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Performance of QPSK System in the Presence of Pulse Interference and Noisy Carrier Reference Signal

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Abstract: The aim of this paper is the determining the system performance in detecting the QPSK signal. The error probability is determined when the signal, Gaussian noise, pulse interference and imperfect carrier phase recovery are taken into consideration. Phase locked loop, as the constituent part of the receiver, is used in providing the synchronization reference signal extraction, which is assumed to be imperfect. The obtained results are based on the PLL non-linear model of the first order, with the emphasis on the degradation in the system performance produced by the imperfect carrier signal extraction. The system performance is determined when the signal corrupted by pulse interference and Gaussian noise is applied at the input of the receiver.

Keywords: QPSK, Phase-locked loop, Pulse interference, Error probability.

1 Introduction

The performance evaluation of binary and *M*-ary (*M*>2) phase-shift-keying communication systems have been analysed in a great variety of papers, which have appeared in the literature [1-7]. Quaternary phase-shift-keying (QPSK or 4-PSK) systems have the greatest practical interest of all no binary (multiposition) systems of digital transmission of messages by phase-modulated signals. Currently, QPSK is one of the prevalent modulations in use for digital communication systems (since bandwidth efficiency) [1, 2]. The only significant penalty factor is an increased sensitivity to carrier phase synchronisation error.

Any successful transmission of information through a digital phase-coherent communication system requires a receiver capable of determining or estimating the phase and the frequency of the received signal with as few errors as possible; any noise associated with carrier leads to degradation of the detection performance of the system. In practice, quite often the phase locked loop (PLL) is used in providing the desired reference signal [3, 4, 5, 6]. Frequently, a PLL system must operate in such conditions where the external fluctuations due to the additive noise are so intense that classical linear PLL theory neither characterises adequately the loop performance, nor explain the loop be-

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haviour [7]. The direct linearization cannot be used in loop performance explanation and characterization in the region of the operation in many practical solutions. So, the analytical approach in developing an exact non-linear theory of PLLs, based on Fockker-Planck theory was investigate in [7,11]. Whenever the important parameter β which characterize the p.d.f. of the phase error is not equal to zero the loop is said to be stressed. It is possible to make some theoretical and experimental results be in agreement within a few tenths of a decibel if the effects, which the loop stress has on the phase-coherent system, are analytically accounted. Numerical results for QPSK system is presented so that this results combined with the characteristic of the phase recovery circuit will enable the best practical design of a QPSK system.

It is well known that the certain components that appear in telecommunication channels are very often with a pulse characteristics, i.e. noise can be described as a sum of peaks of large amplitudes in comparison with the common noise level. This channels are often narrowband, so it follows narrowband systems are considered. Poisson pulse noise model is used for modelling. Samples consist of a random delta functions. This model gives a very good approximation of the most important natural pulse noise features.

In the approach used here the primary emphasis is placed on the degradation in the QPSK system performance produced by the imperfect carrier signal extraction. The contribution of this paper is in determining the error probability in detecting the quaternary phase-modulated signal when the PLL is used for synchronization reference signal extraction, which is imperfect. Our assumption of the imperfect carrier extraction is justified due to the fact that in any practical communication system synchronization signals needed to perform a coherent detection are not exactly known since they are derived at the receiver in the presence of noise. The error probability, as a measure of systems quality, is an important issue and has received much attention in the literature. Pulse interference influence is often fundamental limiting factor in digital transmission systems. An expression for the bit error probability was calculated when the signal and Gaussian noise are applied at the input of the QPSK system [8]. QPSK system performance when the signal, Gaussian noise, pulse interference and imperfect carrier phase recovery are considered as source of degradation, are determined in this paper.

2 System Model

The model for the communication system to be considered in this paper is shown in Fig.1. Let the input signal at QPSK receiver consists of the signal, pulse interference and Gaussian noise:

$$r(t) = A\cos\left(\omega_0 t + \phi_o\right) + A_1 n_i \cos(\omega_0 t + \theta) + n(t), \tag{1}$$

where A is a signal amplitude, ϕ_o can be $\pi/4$, $3\pi/4$, $-\pi/4$ or $-3\pi/4$ depending upon which symbol is transmitted, ω_o is a constant carrier frequency, $n(t)$ is a Gaussian noise, θ is the uniformly distributed phase with the probability density function,

$$p(\theta) = \frac{1}{2\pi}, \quad \{-\pi \le \theta \le \pi\}. \tag{2}$$

A_1 is a pulse interference amplitude, where the cos wave is modulated by the pulses with the form

$$n_i = \sum_{i=-\infty}^{\infty} a_i \sqrt{\frac{2}{T}} \cos\omega_o (T - t_i), \tag{3}$$

where a_i represents a random area where i pulses appears at the random time t_i, $\omega_o = 2\pi n / T$, n is an integer and T is pulse duration. Moments of pulses appearing are presented as a Poisson process.

Now, input signal can be also written with the form

$$r(t) = AR\cos(\omega_0 t + \psi) + n(t),$$

$$R = \sqrt{1+\eta^2 + 2\eta\cos\theta}, \quad \eta = \frac{A_1}{A} n_i, \quad \psi = \text{arctg}\frac{\eta\sin\theta}{1+\eta\cos\theta}, \tag{4}$$

where η is a pulse interference to signal ratio.

It is assumed that the signal is corrupted by the pulse interference with the following probability density function [9],

$$p(n_i) = (1-\gamma)\delta(t) + \left(\frac{\gamma}{\pi}\right)^{3/2} \int_0^{\pi/2} \frac{e^{-\frac{\gamma n_i^2}{4\sigma^2\cos^2(y)}}}{\sigma\cos(y)} \text{d}y , \tag{5}$$

where γ represents average number of pulses which appears in the signal time duration T, $\delta(t)$ – delta function, σ is the spectral density of the pulse modulated interference.

From now on, pulse interference, additive Gaussian noise and imperfect phase carrier recovery, are taken into account in our detection analysis. All other functions are considered ideal. The block diagram of a QPSK system would be adopted is shown in Fig.1. The recovered carrier signal is assumed to be in the form of the sin wave. Also, it would be adopted that an original message is in binary form.

Under the assumption of a constant phase in the symbol interval, the conditional error probability for the given phase error ϕ (the phase error ϕ is the difference between the receiver incoming signal phase and the voltage controlled oscillator output signal phase) can be written as [8],

$$P_{e/\phi}(\phi) = \frac{1}{4}\left\{\text{erfc}\left[\sqrt{2R_b}\cos[(\pi/4)+\phi]\right] + \text{erfc}\left[\sqrt{2R_b}\cos[(\pi/4)-\phi]\right]\right\}, \tag{6}$$

where the function erfc(x) is the well known complementary error function defined as

$$\text{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-z^2/2} \text{d}z. \tag{7}$$

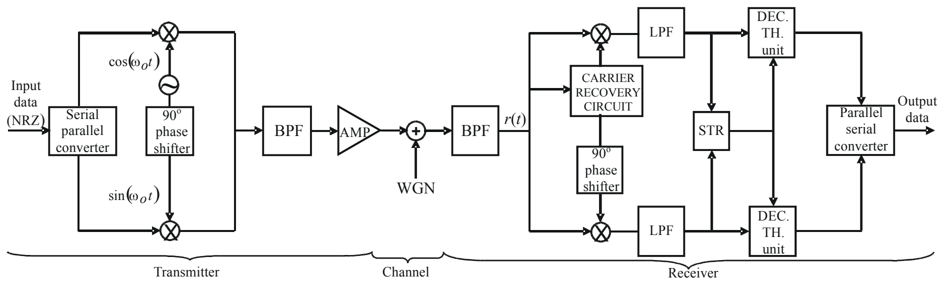

Fig. 1 - *Block diagram of a QPSK receiver.*

The received signal to noise spectral density ratio in the data channel (demodulator) denoted by R_b, is given by $R_b=E/N_o$, where E is a signal energy per bit duration T. N_o represents the normalised noise power spectral density in W/Hz, referenced to the input stage of the demodulator, since the signal to noise ratio is established at that point. The signal detection in receiver is accomplished by cross-correlation-and-sampling operation. The effect of filtering due to $H(f)$ in Fig.1. is not considered here.

The conditional steady state probability density function, for the non-linear PLL model with a known signal and noise at the PLL input, of modulo 2π reduced phase error is given by the following approximation [7]

$$p(\phi) = \frac{e^{\beta\phi+\alpha\cos\phi}}{4\pi^2 e^{-\pi\beta}\,|I_{j\beta}(\alpha)|^2} \int_{\phi}^{\phi+2\pi} e^{-\beta x+\alpha\cos x}\mathrm{d}x, \tag{8}$$

$I_{j\beta}(\alpha)$ is the modified Bessel function of complex order $j\beta$ and real argument α. The range of definition for ϕ in the previous equation is any interval of width 2π centred about any lock point $2n\pi$, with n an arbitrary integer. The parameters α and β, that characterise eq. (8), for the first order non-linear PLL model in this case are:

$$\alpha = \alpha_0 R, \qquad \beta = \beta_0 \Omega, \tag{9}$$

where α_0 and β_0 are constants [7, 11], R refers to eq. (4). The parameter α is a measure of the loop signal to noise ratio in the sense that the larger the value of α, the smaller are the deleterious effects due to noise reference signal. The parameter β is a measure of the loop stress. Ω is the loop detuning, i.e. the frequency offset of the first term in eq. (4) defined by

$$\Omega = \frac{\mathrm{d}}{\mathrm{d}t}(\omega_0 t + \psi) - \omega_0 = \frac{\eta(\eta+\cos\theta)}{R^2}\frac{\mathrm{d}\theta}{\mathrm{d}t}. \tag{10}$$

Since $(d\theta/dt) = 0$, it follows $\Omega = 0$, i.e. $\beta = 0$. Therefore, the average steady-state probability density function of the phase error is

$$p(\phi) = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} p(\phi / \theta, n_i) p(\theta) p(n_i) \mathrm{d}\theta \mathrm{d}n_i \ . \tag{11}$$

Substituting eq. (4) into eq. (11) yields the probability density function of the phase error that is shown in Fig.2.

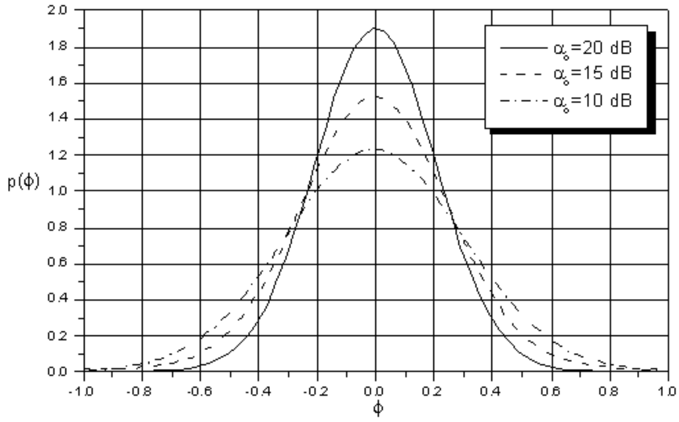

Fig. 2 - *Probability density function of the phase for non-linear first order PLL model.*

3 System Performance

Substituting $R_b = R_1 R^2$ in eq. (6), where R_1 corresponds to the case when there is no interference, the conditional bit error probability, given both ϕ and θ is determined. The total error probability is determined by averaging the conditional error probability over random variables ϕ, θ and n_i

$$P_e = \int_{\phi} \int_{\theta} \int_{n_i} P_{e/\phi} p(\phi / \theta, n_i) p(\theta) p(n_i) \mathrm{d}\phi \mathrm{d}\theta \mathrm{d}n_i \ . \tag{12}$$

The total error probability is computed on the basis of the eq. (12) and is plotted versus signal to noise ratio (R_1 [dB]) at the demodulator input in Fig.3 a), and b). The values are given in figures.

4 Numerical Results Analysis

The average error probability as a function of the signal-to-noise ratio at the receiver input for constant value of signal-to-noise ratio α_o and the various values of the interference-to-noise ratio A_1/A is plotted in Figs.3 a) and b). The curve with α_o=20 dB and for various values of the signal-to-noise ratio is split in two regions (denoted by AB and BC) for the explanation of the signal to noise ratio influence on the error probability variations. In the AB region, the error probability decreases sharply with the parameter R_1 increasing. For example, if the parameter R_1 is changed from 5 to 20 dB, the error probability decreases 4.65 10^3 times. In the BC region the error probability variations with R_1 increasing are fewer than in previous case. If the parameter R_1 is changed from 20 to 25 dB, the error probability decreases only 5.64 times. In this region, the parameter R_1 is relatively large, and in comparison with the value of the parameter α_o, its influence on the error probability decreases. The error probability for the greater values of R_1 tends to the constant value (*BER floor*). This *BER floor* can be reduced by increasing the parameter α_o. The influence of the parameter α_o is noticeable for the values of R_1 greater than 5 dB. The *BER floor* value for α_o=10 dB is greater 9.26 10^3 times than the *BER floor* value for α_o=20 dB (Fig.3b).

The influence of the interference-to signal ratio (A_1/A) on the error probability is evident from the Fig.3 a) and Fig.4. The following observation is significant. From the figures follows that the system error probability decreases with the increase of the signal to noise ratio (R_1). One can see that the system shows the better performance with increases of both, PLL parameter α_0, Fig.3 a), and ratio A_1/A, Fig.3 b). If the pulse interference is present at the receiver input and if the parameter R_1 is in the range of 0 to 10 dB, the bit error probability increases only 11.53 times. But, if the parameter R_1 is in the range of 30 dB or more the BER floor value increases 1.5 10^6 times. On the basis of the shown analysis one can see that the appearing of the pulse interference at the receiver input has a great influence on the QPSK system performance.

5 Conclusion

The quaternary PSK system is analysed by means of the system error probability, in this paper. Noise influence, pulse interference and imperfect carrier phase recovery are the limiting factors in the observed system performance. The pulse interference is represented by cosinusoidal signal with the uniform distributed phase. The influence of the imperfect reference signal extraction is expressed by the probability density function of the PLL phase error.

The detailed analysis of the obtained numerical results is performed in this paper. Case when the signal Gaussian noise and pulse interference is applied at the input of the receiver has been considered in this paper. The influence of the parameters α_0 as well as the influence of the pulse interference to signal ratio A_1/A and the signal to noise ratio R_1 on the system error probability are especially considered. On the basis of the shown analysis one can conclude that the system has better performances if both, PLL parameter, α_0, and A_1/A parameter have a greater values.

However, from all figures, the large signal to noise ratio system error tends to a constant value (BER floor). In the BER floor area, the signal to noise ratio is relatively large with respect to parameter α_0 and has therefore a small influence on the system error probability. It is seen from Fig.3 a) that this BER floor can be reduced by increasing the parameter α_0, which depends on the applied PLL loop. On the basis of the shown analysis it is possible to determine the QPSK system parameter α_0 and useful signal power necessary to compensate the imperfect carrier extraction. This means that the presented conclusions can be useful in practice for the QPSK system design.

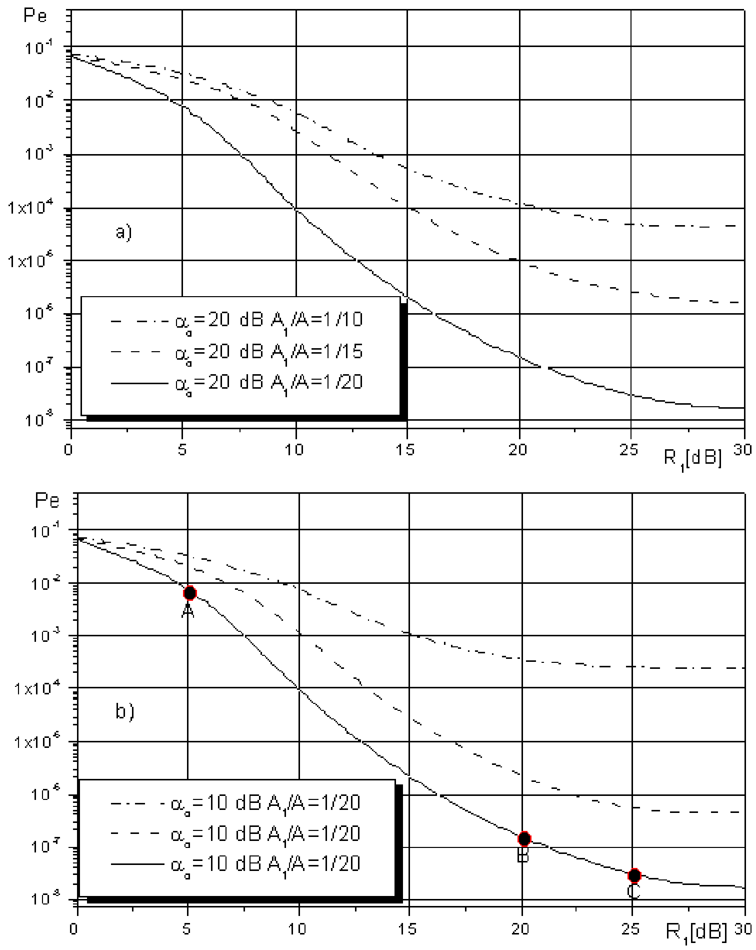

Fig. 3 - *Average error probability performance of a QPSK coherent detector with a noisy carrier synchronization reference signal when α_o is a constant, while A_1/A is parameter a), and when A_1/A is a constant, while α_o is parameter b).*

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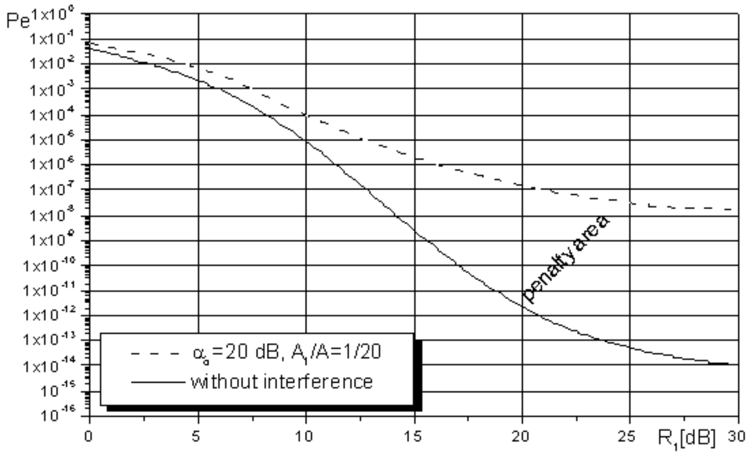

Fig. 4 - *QPSK system performance penalty caused by appearing the pulse interference at the receiver input.*

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