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Finite Impulse Response Digital Filters with Integer Multipliers

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Abstract: In this paper, a family of non-recursive digital filters will be described in which all multipliers are small integers. It is shown that this practical advantage is only available if some rather severe restrictions on the locations of z-plane poles and zeros are accepted. These restrictions have the further advantage that all filters of the family display pure linear-phase characteristics, imposing a pure transmission delay on all frequency components of an input signal.

Keywords: Digital filter, Circuit theory.

1 Introduction

In practice it is often difficult to choose the most appropriate technique for a particular application, and even when a decision has been made in favour of a time-domain operation it may be unclear whether to use a recursive or non-recursive filter. The amount of computation required to achieve a particular filtering action will generally be an important practical consideration; if a general-purpose computer is programmed as a digital filter, the computational economy of the filter design may well determine the feasibility of a real-time operation on the input data. But whether filtering is performed using a general-purpose computer or special-purpose hardware, they are generally the multiplication operations that are the most expensive in terms of time and equipment [1]. In other words, computational economy depends largely upon minimizing the number of multiplication required to calculate each new output sample value. Furthermore, if the coefficients, by which sample values are multiplied, are small integers, multiplication will be far simpler than when those coefficients are floating-point numbers needing specifications to an accuracy of perhaps five or six decimal figures.

In many cases the use of recursive filter dramatically reduces the number of required multiplications, compared to a non-recursive filter having similar frequency-response characteristics. Unfortunately, the coefficients by which sample values must be multiplied in a recursive filtering operation must normally be specified with considerable accuracy. The reason for this is that a recursive design generally uses z-plane poles lying close to the unit circle, and a small error in the coefficient values of the time-domain recurrence equation may in effect move these poles outside the unit circle, causing instability. Apart from the problem of instability, the poles must normally be located with considerable accuracy if the required frequency-response characteristic is realized.

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In this paper, a family of non-recursive digital filters will be described in which all multipliers are small integers what is first proposed by Lynn (2J. It will be shown that this practical advantage is only available if some rather severe restrictions on the locations of z -plane poles and zeros are accepted. These restrictions have the further advantage that all filters of the family display pure linear-phase characteristics, imposing a pure transmission delay on all frequency components of an input signal. This family of non-recursive digital filters is adopted for design digital filter for extraction of data that are transmitting by supply network.

2 Recursive Realization of Linear-Phase Non-recursive Filters

2.1 Low-pass Filters

The symmetrical weighting function of non-recursive filters may be realized with computational economy when cast is in recursive form. Further advantages appear when the coefficients involved in the recursive are integers. This topic is conveniently introduced by referring to the simple "moving average" weighting function (3J illustrated in Fig. 1, which is symmetrical about $t-kT$. Using Z - transform notation, we may write the filter transfer function directly

$$H(z)=\frac{Y(z)}{X(z)}=1+z^{-1}+z^{-2}+\ldots+z^{-2k}=\frac{1-z^{-2k-1}}{1-z^{-1}}. \qquad (1)$$

Transfer function $H(z)$ has zeros where $1-z-2k-1=0$, i.e. there are $(2k+1)$ zeros evenly distributed around the unit circle in the z-plane. There is also pole at $z=1$.

In equation (1) $X(z)$ and $Y(z)$ are the input and output Z - transform respectively, and therefore they obtain the following relationship

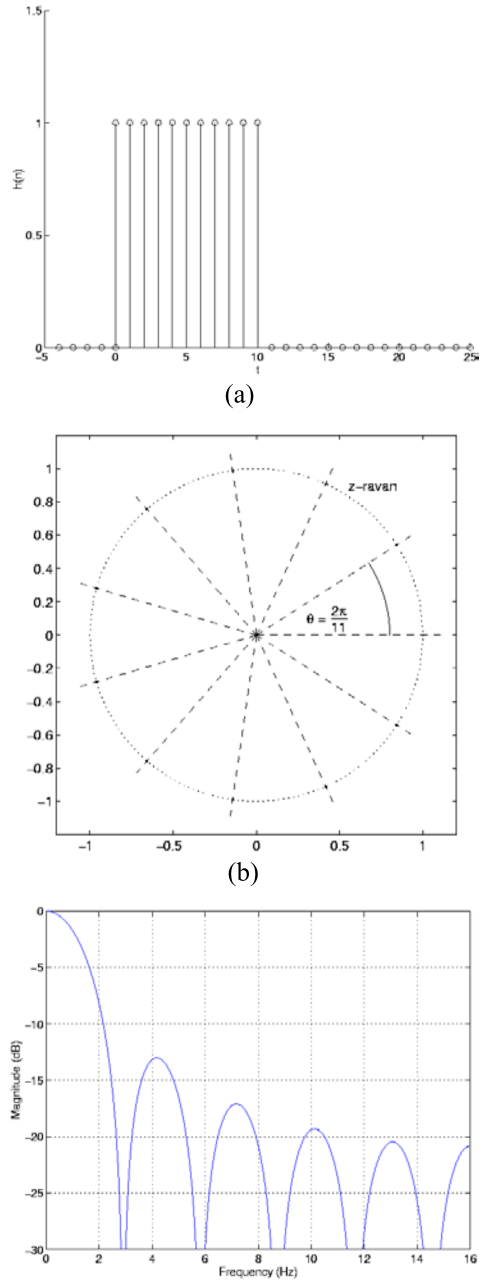
$$Y(z)=z^{-1}Y(z)+(1-z^{-2k-1})X(z), \qquad (2)$$

which yields the time-domain recurrence formula

$$y(n)=y(n-1)+x(n)-x(n-2k-1). \qquad (3)$$

Equation (3) indicates that a recursive operation, involving only 3 terms, is equivalent to a non-recursive "moving-average" filter having any number of terms in its weight function. For example, if $k=5$ the weighting function contains 11 terms and filtering is achieved using the recurrence formula

$$y(n)=y(n-1)+x(n)-x(n-11). \qquad (4)$$



(c)

Fig. 1 - *11-term "moving-average" filter; (a) weighting function; (b) plane pole-zero configuration; (c) frequency-response magnitude function.*

The weighting function, pole-zero configuration and frequency response characteristic of this filter are illustrated in Fig. 2.

Consider next the triangular weighting function shown in Fig. 3. Using the results already derived and considering triangular weighting function function to be addition a number of sets unit-height samples, we may write directly

$$H(z)=\frac{1-z^{-2k-1}}{1-z^{-1}}+z^{-1}\frac{1-z^{-2k+1}}{1-z^{-1}}+\ldots+z^{-k}\frac{1-z^{-1}}{1-z^{-1}} \text{ and} \tag{5}$$

$$H(z)=\frac{(1-z^{-k-1})^2}{(1-z^{-1})^2}\,. \tag{6}$$

The corresponding recurrence formula is therefore

$$z(n)=x(n)-2\boldsymbol{x}(n\text{ - }k\text{ - }1)+x(n\text{ - }2k\text{ - }2)+2y(n\text{ - }1)-y(n-2)\,. \tag{7}$$

These results show that a triangular weight function of any number of terms may be realized by 5-terms recursive filter. For example, $\mathrm{k}=10$ specifies a 21-term triangular weighting function. The z -plane of the filter may be written as

$$H(z)=\frac{(1-z^{-11})^2}{(1-z^{-1})^2}\,. \tag{8}$$

Thus there are 11 second-order zeros spaced equally around the unit circle in z -plane and a second pole at $z=1$. The pole-zero configuration and frequency response of this filter are illustrated in Fig. 3. This filter may be realized by recurrence formula

$$y(n)=x(n)-2x(n\text{ - }11)+x(n\text{ - }22)+2y(n\text{ - }1)-y(n\text{ - }2)\,. \tag{9}$$

We notice that, with $k=10$, the pole-zero pattern is similar to that of the simple moving-average filter previously discussed (with $k=5$), except that double-zeros replace single zeros, and double-pole replace single pole at $z=1$. The transfer function of this 21-term triangular filter is thus the square of that of the 11-term moving average filter. Recalling that multiplication in the frequency domain is equivalent to convolution in time domain, we note that, as would be expected, the self-convolution of the moving-average weighting function does indeed yield the triangular one.

Both the filters so far discussed make use of x-plane zeros equally distributed around the unit circle, which are cancelled in one position by a coincident pole (or poles), giving rise to a pass band. In general it may be shown that the placing of zeros at equal angular intervals on the unit circle, with cancellation by coincident poles in one or more positions, gives rise to recursive filters with the advantages of integer multipliers and linear-phase characteristics. It is always possible to raise a given transfer function

$H(z)$ to an integer power; this has the effect of sharpening the cut-off and reducing the filter side lobe levels.

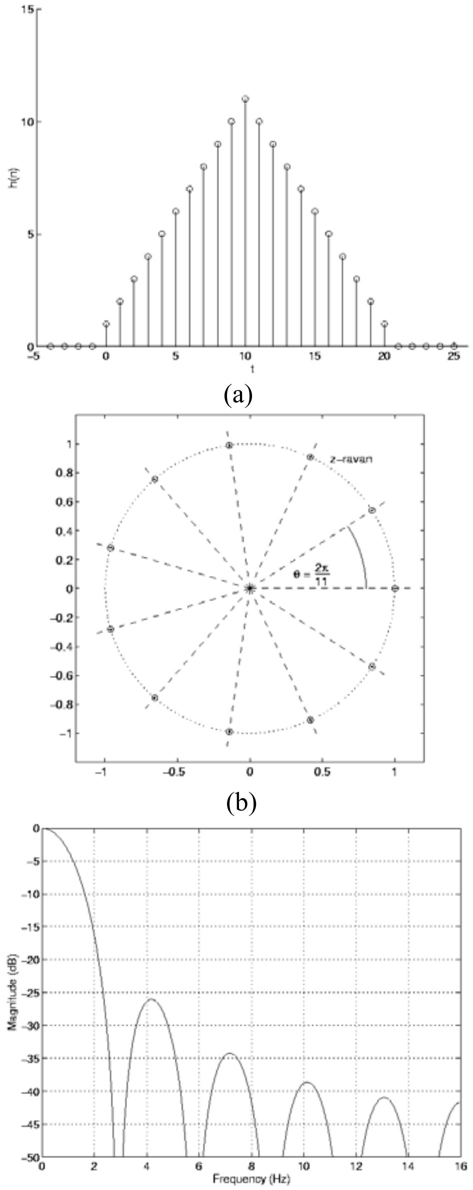
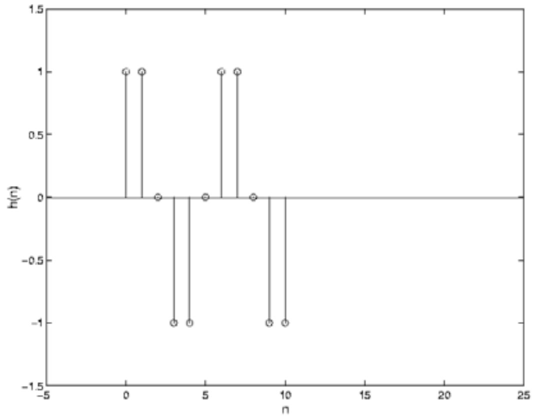


Fig. 2 - *21-term "triangular" filter; (a) weighting function;*
(b) z-plane pole-zero configuration; (c) frequency-response magnitude function.

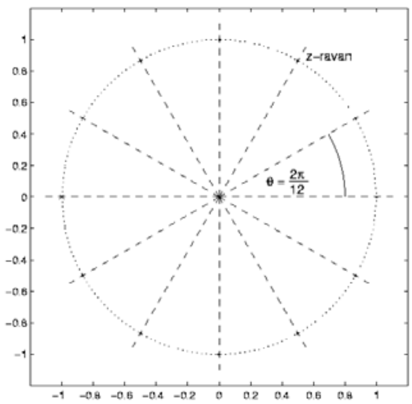
The technique of placing cancelling poles at $z = 1$ may be modified to realize high pass filters. High pass filters are obtained by placing poles at $z = -1$. When a high pass filter having a transmission zero at $\omega = 0$ is required, the numerator of $H(z)$ must be a term of the form $(1 + z^{-k})^n$, where *k* is an even integer.

2.2 Band-pass Filters

Band pass filters may be obtained by placing cancelling poles at $z = \pm j$, giving a pass band centred on the frequency $\omega = \pi/(2T)$, where $T = 1/f_s$ is the sampling period and f_s sampling frequency. Pass bands centred on $\omega_c = \pi/(3T)$, or $\omega_c = 2\pi/(3T)$ may also be obtained by placing three sets of cancelling poles at equal angular intervals around zeros. Fig. 3 shows typical pole zeros configuration, which give band-pass characteristics.



(a)



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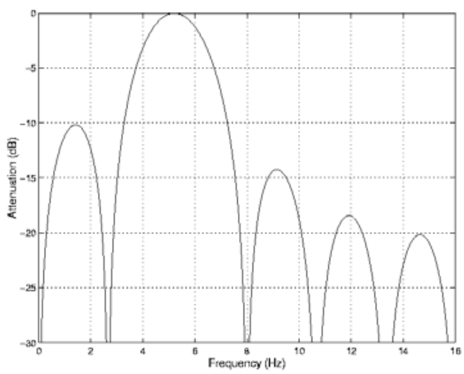

(c)

Fig. 3 - *10-term band-pass filter; (a) weighting function;*
(b) z *-plane pole zero configuration; (c) frequency-response magnitude function.*

The z -plane transfer function of the band-pass filter in Fig. 3 is given by

$$H(z) = \frac{(1 - z^{-12})(1 + z^{-1})}{1 + z^{-3}}. \tag{10}$$

The next example is band pass filter for data extraction, which is transmitting trough supply network [4]. Modulation is amplitude shift keying, which carrier with frequency ω_c is coded with one, but zero value of carrier is coded with zero. Additional specification for this filter is suppression noise of harmonics of line frequency.

Weighting function of this filter, or impulse response, is sampling data of carrier

$$h(n) = \begin{cases} C\sin(\omega_c nT + \phi), & \text{for } n = 0,1,...,N-1 \\ 0, & \text{for } n < 0 \text{ and } n \geq N \end{cases}, \tag{11}$$

where T is sampling interval. Transfer function in z -domain of this filter is

$$G(z) = h(0) + h(1)z^{-1} + ... + h(N-1)z^{-N+1}, \tag{12}$$

where $h(n)$ are:

$$h(0) = C\sin(\phi),$$

$$h(1) = C\sin(\omega_c T_\phi), \tag{13}$$

$$\dots$$

$$h(N-1) = C\sin[(N-1)\omega_c T + \phi].$$

For different values of parameters C, ϕ and T we can evaluated different values of coefficient a_n, $n = 0,1,...2k.$. For $C = 2/\sqrt{3}$, $f_s = 6f_c$ and $\phi = 0$, the coefficients of weighting function of this non-recursive digital filters have only following small integer value 0, 1 and -1. This weighting function is symmetrical about $n = k$ and periodic of period $N_p = 6$. Thus $N = 6m$, where m is an integer.

Thus we may written transfer function (12) in simple form

$$G(z^{-1}) = z^{-1} + z^{-2} - z^{-4} - z^{-5} + ... - z^{-(N-1)}. \tag{14}$$

The equation (14) can be formed by addition of two shifted band-pass functions with $N/3$ unit-weight samples

$$G(z^{-1}) = z^{-1}(1 - z^{-3} + z^{-6} + ... - z^{-(N-3)}) + z^{-2}(1 - z^{-3} + z^{-6} + ... - z^{-(N-3)}). \tag{15}$$

We may written directly

$$G(z^{-1}) = z^{-1}\frac{1-(-z^{-3})^{N/3}}{1+z^{-3}} + z^{-2}\frac{1-(-z^{-3})^{N/3}}{1+z^{-3}}, \tag{16}$$

that reduces to

$$G(z^{-1}) = z^{-1}\frac{1-z^{-N}}{1-z^{-1}+z^{-2}}, \tag{17}$$

because $N/3 = 2m$ is even integer.

The corresponding recurrence formula is therefore

$$y(n) = x(n-1) - x(n-N-1) + y(n-1) - y(n-2). \tag{18}$$

It is shown that these transfer function of any number of terms may be realized by a 4-terms recursive filter.

The pole-zero pattern is similar to that of the previous discussed examples. There are N zeros spaced equally around the unit circle in the z-plane,

$$z_{zk} = \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}k}. \tag{19}$$

The cancelling poles are placing at

$$z = \mathrm{e}^{\pm\mathrm{j}\frac{\pi}{3}}, \tag{20}$$

giving pass band centred on the frequency $f_\mathrm{c} = f_\mathrm{s}/6$, where f_s is sampling frequency.

The zeros of transfer function are on the frequencies

$$f_{zk} = k\frac{f_s}{N}, \text{ for } k = 0,1,2,..., k \le \frac{N}{2}. \tag{21}$$

This filter suppresses all signals on the frequencies f_{zk}. If $f_s / N = f_n / r$ where f_n is frequency of power supplies voltage and r is an integer, then this filter suppress harmonics of power network voltage. For $f_s = 6f_c$ and $N = 6m$ we may compute the integer m if integer r is known

$$m = r\frac{f_c}{f_n}. \tag{22}$$

For example, frequency $f_c = 216\frac{2}{3}$ Hz is applied for data transmission over power supplies network, and pass band is centred on this frequency. Power supplies voltage frequency is $f_n = 50$ Hz, the integer m is calculated

$$m = r\frac{f_c}{f_n} = r\frac{13}{3}. \tag{23}$$

The smallest value for r is $r = 3$ and we calculated $m = 13$ and $N = 6m = 78$.

The frequency response of the filter with transfer function (17) is

$$G(e^{-j\omega T}) = e^{-j\frac{N-1}{2}\omega T 2\frac{\sin\frac{N\omega T}{2}\cos\frac{\omega T}{2}}{\cos\frac{3\omega T}{2}}}. \tag{24}$$

Only frequency response of magnitude function in Fig. 4 is given, but frequency response of phase function is linear. Fig. 4 shows that magnitude of transfer function has ripples in the frequency response, side lobes. Ripple frequencies increase with N, but maximum high of ripple is almost constant.

3 Realization Structures for Digital Transfer functions

The simplest realization of obtained transfer functions is a cascade arrangement of recursive and non-recursive part of transfer function (17). Thus, the transfer function $G(z^{-1})$ may be written in the form

$$G(z^{-1}) = (z^{-1} - z^{-N-1}) \times \frac{1}{1 - z^{-1} + z^{-2}}. \tag{25}$$

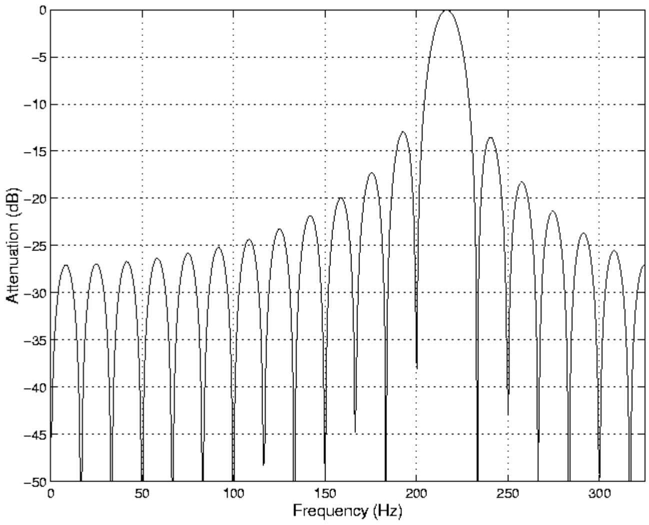

Fig. 4 - *Frequency response magnitude function of filter for data extraction, which are transmitted over supplies network.*

The overall transfer function $G(z^{-1})$ is obtained by cascading two realization structures as it is shown in Fig. 5. This structure requires three adders and $N+3$ delay lines and it is non-canonical. The structure shown in Fig. 6 forms canonical structure and it is known as "direct form realization". This realization structure requires only two adders and $N+1$ delay lines, or two delay lines less than cascade form structure.

Oscillation in this type of digital filter occurs due to adder overflow, because the multiplication is only with 1 or -1. Due to eliminated adder overflow if the input sequence is encoded in 2's complement binary fixed point arithmetic using 8 bits, one of them is the sign bit, the addition of two 8 bits number results in a very undesirable overflow. Using 16 bits 2's complement adder may eliminate the adder overflow limit cycles.

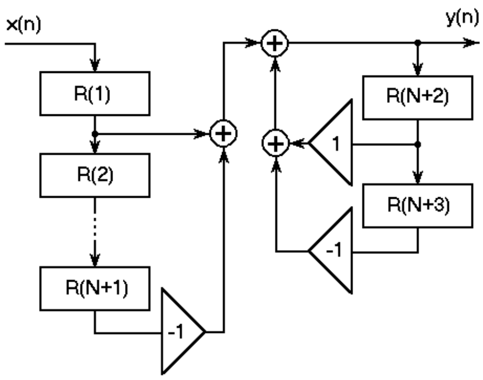

Fig. 5 - *Cascade form.*

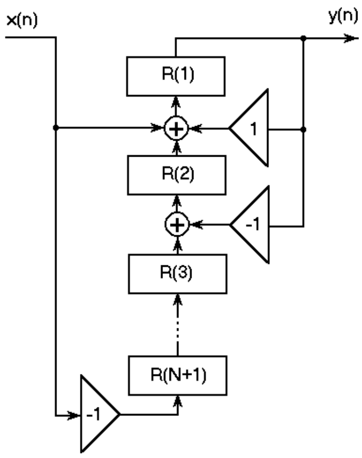

Fig. 6 - *Direct form.*

4 Conclusion

The family of non-recursive digital filters in which all multiplier is small integers is described. It is shown that this practical advantage is only available if some rather severe restrictions on the locations of z -plane poles and zeros are accepted. These restrictions have the further advantage that all filters of the family display pure linear-phase characteristics, imposing a pure transmission delay on all frequency components of an input signal.

Proposed technique is adopted for design FIR digital filter for extraction of data, which are transmitting by supply network. Impulse response of this filter has coefficient, which is equal 1 or -1.

References

[1] Y. C. Lim, S. R. Parker: FIR filter design over a discrete power-of two coefficient space, IEEE Trans. Acoust. Speech, Signal Processing, vol. ASSP - 31, pp. 583 - 591, June 1983.

[2] P. A. Lynn: Economic linear-phase recursive digital filters, Electron. Lett., vol. 6, no 8, pp. 143 - 145, 1970.

[3] N. K. Bose: Digital Filters, Theory and Applications, Elsevir Sc. Publishing Co., 1985.

[4] W. Landau, G. Thielscher: Dritte Generation Elektronischer Tonfrequenz-Rundsteuerempianger, Manheim BBC, 1980 Nr. D SI 115180 D.