_b = 1000$, $g_a = 10500$, $C_0 = 0.2$, $I_{pa} = 5\text{mW}$, $I_{pb} = 5\text{mW}$ [3]. In our simulations parameter g_b changes in the range of 4300 to 5200. So bifurcation parameter is the gain coefficient g_b. In this scenario of transition to chaos, the equilibrium point loses stability and the stable limit cycle emerges through the Andronov-Hopf's bifurcation when increasing the value of the parameter g_b. Fig. 5 shows the phase portraits corresponding to period-doubling route to chaos. For $g_b = 4370$ the Fig. 5a, we see period 1 limit cycle.

If we increase the gain g_b further, period 1 limit cycle loses stability and a stable limit cycle occurs approximately with twice the period, which we call the period 2 limit cycle ($g_b = 4470$, Fig. 5b). If the gain g_b is further increased, the period 2 limit cycle loses stability and there is a period 4 limit cycle ($g_b = 4515$, Fig. 5c). These bifurcations are repeated infinitely many times: arise period 8 cycle ($g_b = 4524$, Fig. 5d), the period 16 cycles and at the end for $g_b \geq 4535$ (Fig. 5e and 5f), there is chaos.

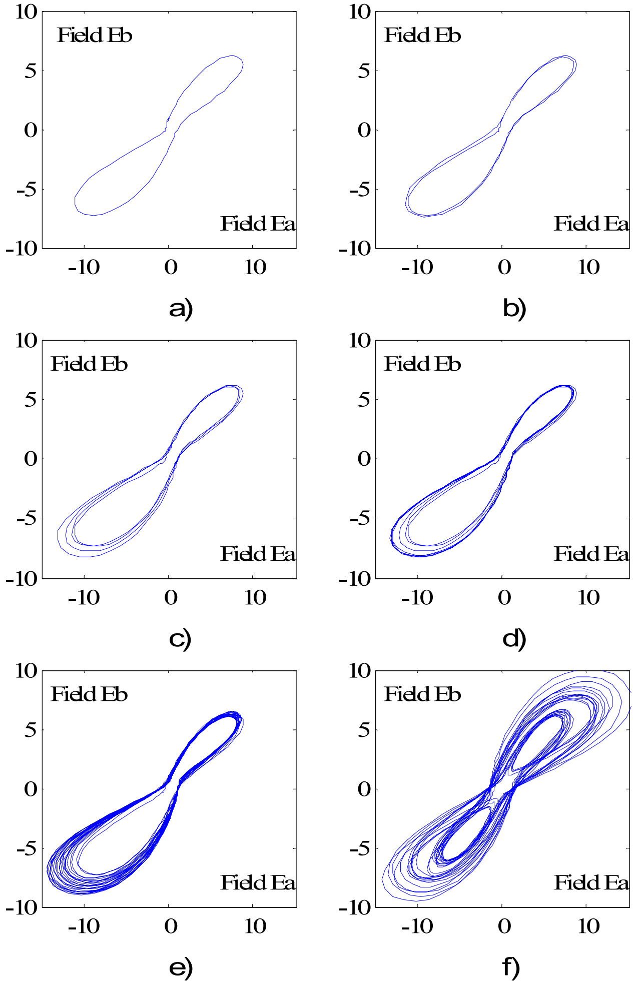

Fig. 5 – *Period-doubling route to chaos and strange attractors:*
(a) $g_b = 4370$ (*period 1 cycle*), (b) $g_b = 4470$ (*period 2 cycle*),
(c) $g_b = 4570$ (*period 4 cycle*), (d) $g_b = 4524$ (*period 8 cycle*),
(e) $g_b = 4535$ (*chaos*), (f) $g_b = 4520$ (*chaos*).

5 Conclusion

In this paper, different scenarios for generating chaos in erbium-doped fiber laser systems, are considered. Using different types of modulation of losses, chaos is achieved through quasiperiodic and intermittent scenarios in erbium-doped fiber laser systems with a single ring. In the erbium-doped fiber laser with two rings chaos is through period-doubling scenario generated by changing the value of a one parameter. The dependence of chaotic dynamics in EDFRL of value of bifurcation parameter, is examined. Simulation results are presented in the time domain and in the phase space. Optical chaos generated in fiber lasers is an excellent mask to hide the message signal due to the complexity of the waveforms and longtime unpredictability. Furthermore, it may be considered dependence of the chaotic dynamics on the other system parameters. Also, the other waveform of the modulation signals and their effect on the degree of chaos, and thus the security of communication, can be considered.

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