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Second Order Statistics of SC Receiver over k-μ Multipath Fading Channel

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Abstract: In this paper wireless communication system with dual branch selection combining (SC) diversity receiver operating over k-μ multipath fading environment is considered. Closed form expressions for average level crossing rate of SC receiver output signal envelope and average fade duration of proposed system are evaluated. Numerical results are presented graphically to show the influence of Rican factor k and fading severity μ on average level crossing rate and average fade duration.

Keywords: Multipath fading, Level crossing rate, Average fade duration, Selection diversity.

1 Introduction

Short term fading degrades system performance and limits channel capacity. Received signal experiences multipath fading result in signal envelope variation. There are more distributions which can be used to describe signal envelope variation in fading channels depending on propagation environment and communication scenario. The most frequently used statistical models are Rayleigh, Rican, Nakagami-m and α-μ distributions. Rayleigh distribution can be used to describe small scale signal envelope variation in linear non line-of-sight multipath environments. Rican distribution can be used to describe signal envelope variation in linear line-of-sight multipath fading environments. In linear non line-of-sight multipath fading environments, in the presence of more number of clusters, small scale signal envelope variation can be described by using Nakagami-q distribution [5].

Parameter m in Nakagami-m distribution is fading severity. For lower values of parameter m, fading is more severe. By setting $m = 1$, Nakagami-m distribution reduces to Rayleigh distribution and one sided Gaussian distribution can be derived from Nakagami-m distribution by setting $m = 0.5$. The Weibull

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and α-μ distribution can be used to describe small scale signal envelope variation in non-linear and line-of-sight multipath fading environments. The α-μ distribution is general distribution. Rayleigh, Nakagami-m and Weibull distributions can be obtained from α-μ distribution as special cases. By setting $\alpha = 2$, α-μ distribution reduces to Nakagami-m distribution and Weibull distribution can be derived from α-μ distribution by $\mu = 1$ [6]. The α-μ distribution approximates Rayleigh distribution by setting $\alpha = 2$ and $\mu = 1$. Weibull distribution approximates Rayleigh distribution by setting $\alpha = 2$. Log-normal distribution and Gamma distribution can be used to describe long scale signal envelope variation in shadowing fading environments. In shadowed multipath fading channel, signal envelope variation can be described by using K or K_G distributions. Nakagami-q distribution can be used to describe small scale signal envelope variation in linear non line-of-sight multipath fading environments where power of in-phase component and power of quadrature component are different [6].

There are several combining techniques that can be used to reduce multipath fading effects on system performance depending on implementation complexity of communication system and amount of channel state information available at the receiver. The most frequently used combining techniques are maximal radio combining (MRC), equal gain combining (EGC), selection combining (SC), switch and stay combining (SSC) and general selection combining (GSC). The MRC combining enables the highest system capacity and the best performances and has the highest implementation complexity. The EGC provides performances better than SC diversity and implementation complexity simpler than MRC. In SC receiver the processing is performed only on one diversity branch and no channel state information is required at receiver. The SC receiver has implementation complexity simpler than MRC receiver and EGC receiver [9, 10].

The outage probability, bit error probability and system capacity are performance measures of the first order. The second order performance measures are the average level crossing rate and average fade duration. The average level crossing rate can be calculated as average value of the first derivative of communication system output signal. The average fade duration can be calculated as ratio of outage probability and average level crossing rate. There are lots of papers in the open technical literature which consider second order statistics of wireless communication systems operating over multipath fading channels [8, 12, 13].

In paper [1, 11], macro diversity system with two MRC micro diversity is presented. Independent multipath Nakagami-m fading is presented at inputs of MRC diversity receiver and shadowing Gamma fading is presented at inputs of macro diversity system. The MRC receivers are used to reduce small scale

Nakagami-m fading and macro diversity system is used to reduce large scale correlated Gamma shadowing fading. The closed form expressions for average level crossing rate of SC macro diversity receiver output envelope and average fade duration of macro diversity system are calculated.

In paper [2], the wireless communication system with SC receiver operating over multipath fading environment are considered. The cases when signal envelope variation is described by using Rayleigh, Rican and Nakagami-m distributions are analyzed. Average level crossing rate of SC receiver output signal and average fade duration wireless system are calculated.

In paper [3], the ratio of random variable and product of two random variables is considered. The random variable in the nominator of the ratio can represent desired signal envelope subjected to multipath fading. The product of two random variables in the denominator of the ratio can represent co-channel interference envelope affected simultaneously to two multipath fadings. For this ratio in the paper, the level crossing rate and average fade are calculated. These results can be used in performance analysis of wireless communication system operating over multipath fading channels in the presence of co-channel interference affected to composite multipath fading.

In this paper the communication system with SC receiver operating over multipath fading channel is considered. Received signal experiences k-μ multipath fading resulting in system performance degradation. The SC receiver is used to reduce influence of k-μ multipath fading on average fade duration. The closed form expressions for level crossing rate and average fade duration on proposed system are calculated. To the best of authors' knowledge, the level crossing rate and average fade duration of communication system with SC receiver operating over k-μ multipath fading channel is not considered in open technical literature. Numerical results are presented graphically to show the influence of Rican factor k and fading severity m on level crossing rate of analyzed system. The expressions for evaluation of level crossing rate and average fade duration can be used in performance analyses and designing of wireless communication system in the presence of k-μ multipath fading.

2 Level Crossing Rate of Random k-μ Process

The k-μ random variable can be used to describe small scale signal average variation in linear line-of-sight multipath fading environment. There are several parameters in the expression for probability density function of k-μ random variable. The parameter k is Rice factor. Rice factor can be calculated as ratio of dominant components power and scattering components power. For lower values of the Rice factor k, fading severity increases, resulting in system performances degradation. The parameter μ is fading severity. For lower values of parameter μ, fading is more severe.

The k-μ distribution is general distribution. Rayleigh, Nakagami-m and Rican distribution can be derived from k-μ distribution. By setting $k = 0$, the k-μ distribution reduces to Nakagami-m distribution and for $\mu = 1$, Rician distribution can be derived from k-μ distribution. By setting $k = 0$ and $\mu = 1$ the k-μ distribution approximates Rayleigh distribution. The probability density function of k-μ distribution is

$$\begin{aligned} P_x(x) &= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{\mu k} r^{\frac{\mu-1}{2}}} x^{\mu} e^{-\frac{2\mu(k+1)x^2}{r}} I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{r}}x\right) = \\ &= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{\mu k} r^{\frac{\mu-1}{2}}} x^{\mu} e^{-\frac{2\mu(k+1)x^2}{r}} \sum_{i=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{r}}\right)^{2i+\mu-1} \frac{1}{i!\Gamma(i+\mu)} x^{2i+\mu-1}. \end{aligned} \tag{1}$$

Commutative distribution function of k-μ random variable is [7]

$$\begin{aligned} F_x(x) &= \int_0^x \mathrm{d}x P_x(x)\,\mathrm{d}x = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{\mu k} r^{\frac{\mu-1}{2}}} x^{\mu} \sum_{i=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{r}}\right)^{2i+\mu-1} \cdot \\ &\cdot \frac{1}{i!\Gamma(i+\mu)} \int_0^x x^{2i+\mu-1} e^{-\frac{2\mu(k+1)x^2}{r}} \mathrm{d}x = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{\mu k} r^{\frac{\mu-1}{2}}} \sum_{i=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{r}}\right)^{2i+\mu-1} \cdot \\ &\cdot \frac{1}{i!\Gamma(i+\mu)} \frac{1}{2}\left(\frac{r}{\mu(k+1)}\right)^{i+\mu} \gamma\left(i+\mu, \mu(k+1)x^2/r\right). \end{aligned} \tag{2}$$

Squared k-μ random variable can be written as sum of 2μ squared Gaussian random variables

$$x^2 = x_1^2 + x_2^2 + \cdots + x_{2\mu}^2, \tag{3}$$

where x_i, $i = 1, 2, \ldots, 2\mu$ are Gaussian random variables

$$P_{x_i}(x_i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i - A_i)^2}{2\sigma^2}}, \quad i = 1, 2, \ldots, 2\mu. \tag{4}$$

The first derivative of k-μ random variable x is

$$\dot{x} = \frac{1}{x}\left(x_1\dot{x}_1 + x_2\dot{x}_2 + \cdots + x_{2\mu}\dot{x}_{2\mu}\right). \tag{5}$$

The first derivate of Gaussian random variable is Gaussian random variable. Linear transformation of Gaussian random variable is Gaussian random variable. Therefore, for the first derivative of k-μ random variable is Gaussian random variable. The mean of the first derivative of k-μ random variable is

$$\bar{\dot{x}} = \frac{1}{x}\left(x_1 \bar{\dot{x}}_1 + x_2 \bar{\dot{x}}_2 + \cdots + x_{2\mu} \bar{\dot{x}}_{2\mu} \right) = 0 , \tag{6}$$

where is

$$\bar{\dot{x}}_1 = \bar{\dot{x}}_2 = \cdots = \bar{\dot{x}}_{2\mu} = 0 . \tag{7}$$

The variance of the first derivative of k-μ random variable is

$$\sigma_{\dot{x}}^2 = \frac{1}{x^2}\left(x_1^2 \sigma_{\dot{x}_1}^2 + x_2^2 \sigma_{\dot{x}_2}^2 + \cdots + x_{2\mu}^2 \sigma_{\dot{x}_{2\mu}}^2 \right) \tag{8}$$

where is

$$\sigma_{\dot{x}_1}^2 = \sigma_{\dot{x}_2}^2 = \cdots = \sigma_{\dot{x}_{2\mu}}^2 = \pi^2 f_m^2 \frac{r}{m} . \tag{9}$$

By substituting, the expression for variance becomes

$$\sigma_{\dot{x}}^2 = \frac{1}{x^2} \pi^2 f_m^2 \frac{r}{m}\left(x_1^2 + x_2^2 + \cdots + x_{2\mu}^2 \right) = \frac{1}{x^2} \pi^2 f_m^2 \frac{r}{m} x^2 = \pi^2 f_m^2 \frac{r}{m} . \tag{10}$$

The probability diversity function of $\dot{x}$ is

$$P_{\dot{x}}(\dot{x}) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} \mathrm{e}^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} . \tag{11}$$

The joint probability density function of k-μ random variable and the first derivative of k-μ random variable is

$$\begin{aligned} P_{x\dot{x}}(x\dot{x}) = P(x) P_{\dot{x}}(\dot{x}) = \frac{2\mu (k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} \mathrm{e}^{\mu k} r^{\frac{\mu-1}{2}}} x^{\mu} \mathrm{e}^{-\frac{\mu(k+1)x^2}{r}} . \\ \cdot I_{\mu-1}\left(2\mu \frac{\sqrt{k(k+1)}}{r} x \right) \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} \mathrm{e}^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} . \end{aligned} \tag{12}$$

The average level crossing rate of k-μ random variable can be calculated as average value of the first derivation of k-μ random variable

$$\begin{aligned} N_x = \int_0^{\infty} \mathrm{d}\dot{x} \dot{x} P_{x\dot{x}}(x\dot{x}) = \frac{2\mu (k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} \mathrm{e}^{\mu k} r^{\frac{\mu-1}{2}}} x^{\mu} \mathrm{e}^{-\frac{2\mu(k+1)x^2}{r}} \cdot I_{\mu-1}\left(2\mu \frac{\sqrt{k(k+1)}}{r} \right) \cdot \\ \cdot \int_0^{\infty} \mathrm{d}\dot{x} \dot{x} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} \mathrm{e}^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} = \frac{2\pi f_m}{\sqrt{2\pi}} \frac{\mu^{1/2} (k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{\mu k} r^{\frac{\mu-1}{2}}} x^{\mu} \mathrm{e}^{-\mu(k+1)} I_{\mu-1}\left(2\mu \sqrt{\frac{k(k+1)}{r}} \right) . \end{aligned} \tag{13}$$

This expression can be used for evaluation average fade duration of wireless communication system with SC receiver operating over k-μ multipath fading environment.

3 SC Receiver Average Fade Duration in the Presence of *k*-μ Multipath Fading

In this paper SC receiver with two inputs is used to reduce the influence of k-μ multipath fading on average fade duration of wireless communication system. The SC receiver selects diversity branch with the strongest signal. Therefore, joint probability density function of SC receiver output signal envelope and the first derivative of SC receiver output signal envelope is

$$P_{z\dot{z}}(z\dot{z}) = P_{x\dot{x}}(z\dot{z})F_y(z) + P_{y\dot{y}}(z\dot{z})F_x(z), \tag{14}$$

where $P_{x\dot{x}}(x\dot{x})$ is the joint probability density function of signal envelope at the first input of SC receiver and its first derivative, $P_{y\dot{y}}(y\dot{y})$ is the joint probability density function of signal envelope at the second input of SC receiver, and its first derivative and $F_y(y)$ is cumulative distribution function of signal envelope at the second input of SC receiver. For identical k-μ fading, previous expression becomes

$$P_{z\dot{z}}(z\dot{z}) = 2P_{x\dot{x}}(z\dot{z})F_y(z). \tag{15}$$

The level crossing rate of SC receiver output signal envelope is

$$\begin{aligned} N_z &= \int_0^\infty \mathrm{d}\,\dot{z}P_{z\dot{z}}(z\dot{z})\dot{z} = 2F_y(z)N_x = 2\frac{2\pi f_m}{\sqrt{2\pi}}\frac{\mu^{1/2}(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}\,\mathrm{e}^{\mu k}\,r^{\frac{\mu-1}{2}}}x^{\mu}\,\mathrm{e}^{-\mu(k+1)}\cdot \\ &\cdot I_{\mu-1}\left(2\mu\frac{\sqrt{k(k+1)k}}{r}x\right)\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}\,\mathrm{e}^{\mu k}\,r^{\frac{\mu-1}{2}}}\sum_{i=0}^{\infty}\left(\sqrt{\frac{k(k+1)}{r}}\right)^{2i+\mu-1} = \\ &= \frac{1}{i!\Gamma(i+\mu)}\frac{1}{2}\left(\frac{r}{\mu(k+1)}\right)^{i+\mu}\gamma\left(i+\mu,\frac{\mu(k+1)x^2}{r}\right). \end{aligned} \tag{16}$$

The SC receiver output signal envelope is

$$z = \max(x, y), \tag{17}$$

where x and y are SC receiver input signals envelope. The cumulative distribution function of SC receiver output signal envelope is

$$F_z(z) = F_{xy}(z,z) = F_x(z)F_y(z) = F_x^2(z). \tag{18}$$

The outage probability of wireless communication system with dual SC receiver is equal to cumulative distribution function of SC receiver output signal

$$P_{z_0} = F_z(z_0) = F_x^2(z_0)\,, \tag{19}$$

where outage probability is defined as

$$P_{z_0} = \int_0^{z_0} P_z(z)\,\mathrm{d}z\,. \tag{20}$$

The average fade duration of wireless communication system with dual SC receiver is

$$AFD = \frac{P_{z_0}}{N_{z_0}} = \frac{F_x(z)}{2N_x(z)}\,. \tag{21}$$

In Fig. 1, normalized average level crossing rate is plotted versus SC receiver output signal envelope for several values of Rician factor k, fading severity m and SC receiver output signal envelope power. For values of SC receiver output signal envelope less than one, average level crossing rate increases as output signal envelope increases. For values of output signal envelope more than one level crossing rate decreases as output signal envelope decreases. Influence of output signal envelope on level crossing rate is the greatest for intermediate values of output signal envelope. The influence of SC receiver output signal envelope on level crossing rate is grater for lower values of Rician factor k and for lower values of output signal envelope power. System performance is better for higher values of fading severity m.

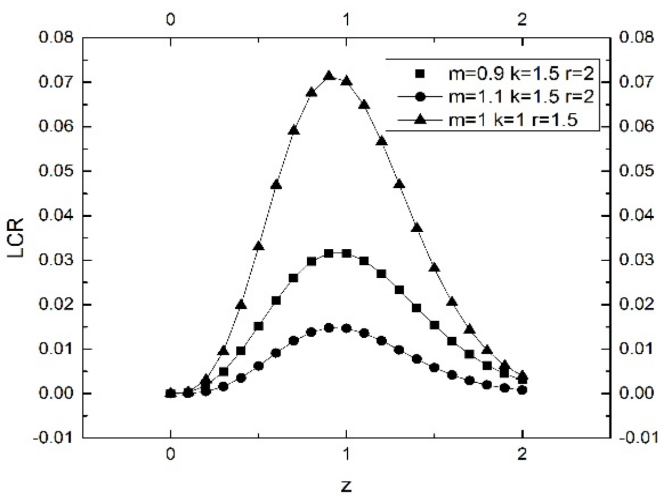

Fig. 1 – *Normalized LCR versus SC receiver output signal envelope.*

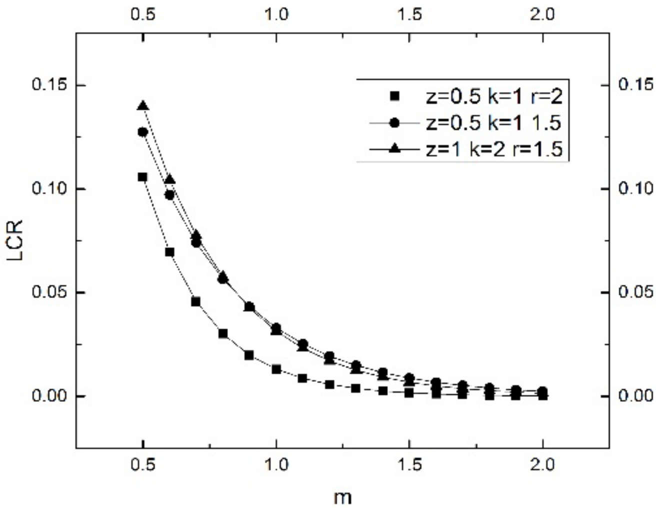

Fig. 2 – *Normalized LCR versus fading severity.*

In Fig. 2. normalized LCR is shown versus fading severity *m* for several values of signal envelope *z*, Rician factor *k* and SC receiver output signal envelope power *r*. Average level crossing rate of SC receiver output signal envelope decreases as fading severity increases. The influence of fading severity on LCR is greater for lower values of fading severity. LCR decreases as output signal envelope power increases.

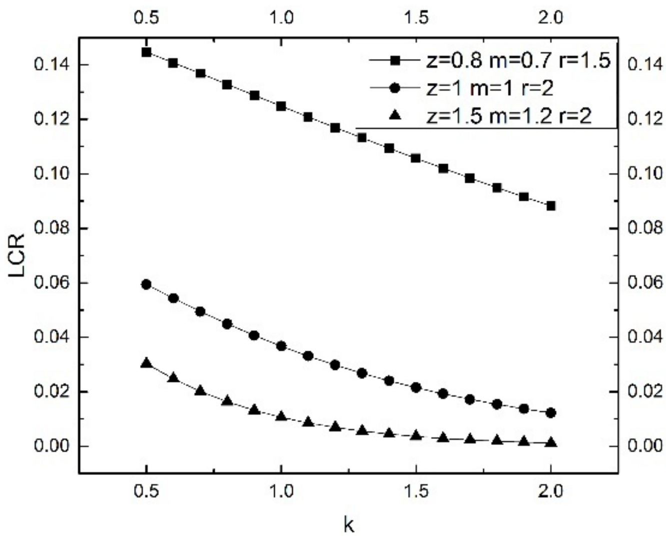

Fig. 3 – *Normalized LCR versus Rican factor.*

In Fig. 3, normalized LCR of SC receiver output signal envelope is plotted versus Rician factor k. LCR decreases as Rician factor decreases. System performance is better for higher values of Rician factor. Dominant component power increases for higher values of Rician factor. For higher values of output signal envelope power, LCR has lower values. The influence of Rician factor on LCR is grater for higher values of output signal envelope power.

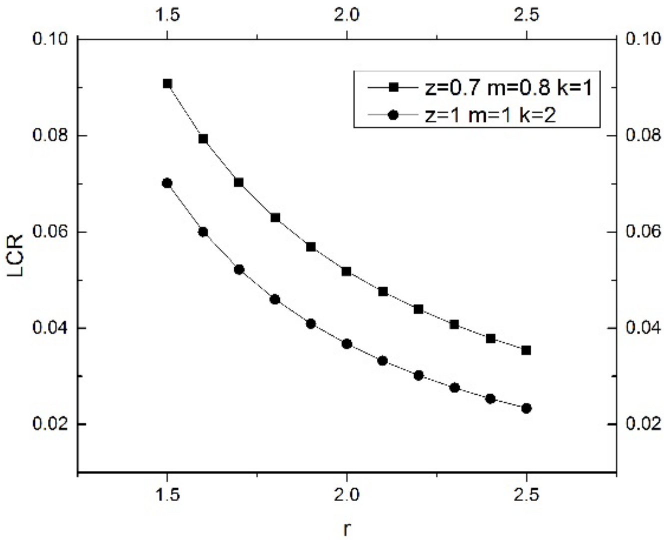

Fig. 4 – *Normalized LCR versus SC receiver output signal envelope.*

In Fig. 4, normalized LCR is plotted versus SC receiver output signal envelope power for several values of SC receiver output signal envelope, fading severity and Rician factor. LCR decreases as output signal envelope power increases. System has better performance for higher values of output signal envelope power. The influence of output signal envelope power on average level crossing rate is greater of lower values of output signal envelope power. System performances are better for higher values of Rician factor and fading severity.

4 Conclusion

In this paper, wireless communication system with dual SC receiver operating over multipath fading environment is considered. Received signal experiences k-μ multipath fading resulting in signal envelope variation and system performance degradation. The SC received is used to reduce fading effect on average fade duration and outage probability. The SC receiver is simple for practical realization. The k-μ distribution can be used to describe small scale signal envelope variation in multipath, line-of-sight fading channels. Rayleigh, Rican and Nakagami-m distributions can be derived from k-μ distribution as special cases. Closed form expressions for average level crossing rate of SC receiver output signal and average fade duration of proposed wireless communication system are evaluated. The level crossing rate is obtained as average value of the first derivative of SC receiver output signal envelope. The average fade duration of wireless system is calculated as the ratio of outage probability and average level crossing rate. Closed form expressions for level crossing rate of k-μ random variable and cumulative distribution function of SC receiver output signal are also calculated. The numerical results are presented graphically to show influence of Rican factor and k-μ fading severity on level crossing rate and average fade duration. The level crossing rate increases as fading severity m decreases. Fading severity has greater influence on average fade duration for lower values of Rican factor. Average level crossing rate decreases as Rican factor increases. Rican factor is defined as ratio of dominant components power and scattering components power. Obtained results can be used for performance analysis and designing wireless communication system in the presence of k-μ multipath fading.

5 References

[1] D. Stefanovic, S.R. Panic, P. Spalevic: Second-order Statistics of SC Macrodiversity System Operating over Gamma Shadowed Nakagami-m Fading Channels, AEU - International Journal of Electronics and Communications, Vol. 65, No. 5, May 2011, pp. 413 – 418.

[2] E. Mekic, N. Sekulovic, M. Bandjur, M. Stefanovic, P. Spalevic: The Distribution of Ratio of Random Variable and Product of Two Random Variables and its Application in Performance Analysis of Multi-hop Relaying Communications over Fading Channels, Przeglad Elektrotechniczny, Vol. 88, No. 7A, 2012, pp. 133 – 137.

[3] M.D. Yacoub: The k-μ Distribution and the η-μ Distribution, IEEE Antennas and Propagation Magazine, Vol. 49, No. 1, Feb. 2007, pp. 68 – 81.

[4] J. Proakis: Digital Communications, McGraw-Hill, NY, USA, 2001.

[5] M.K. Simon, M.S. Alouini: Digital Communication over Fading Channels, John Wiley and Sons, Hoboken, NJ, USA, 2000.

[6] I. Gradshteyn, I. Ryzhik: Tables of Integrals, Series and Products, Academic Press, NY, USA, 1994.

[7] A.S. Panajotovic, N.M. Sekulovic, M.C. Stefanovic, D.L. Draca: Average Level Crossing Rate of Dual Selection Diversity over Correlated Unbalanced Nakagami-m Fading Channels in the Presence of Cochannel Interference, IEEE Communications Letters, Vol. 16, No. 5, May 2012, pp. 691 – 693.

[8] G.L. Stüber: Principles of Mobile Communications, Kluwer Academic Publishers, Norwell, MA, USA, 1996.

[9] W.B. Davenport, W.L. Root: An Introduction to the Theory of Random Signals and Noise, John Wiley and Sons, NY, USA, 1987.

[10] S. Panic, M. Stefanovic, J. Anastasov, P. Spalevic: Fading and Interference Mitigation in Wireless Communications, CRC Press, NY, USA, 2013.

[11] A. Matovic, E. Mekic, N. Sekulovic, M. Stefanovic, M. Matovic, C. Stefanovic: The Distribution of the Ratio of the Products of Two Independent a-μ Variates and its Application in the Performance Analysis of Relaying Communication Systems, Mathematical Problems in Engineering, Vol. 2013, Article ID 147106.

[12] A.S. Panajotovic, M.C. Stefanovic. D.LJ. Draca, N.M. Sekulovic: Average Level Crossing Rate of Dual Selection Diversity in Correlated Rician Fading with Rayleigh Cochannel Interference, IEEE Communications Letters, Vol. 14, No. 7, July 2010, pp. 605 – 607.