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Optimal Design of a DC MHD Pump by Simulated Annealing Method

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Abstract: In this paper a design methodology of a magnetohydrodynamic pump is proposed. The methodology is based on direct interpretation of the design problem as an optimization problem. The simulated annealing method is used for an optimal design of a DC MHD pump. The optimization procedure uses an objective function which can be the minimum of the mass. The constraints are both of geometrics and electromagnetic in type. The obtained results are reported.

Keywords: Simulated annealing algorithm, Design, Optimization, Constraints, DC MHD pump.

1 Introduction

Magnetohydrodynamics (MHD) is the theory of interaction of electrically conducting fluids and electromagnetic fields. Applications arise in astronomy and geophysics as well as in numerous engineering problems, such as liquid metal cooling of nuclear reactors, electromagnetic casting of metals, MHD power generation and propulsion [1, 2].

Applications of MHD are numerous and used in many domains, such as metallurgical industry, the transport or pumping of liquid metals in fusion and marine propulsion [3, 4].

The interaction of moving conducting fluids with electric and magnetic fields provides a rich variety of phenomena associated with electro-fluid-mechanical energy conversion [5].

The pumping of liquid metal may require an electromagnetic device which induces currents, and their associated magnetic field generates the Lorentz force whose effect ensures the pumping of the liquid metal [6, 7].

The determination of geometry and electrical configuration of an MHD device gives rise to an optimization problem. When the requirements of the design are defined, this problem can be solved by optimization technique.

The objective function of the optimization problem is derived from the main design requirement. The other design requirements can be taken into

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account by the constraints. The optimal solution to the problem corresponds to the minimum (or maximum) of the functional cost.

Many studies pertaining to the optimal design of MHD devices have been done, with different techniques employed. The finite element technique was considered to solve constrained optimal control problem in the design of a MHD pump as described in [8, 9]. The successive quadratic programming method is used to solve optimal design of a superconducting MHD saddle magnet. The global evolution strategy technique (GES) was used as global minimization algorithm for the optimal design of a liquid metal induction pump. In [10], the determination of geometry and electrical configuration of an MHD device yields to the inverse magnetohydrodynamic field problem. Stochastic optimization procedures for design problems in electrical engineering have received considerable attention over the past few years because they are rather simple to implement, stable in convergence and able to find the desired region with quite a good probability.

Simulated annealing is a higher level heuristic algorithm for solving optimization problems. It is an iterative improvement procedure that starts from any initial solution and attempts to determine a better solution. It has now become an established optimization approach that is rapidly spreading to many new fields. With the other heuristic search algorithms, such as Genetic algorithm, simulated annealing has been singled out as 'extremely promising' for the future treatment of practical applications.

The presented approach has potential to find the global optimum and it does not need any additional information than the dynamic model itself. The developed method incorporates penalty methods for constraint handling, which ensures reliability in the worst case performance.

In this paper, simulated annealing method (SA) is used for optimization of the liquid metal conduction pump. In order to complete this task, the pump's mass is adopted as the cost function. The magnetic induction and the current density are used as constraints.

The paper is organized as follows**:** **Section 2** provides the description of the MHD pump. The optimization problem formulation is described in **Section 3**. Simulated annealing as well as the control parameter are introduced in **Section 4**, while **Section 5** presents the optimization results.

2 Description of the Conduction MHD Pump

Figs. 1 and 2 show schematic view and geometry of the considered DC electromagnetic pump in torus shape with two coils, four electrodes and a channel.

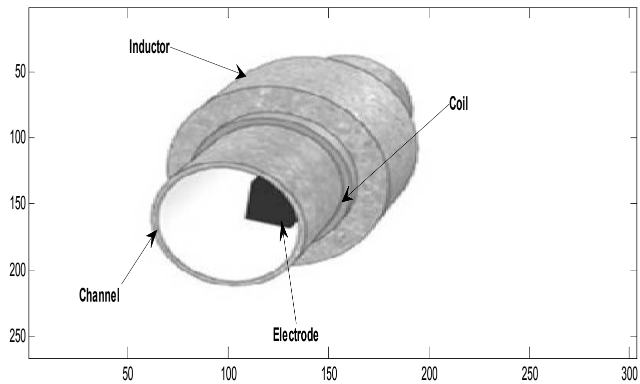

Fig. 1 – *Schematic view of the DC pump.*

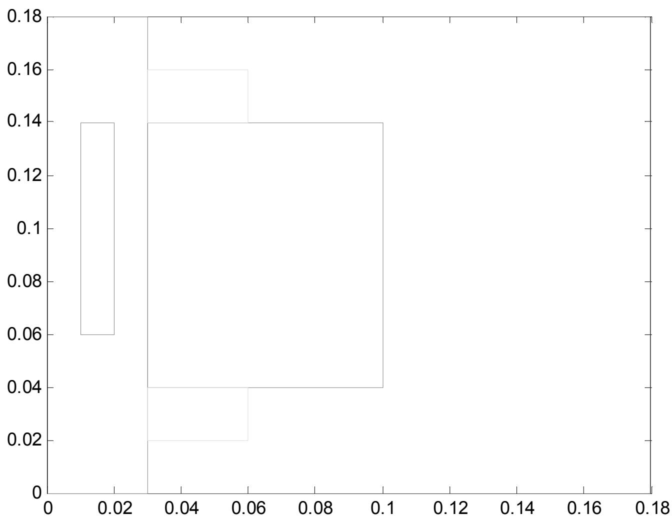

Fig. 2 – *Model of the conduction MHD pump.*

3 Optimization Problem Formulation

The optimization model is based on the solution of a nonlinear constrained optimization problem stated as follows:

$$\begin{cases} \text{Min} f(x), & x \in R^n \\ p_j(x) = 0, & j = 1, \ldots, m \qquad x_{lower} \le x \le x_{upper} \\ g_i(x) = 0, & i = 1, \ldots, n \end{cases} \tag{1}$$

where $f(x)$ is the cost function, $p_j(x)$ and $g_i(x)$ are equalities and inequalities constraint functions. The vector x contains the unknowns of the problem which are the geometrical parameters defining the structure of the pump, while x_{lower} and x_{upper} are the thrusts fixing the acceptable field.

To formulate the optimization problems requires defining an objective function to be optimized. In this case, we have considered the mass of a DC MHD pump where a function of external penalty is used [11], according to which the function to be minimized becomes equal to:

$$W(x) = f(x) + r\sum_{i=1}^{m}\left[\max\left[1, g_j(x)\right]\right]^2, \qquad (2)$$

where $f(x)$ is the objective function without constraints, $g_j(x)$ represents the constraints function and *r* is penalty coefficient.

In this case, the optimization problem is resumed as follows: $f(x) = Minmass$ subject to the inequalities constraints:

$$\begin{cases} B(x) \le 1.5\,\mathrm{T} \\ j(x) \le 4.10^6\ \mathrm{A/m^2} \\ x_{lowr} \le x \le x_{upper} \\ x = \{x_1, x_2, x_3, x_4, x_6, x_7, x_8\} \end{cases} \qquad (3)$$

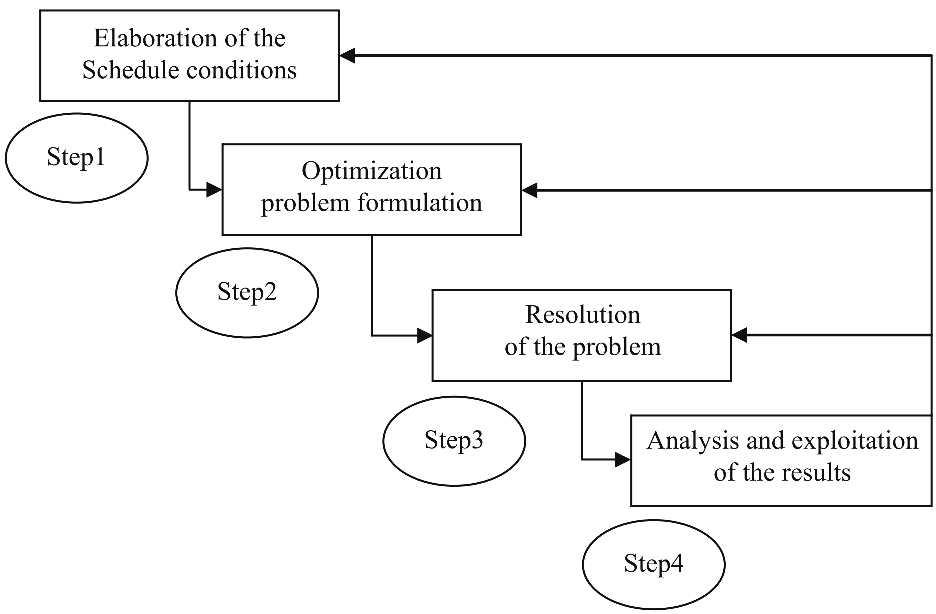

Fig. 3 – *Optimization procedure for the design problem.*

Modeling is important to achieve the design, therefore we have used the electromagnetic model of the DC pump obtained by the finite volume method. Fig. 3 shows the adopted optimization procedure [12].

4 Simulated Annealing Method

The simulated annealing method was proposed in 1983 by Kirkpatrick, Gelatt and Vecchi and originates in thermodynamics [13]. This method is based on slow cooling of a material at state fusion, which leads it to a solid state with low energy.

The same basic principle can be used in an optimization algorithm. The objective function to be minimized can be considered as the system energy, while different combinations of the optimization are the configurations of the system given its degrees of freedom.

4.1 Simulated annealing and physical system analogy

The analogy between a physical system made up of several particles and an optimization problem is based on the equivalences summarized in **Table 1**.

Table 1
Physique and optimization problem (simulated) analogy.

Physical system	Optimization Problem
Free energy	Objective function
Coordinated particles	The problem Parameters
State of low energy	Optimal configuration
Temperature (T)	Control Parameter
Physical system	Optimization Problem

4.2 Simulated Annealing (SA) algorithm

SA's major advantage over other methods is an ability to avoid becoming trapped in local minimum. The algorithm employs a random search which not only accepts changes that decrease the objective function $f(x)$ (assuming a minimization problem), but also some changes that increase it. The latter are accepted with a probability:

$$P = \mathrm{e}^{-\Delta E/T}, \tag{4}$$

where ΔE is the increase in f and T is a control parameter which by analogy with the original application is known as the system 'temperature' irrespective of the objective function involved. The implementation of the basic SA algorithm is simple.

The algorithm for the general procedure for implementing SA is presented in the following steps [13]:

- Step1: Select an initial temperature $T_0 \geq 0$;
- Step2: Select an initial solution S_0 and make it the current solution S and the current best solution S^*;
- Step3: repeat;
- Step4: set repetition counter $n = 1$;
- Step5: repeat;
- Step6: generates solution S_n in the neighborhood of S^*;
- Step7: calculate $\Delta E = f(S_n) - f(s^*)$;
- Step8: if $\Delta E \leq 0$ then $S_n = s^*$;
- Step9: else $S_n = s^*$ with the probability of $P = e^{(-\Delta E/T)}$;
- Step10: if $f(S_n) \leq f(s^*)$ then $S_n = s^*$;
- Step11: $n = n + 1$;
- Step12: until $n \geq$ number of repetitions allowed at each temperature level (L);
- Step13: reduce the temperature T until the stop criterion becomes true.

The principle underlying the choice of a suitable annealing schedule is easily stated: the initial temperature should be high enough to 'melt' the system completely and should be reduced towards its 'freezing point' as the search progresses.

Therefore, the following parameters should be specified:

- an initial temperature T_0,
- a final temperature T_f or a stopping criterion,
- a rule for decrementing the temperature.

4.2.1 Initial temperature determination

For the determination of the initial temperature, several methods are proposed in literature. The method used in this paper involves generating a certain number of random configurations *X*, for which the objective function is evaluated and their average value *M* calculated. This average value divides the distribution into two parts of equal probability of 0.5. Finally, the initial temperature is deduced from the criterion of Métropolis given by [14]:

$$P = e^{-\frac{M}{T_0}} = 0.5\,, \tag{5}$$

$$\ln P = -\frac{M}{T_0}\,, \tag{6}$$

$$T_0 = -\frac{M}{\ln P}\,, \tag{7}$$

so

$$T_0 = 1.44M \ . \tag{8}$$

4.2.2 Final temperature

In some simple implementations of the SA algorithm the final temperature is determined by fixing:

- the number of temperature values to be used, or
- the total number of solutions to be generated.

Alternatively, the search can be halted when it ceases to make progress.

4.2.3 Decrementing the temperature

The simplest and most common temperature decrement rule is:

$$T_{k+1} = \alpha T_k \tag{9}$$

where T_k is the current temperature, T_{k+1} is the new temperature and α is reduction factor ($0 < \alpha < 1$).

Reduction factor α is a constant close to, but smaller than 1. This exponential cooling scheme was first proposed with $\alpha = 0.95$ in which T is reduced after every L trials. In general, the final value of f is improved with slower cooling rates at the expense of greater computational effort.

SA has been applied to such problems as the well-known travelling salesman problem and optimization of wiring on computer [15 – 17].

4.3 Comparison with other methods

Any efficient optimization algorithm necessitates two techniques to find the global maximum or minimum, i.e. *exploration,* to investigate new and unknown areas in the search space and *exploitation* of the knowledge found at previous points to help find better ones. These two requirements are contradictory and a good search algorithm is needed for a compromise between the two.

4.3.1 Genetic algorithm

Genetic algorithms can be a powerful tool for solving problems and simulation of natural systems in a wide range of scientific fields [18]. Genetic Algorithms are not only efficient in their search strategy but also are also statistically guaranteed to find the function optima. They have been demonstrated to be competitive with other standard Boltzmann-type simulated annealing techniques [19].

4.3.2 Random search

The random method does not require the calculation of derivatives. They are simpler and applicable to the optimization of non differentiable function [20].

4.3.3 Gradient methods

In order to use a gradient method for optimization procedure, we need to know the information about the gradient of the objective function. If the derivative of the function cannot be computed due to its discontinuousness, for example, these methods are often unsuccessful. They are efficient for unimodal functions. However, for the multimodal functions they can not reach the top of a local maximum.

4.4 Effects of the reduction factors α on the convergence of optimal design method

Table 2 shows the effect of the reduction factor on the objective function. The initial temperature T_0 is calculated by the criterion of Métropolis (4). The objective function (Pump's mass [kg]) may not reach the value of the global minimum if the reduction factor is small. The table suggests that $\alpha = 0.99$ may be suitable.

Table 2
Pump's Performances.

α	0.7	0.8	0.9	0.99
Pump's mass [kg]	11.69	11.49	11.16	10.91

5 Optimization Results

Table 3 shows the dimensions of the pump obtained by the simulated annealing method.

Table 3
Optimal dimensions of the pump.

parameter	simulated annealing α = 0.99
channel's length [m]	0.1799
channel's radius [m]	0.0294
electrode's length [m]	0.0094
electrode's width [m]	0.1000
inductor's length [m]	0.1000
inductor's radius [m]	0.0703
coil's length [m]	0.0200
coil's radius [m]	0.0300
pump's mass [kg]	10.91

In order to verify the efficiency of the method, the obtained vector dimension is used in finite volume method as a dimension of the pump and the results are illustrated in Figs. 4 – 7.

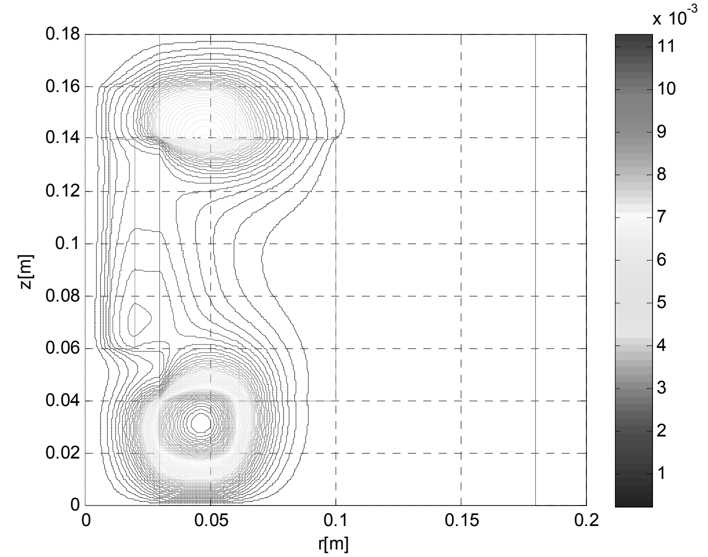

Fig. 4 – *The equipotential lines in the MHD pump.*

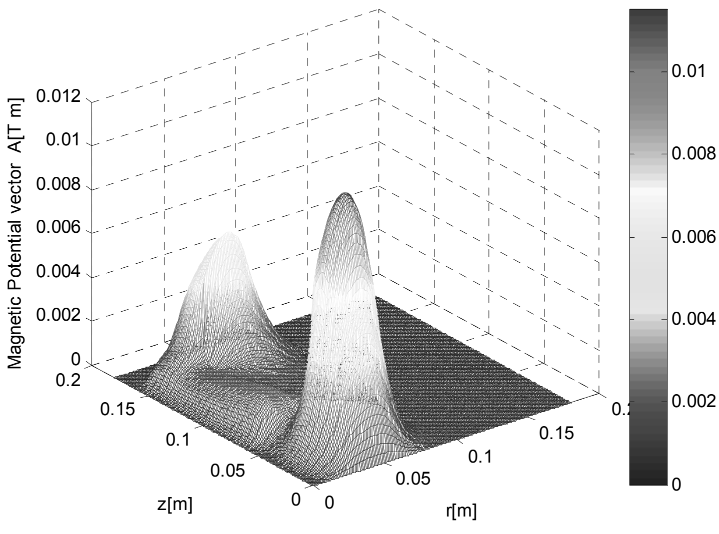

Fig. 5 – *The magnetic vector potential in the MHD pump.*

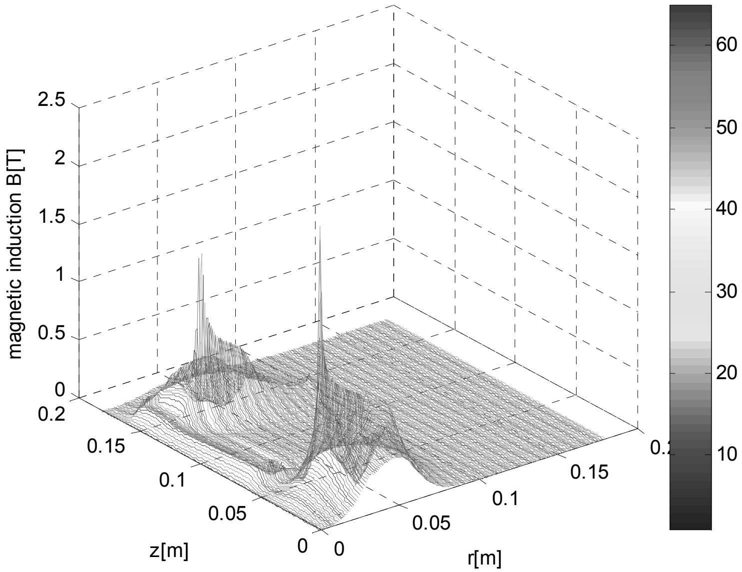

Fig. 6 – *The magnetic induction in the MHD pump.*

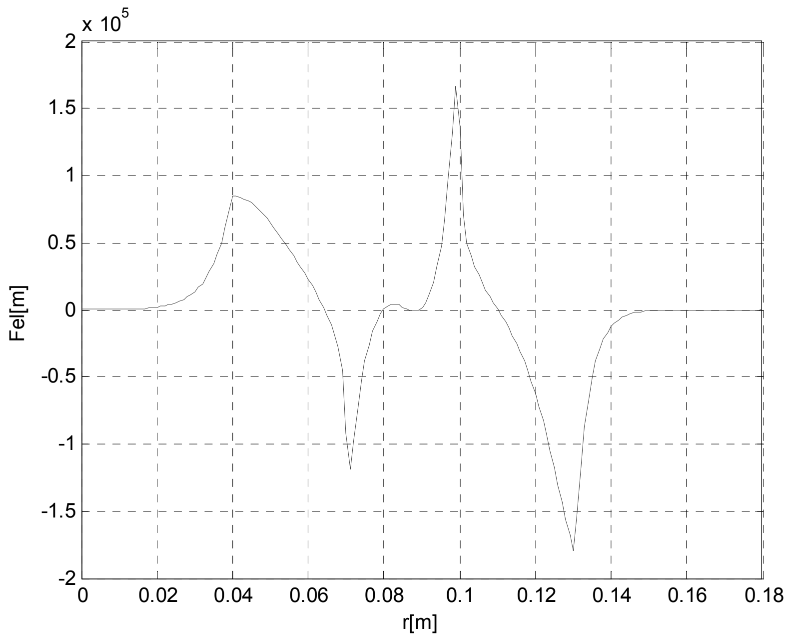

Fig. 7 – *The electromagnetic force in the channel.*

6 Conclusion

The calculation of the optimal dimension of a DC pump via global optimization is presented by using the simulated annealing algorithm. The stochastic optimization method is suitable as it does not depend on any initial conditions on the objective function, such as the derivability and the convexity. In addition, it gives preference to global minima.

The proposed method represents a means to obtain satisfactory pump design geometry. Also, it strictly fulfills the design requirements where constraints are satisfied regarding the obtained optimal mass.

It is important to highlight that the simulated annealing results depend effectively on the control parameters. The best one is obtained with initial temperature $T_0 = 100$ and $\alpha = 0.99$.

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