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Robust Linear Parameter Varying Induction Motor Control with Polytopic Models

Khamari Dalila[1], Makouf Abdessalem[1], Drid Said[1], Larbi Chrifi-Alaoui[2]

Abstract: This paper deals with a robust controller for an induction motor which is represented as a linear parameter varying systems. To do so linear matrix inequality (LMI) based approach and robust Lyapunov feedback controller are associated. This new approach is related to the fact that the synthesis of a linear parameter varying (LPV) feedback controller for the inner loop take into account rotor resistance and mechanical speed as varying parameter. An LPV flux observer is also synthesized to estimate rotor flux providing reference to cited above regulator. The induction motor is described as a polytopic model because of speed and rotor resistance affine dependence their values can be estimated on line during systems operations. The simulation results are presented to confirm the effectiveness of the proposed approach where robustness stability and high performances have been achieved over the entire operating range of the induction motor.

Keywords: Induction motor, LMI, LPV controller, Lyapunov feedback controller, Polytopic representation.

1 Introduction

The Induction motor is widely used in industry due to the simple mechanical structure and easy maintenance. However this motor presents a challenging control problem for three reasons. The dynamical system is highly non linear, the rotor flux is not usually measurable and finally the rotor resistance value varies considerably with a significant impact on the system dynamics. The trends in induction motor control system is to use effective robust controller design such as H_∞ and other robust control approaches [3, 4, 5, 21]. Furthermore the main advantage of using field-oriented control of voltage-controlled induction motor is that good performance can be achieved via non-linear state feedback [2].

[1]Department of Electrical Engineering, Batna University, Université de Batna, Avenue Chahid Mohamed Boukhlouf – 05000 – Batna, Algeria; E-mails: khamari.dalila@yahoo.com; a_makouf@yahoo.fr; s_drid@yahoo.fr
[2]I.U.T de l'Aisne Département GEII. 13 avenue François Mitterrand, 02880 Cuffies, Laboratoire des Technologies Innovantes (L.T.I), Université de Picardie Jules Verne, Soissons, France;
E-mail: larbi.alaoui@u-picardie.fr

In this work, the synthesis of a LPV system controller use a linear matrix inequality (LMI) approach [6, 7] and Lyapunov theory [8]. LPV systems are a special class of systems, which are linear time invariant (LTI) system for every fixed value of the parameter vector $\theta(t)$ This parameter can be measured on line during control operation. Also the LPV control technique can eliminate the tedious process of manually tuning control, gain and can provide a systematic gain-scheduling method [9, 10, 11, 23]. In our case it is assumed that only the stator current and the rotor speed are available for measurement, the rotor resistance estimation is beyond the scope of this work. The control law consists of fast inner loop used to track stator current reference generated by the Lyapunov theory associated to a sliding mode control of the flux and the speed of the motor. This approach shows good robustness and high performance with respect parameter and load torque variation.

The induction motor model described in the (α,β) frame can be written as an LPV system wich can be translated in polytopic representation because of affine dependence with the rotor speed and the rotor resistance. This feature will be exploited in designing a self gain scheduled LPV feedback controller for the inner loop [12, 13, 14]. Also, the LPV motor structure can be used to improve the robustness of the flux observer and to compute the worst case flux estimation error in terms of H_∞ norm respecting parameter variation [17]. The paper is organized as follows. In Section 2, the LPV modeling and control synthesis conditions are derived for affine parameter dependent systems. In Section 3 the control structure of the stator current is given. In Section 4 robust non linear controls is obtained for the speed and flux control. In Section 5 the robust flux observer synthesis with mixed sensitivity structure is given. Validation with numerical simulations for all theoretical resulting and interpretations are presented in Sections 6 and 7.

2 Induction Motor LPV Model

2.1 Induction motor models

The state space representation of the induction motor in the stator reference frame is given as follows:

$$\begin{bmatrix} \dot{\varphi}_{r\alpha} \\ \dot{\varphi}_{r\beta} \\ \dot{i}_{s\alpha} \\ \dot{i}_{s\beta} \end{bmatrix} = \begin{bmatrix} a_1\theta_2 & p\theta_1 & a_2\theta_2 & 0 \\ p\theta_1 & a_1\theta_2 & 0 & a_2\theta_2 \\ a_3\theta_2 & pa_3 & a_4 + a_5\theta_2 & 0 \\ pa_3\theta_1 & a_3\theta_2 & 0 & a_4 + a_5\theta_2 \end{bmatrix} \cdot \begin{bmatrix} \varphi_{r\alpha} \\ \varphi_{r\beta} \\ i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b & 0 \\ 0 & b \end{bmatrix} \cdot \begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \end{bmatrix}, \quad (1)$$

where

$$a_1 = -\frac{1}{L_r}, \quad a_2 = \frac{M}{L_r}, \quad a_3 = \frac{M}{L_s L_r^2 \sigma}, \quad a_4 = \frac{-L_r^2 R_s}{L_s L_r^2 \sigma},$$

$$a_5 = -\frac{M^2}{L_s L_r^2 \sigma}, \quad \sigma = 1 - \frac{M^2}{L_s L_r^2 \sigma}, \quad b = \frac{1}{\sigma L_s},$$

with $\theta_1 = \omega$, $\theta_2 = R_r$, $(i_{s\alpha}, i_{s\beta})$ are the stator current components, $(\varphi_{r\alpha}, \varphi_{r\beta})$ are the rotor flux components and $(V_{s\alpha}, V_{s\beta})$ is the stator voltage components.

The electromagnetic torque is given by:

$$\boldsymbol{T}_e = p\frac{M}{L_r}\left(\boldsymbol{i}_s \times \boldsymbol{\varphi}_r\right). \tag{2}$$

2.2 Polytopic induction motor representation

LPV model of induction motor is described by state space representation of the form

$$\boldsymbol{G}(\boldsymbol{\theta}): \begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{\theta}(t))\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases} \tag{3}$$

where $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T = \begin{bmatrix} \omega(t) & R_r(t) \end{bmatrix}^T$ is a time varying parameter. According to [22] and based on the theory of heating materials, the rotor resistance R_r can be taken as time varying parameter since it can be accurately estimated on line [20]. Thus:

$$\boldsymbol{A}(\boldsymbol{\theta}) = \boldsymbol{A}(\omega, R_r) = \boldsymbol{A}_0 + \omega\boldsymbol{A}_1 + R_r\boldsymbol{A}_2. \tag{4}$$

Specifically for our problem, the parameter vector $\boldsymbol{\theta}(t)$ has the following convex decomposition:

$$\boldsymbol{\theta}(t) = \alpha_1\theta_{11} + \alpha_2\theta_{12} + \alpha_3\theta_{21} + \alpha_4\theta_{22}, \quad \sum_{i=1}^{4}\alpha_i = 1, \quad \alpha_i \geq 0, \tag{5}$$

where α_i gives the corner of polytopic parameter range. The corner values of parameter range are:

$$\theta_{11} = (0, \omega_{\min}), \quad \theta_{12} = (0, \omega_{\max}), \quad \theta_{21} = (0, R_{r\min}), \quad \theta_{22} = (0, R_{r\max}). \tag{6}$$

At the vertices values of $\boldsymbol{\theta}$ the plant matrix is:

$$\boldsymbol{G}(\boldsymbol{\theta}) = \alpha_1\boldsymbol{G}(\theta_{11}) + \alpha_2\boldsymbol{G}(\theta_{12}) + \alpha_3\boldsymbol{G}(\theta_{21}) + \alpha_4\boldsymbol{G}(\theta_{22}), \tag{7}$$

with:

$$\alpha_1 = \frac{\omega(t) - \omega_{\min}}{\omega_{\max} - \omega_{\min}}, \quad \alpha_3 = \frac{R_r(t) - R_{r\min}}{R_{r\max} - R_{r\min}}, \quad \alpha_2 = \frac{\omega(t) - \omega_{\max}}{\omega_{\max} - \omega_{\min}}, \quad \alpha_4 = \frac{R_r(t) - R_{r\max}}{R_{r\max} - R_{r\min}}.$$

The structure of the LPV controller of the system (7) is than given by polytopic representation as following:

$$\boldsymbol{K}(\boldsymbol{\theta}) = \sum_{j=1}^{4} \alpha_i \begin{bmatrix} \boldsymbol{A}_k(\boldsymbol{\theta}_i) & \boldsymbol{B}_k(\boldsymbol{\theta}_i) \\ \boldsymbol{C}_k(\boldsymbol{\theta}) & D_k(\boldsymbol{\theta}_i) \end{bmatrix}. \tag{8}$$

3 LPV Stator Current Control

LPV stator current controller is designed in the stator frame. Its main advantage is that the inconveniences related to the Park transformation which could significantly affect the performances are avoided [18].

3.1 LPV control background

L_2 Gain performance

Consider an open loop LPV system P described by

$$P: \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{\theta}(t))\boldsymbol{x}(t) + \boldsymbol{B}_1(\boldsymbol{\theta}(t))\boldsymbol{w}(t) + \boldsymbol{B}_2(\boldsymbol{\theta}(t))\boldsymbol{u}(t), \\ \boldsymbol{z}(t) = \boldsymbol{C}_1(\boldsymbol{\theta}(t))\boldsymbol{x}(t) + \boldsymbol{D}_{11}(\boldsymbol{\theta}(t))\boldsymbol{w}(t) + \boldsymbol{D}_{12}(\boldsymbol{\theta}(t))\boldsymbol{u}(t), \\ \boldsymbol{y}(t) = \boldsymbol{C}_2(\boldsymbol{\theta}(t))\boldsymbol{x}(t) + \boldsymbol{D}_{21}(\boldsymbol{\theta}(t))\boldsymbol{w}(t), \end{cases} \tag{9}$$

where $\mathbf{y}$ denote the measured output, $\mathbf{z}$ the controlled output, $\mathbf{w}$ the reference and disturbance inputs and $\mathbf{u}$ the control inputs.

The matrices in (9) are affine functions of the parameter vector that varies in polytope $\boldsymbol{\Theta}$ with vertices $\boldsymbol{\theta}_1,...,\boldsymbol{\theta}_j$, that is:

$$\boldsymbol{\theta}(t) \in \boldsymbol{\Theta} = \text{conv}\left\{\boldsymbol{\theta}_1,...,\boldsymbol{\theta}_j\right\} \triangleq \left\{\sum_{j=1}^{r} \alpha_i \boldsymbol{\theta}_j, \quad \alpha_j \geq 0, \quad \sum_{j=1}^{r} \alpha_j = 1\right\}.$$

The LPV synthesis problem consists in finding a controller $\boldsymbol{K}(\boldsymbol{\theta})$ described by:

$$\boldsymbol{K}(\boldsymbol{\theta}): \begin{cases} \dot{\boldsymbol{x}}_K(t) = \boldsymbol{A}_K(\boldsymbol{\theta}(t))\boldsymbol{x}_K(t) + \boldsymbol{B}_K(\boldsymbol{\theta}(t))\boldsymbol{y}(t) \\ \boldsymbol{u}(t) = \boldsymbol{C}_K(\boldsymbol{\theta}(t))\boldsymbol{x}_K \end{cases} \tag{10}$$

such that the closed- loop system (11) (with input $\boldsymbol{w}$ and output $\boldsymbol{z}$ is internally stable and the induced L_2-norm of $\boldsymbol{w} \rightarrow \boldsymbol{z}$ is bounded by a given number $\gamma \succ 0$ for all possible parameter trajectories:

$$P_{cl}: \begin{bmatrix} \dot{\xi}(t) \\ \boldsymbol{z}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cl}(\boldsymbol{\theta}(t)) & \boldsymbol{B}_{cl}(\boldsymbol{\theta}(t)) \\ \boldsymbol{C}_{cl}(\boldsymbol{\theta}(t)) & \boldsymbol{D}_{cl}(\boldsymbol{\theta}(t)) \end{bmatrix} \cdot \begin{bmatrix} \xi(t) \\ \boldsymbol{w}(t) \end{bmatrix}. \tag{11}$$

The characterization of robust stability and performance for the closed-loop system P_{cl} (11) is proved by the following theorem.

Theorem: The LPV system (11) has a quadratic stability and L_2 gain level $\gamma \succ 0$ if there exists a matrix $\mathbf{X} \succ 0$ such that:

$$\begin{bmatrix} \boldsymbol{A}_{cl}^T(\boldsymbol{\theta})\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A}_{cl}(\boldsymbol{\theta}) & \boldsymbol{X}\boldsymbol{B}_{cl}(\boldsymbol{\theta}) & \boldsymbol{C}_{cl}(\boldsymbol{\theta})^T \\ \boldsymbol{B}_{cl}(\boldsymbol{\theta})^T \boldsymbol{X} & -\gamma \boldsymbol{I} & \boldsymbol{D}_{cl}(\boldsymbol{\theta})^T \\ \boldsymbol{C}_{cl}(\boldsymbol{\theta}) & \boldsymbol{D}_{cl}(\boldsymbol{\theta}) & -\gamma \boldsymbol{I} \end{bmatrix} \prec 0. \tag{12}$$

This implies for synthesis inequalities (12) that, without loss of generality, we can replace the search over the polytope $\boldsymbol{\Theta}$ by the search over the vertices of this set Consequently, condition (12) can be reduced to a finite set of linear matrix inequalities (LMI).

3.2 Computation of self-scheduled LPV controller

We assume that parameter dependence of the plant P is affine and $\boldsymbol{\Theta}$ is polytope with vertices $\boldsymbol{\theta}_j$, $j = 1,2,...,r$. According to the result in [6, 7], one LPV controller $\boldsymbol{K}(\boldsymbol{\theta})$ can be computed through the following steps:

*Compute the vertex controllers $\boldsymbol{K}_j = (\boldsymbol{A}_{K_j}, \boldsymbol{B}_{K_j}, \boldsymbol{C}_{K_j}, 0)$, $(1 \le j \le r)$ as follows:

Solve the set of LMIs (13) and (14)

$$\begin{bmatrix} \boldsymbol{X}\boldsymbol{A}_j + \hat{\boldsymbol{B}}_{K_j}\boldsymbol{C}_{2_j} + * & * & * & * \\ \hat{\boldsymbol{A}}_{K_j}^T + \boldsymbol{A}_j & \boldsymbol{A}_j\boldsymbol{Y} + \boldsymbol{B}_{2_j}\hat{\boldsymbol{C}}_{K_j} + * & * & * \\ (\boldsymbol{X}\boldsymbol{B}_{1_j} + \hat{\boldsymbol{B}}_{K_j}\boldsymbol{D}_{21_j})^T & \boldsymbol{B}_{1j}^T & -\gamma\boldsymbol{I} & * \\ \boldsymbol{C}_{1_j} & \boldsymbol{C}_{1j}\boldsymbol{Y} + \boldsymbol{D}_{12_j}\hat{\boldsymbol{C}}_{K_j} & \boldsymbol{D}_{11_j} & -\gamma\boldsymbol{I} \end{bmatrix} \prec 0, \tag{13}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{Y} \end{bmatrix} \succ 0. \tag{14}$$

where $(*)$ denotes terms whose expressions follow the requirement that the matrix is self-adjoint. This step gives $(\hat{\boldsymbol{A}}_{K_j}, \hat{\boldsymbol{B}}_{K_j}, \hat{\boldsymbol{C}}_{K_j})$ and symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$.

Compute $\boldsymbol{A}_{K_j}$, $\boldsymbol{B}_{K_j}$ and C_{K_j} by:

$-\boldsymbol{A}_{Kj} = \boldsymbol{N}^{-1}(\hat{\boldsymbol{A}}_{Kj} - \boldsymbol{X}\boldsymbol{A}_j\boldsymbol{Y} - \hat{\boldsymbol{B}}_{Kj}\boldsymbol{C}_{2j}\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{B}_{2j}\hat{\boldsymbol{C}}_{Kj})\boldsymbol{M}^{-T}$,

$-\boldsymbol{B}_{Kj} = \boldsymbol{N}^{-1}\hat{\boldsymbol{B}}_{Kj}$,

$-\boldsymbol{C}_{Kj} = \hat{\boldsymbol{C}}_{Kj}\boldsymbol{M}^{-T}$, where $\boldsymbol{N}$ and $\boldsymbol{M}$ are matrices such that $\boldsymbol{I} - \boldsymbol{X}\boldsymbol{Y} = \boldsymbol{N}\boldsymbol{M}^T$.

Finally the state space matrices of the LPV polytopic controller $\boldsymbol{K}(.)$ as a convex combination of the vertex controllers is given by:

$$\begin{bmatrix} \boldsymbol{A}_K & \boldsymbol{B}_K \\ \boldsymbol{C}_K & 0 \end{bmatrix}(\boldsymbol{\theta}) = \sum_{j=1}^{r} \alpha_j \begin{bmatrix} \boldsymbol{A}_{K_j} & \boldsymbol{B}_{K_j} \\ \boldsymbol{C}_{K_j} & 0 \end{bmatrix}. \tag{15}$$

3.3 Loop shapping-mixed sensitivity structure

To reach objectives in terms of performances and robustness of system control we have to introduce weighting functions acting as frequency filters on the I/O signals of the systems [15, 16]. It can be shown that robust stability, reference tracking, disturbance and noise attenuation can be defined with sensitivity function $\boldsymbol{S} = (\boldsymbol{I} + \boldsymbol{GK})^{-1}$ complementary sensitivity function $\boldsymbol{T} = \boldsymbol{I} - \boldsymbol{S}$ and the closed loop transfer function KS. Thus, H_∞ mixed sensitivity criterion respect following inequality: $\left\| \begin{matrix} W_s S \\ W_T KS \end{matrix} \right\|_\infty \prec 1$, where W_T must be a high-pass filter function to insure robustness against neglect dynamics and W_s a low-pass filter to guarantee good tracking accuracy.

3.4 LPV current controller design

The $K(\omega, R_r)$, as it is shown in Fig. 1, is a current LPV feedback controller allowing to tracks the set point reference i_{sref}. The input of controller is the difference between i_{sref} and i_s obtained from $G(\omega, R_r)$ representing the induction motor.

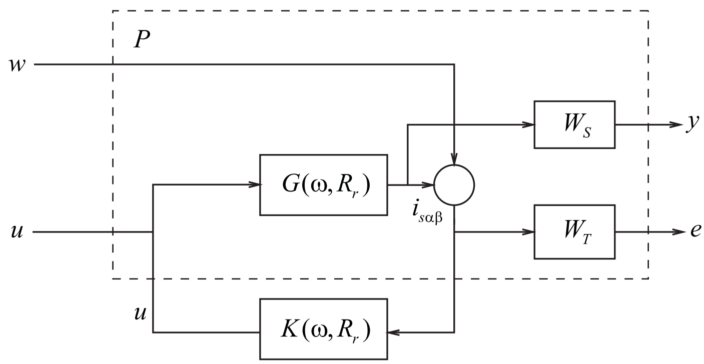

Fig. 1 – *Mixed sensitivity structure of H_∞ traking.*

The current feedback controller is obtained using polytopic representation of induction motor given by (1). The measured output is considered as

$y = [i_{s\alpha} \quad i_{s\beta}]$ and the external inputs is of reference current $w = [i_{s\alpha ref} \quad i_{s\beta ref}]$. The controller outputs are the stator voltage components $u = [V_{s\alpha} \quad V_{s\beta}]$. The robust multivariable LPV controller has to provide satisfactory performance over the whole operating range of the motor. The LPV controller in polytopic representation with four vertices is computed using the LMI toolbox. Each vertex can be considered as an LTI controller with eight states. The L_2-gain bound γ guaranteeing the closed loop system performance and stability is equal for our case to ($\gamma = 1.0002$). In order to obtain the optimal controller the weighting function used are as following:

$$\begin{aligned} W_s &= \mathrm{diag}\left(\frac{200}{s+0.0002}, \frac{200}{s+0.0002}\right), \\ W_T &= \mathrm{diag}\left(\frac{200}{s+0.0002}, \frac{200}{s+0.0002}\right). \end{aligned} \tag{16}$$

4 Speed and Flux Controller

The Lyapunov theory associate to sliding mode control technique is used to design speed and flux controller. This control system allows robust control of all transient electromagnetic phenomena in a motor. To simplify the synthesis procedure of controller the rotor flux is oriented on the *d* axis as it is given by following relations:

$$\varphi_{rd} = \phi_r, \qquad \varphi_{rq} = 0. \tag{17}$$

The induction motor model can be expressed in the synchronous frame and specifically the dynamics of flux and speed are given by the following equation:

$$0 = -\frac{R_r}{L_r} i_{sd} + \frac{R_r}{L_r}\varphi_{rd} + \frac{\mathrm{d}\varphi_{rd}}{\mathrm{d}t}, \tag{18}$$

$$T_e - T_l = J\frac{\mathrm{d}\omega}{\mathrm{d}t} + f\omega, \tag{19}$$

where T_l is the load torque and T_e the electromagnetic torque given by

$$T_e = p\frac{M}{L_r}\varphi_{rd} i_{sq}. \tag{20}$$

The equations described by (18) and (19) can be rewritten as

$$\begin{cases} \dfrac{\mathrm{d}\varphi_{rd}}{\mathrm{d}t} = f_1 + \dfrac{1}{T_r} i_{sd} \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = f_2 + T_e \end{cases} \tag{21}$$

where $f_1 = \dfrac{R_r}{L_r}\varphi_{rd}$, $f_2 = -T_l - f\omega$.

Practically, are f_i non linear functions and strongly affected by temperature, saturation skin effects and different nonlinearities induced by harmonic pollution due to converters of frequencies and noise measurements. The objectives is to determine a control law making possible to maintain flux orientation, tracking reference speed and flux even in the presence of parameter variations and measurement noises [20].

To do so we can write the following expression $f_i = \dot{f}_i + \Delta f_i$, where $\hat{f}_i$ is the true non linear feedback function (NLFF), f_i is the effective NLFF and Δf_i is the NLFF variation around. f_i . The Δf_i can be generated from the variations of parameters as indicated above. We assume that all of the $|\Delta f_i| \prec \beta_i$, where the β_i are known bounds. Knowledge of the β_i is not difficult obtain, since one can use a sufficiently large number to satisfy the constraint $|\Delta f_i| \prec \beta_i$:

$$\begin{cases} \dfrac{\mathrm{d}\varphi_{rd}}{\mathrm{d}t} = \hat{f}_1 + \Delta f_1 - \dfrac{1}{T_r} i_{sd}, \\ J\dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \hat{f}_2 + \Delta f_2 + T_e. \end{cases} \tag{22}$$

Proposition: In the case of all flux state model, if the flux orientation constraints is satisfied the following control laws are used

$$\begin{gathered} i_{sd} = -\frac{T_r}{L_m}\Big(\hat{f}_1 - \dot{\varphi}_{rd} + K_1(\varphi_{rd} - \varphi_r) + K_{11}\,\mathrm{sgn}(\varphi_{rd} - \varphi_r)\Big), \\ T_e = J\dot{\omega}_{ref} - K_2(\omega - \omega_{ref}) - K_{22}\,\mathrm{sgn}(\omega - \omega_{ref}), \end{gathered} \tag{23}$$

where $K_{ii} \geq \beta_i$ and $K_{ii} \succ 0$ for $i = 1,...,3$.

Proof: Let the Lyapunov function related to the flux and speed dynamics defined by

$$V = \frac{1}{2}(\varphi_{rd} - \varphi_r) + \frac{1}{2}J(\omega - \omega_{ref}) > 0\ . \tag{24}$$

This function is globally positive-definite over the whole state space. Its derivative is given by

$$\dot{V} = (\varphi_{rd} - \varphi_r)(\dot{\varphi}_{rd} - \dot{\varphi}_r) + J(\omega - \omega_{ref})(\dot{\omega} - \dot{\omega}_{ref}) . \tag{25}$$

Substituting (22) in (25), it results:

$$\begin{gathered} \dot{V} = (\varphi_{rd} - \varphi_r^*)(\hat{f}_1 + \Delta f_1 + \frac{L_m}{T_r} i_{sd} - \dot{\varphi}_r) + \\ + J(\omega - \omega_{ref})(T_e + \hat{f}_2 + \Delta f_2 - J\dot{\omega}_{ref}). \end{gathered} \tag{26}$$

Let us replace the control law (24) in (26) we obtain

$$\begin{gathered} \dot{V} = (\varphi_{rd} - \varphi_r^*)\left[(\Delta f_1 + \dot{\varphi}_{rd} K_{11} \operatorname{sgn}(\varphi_{rd} - \dot{\varphi}_r^*) \right] + \\ +(\omega - \omega_{ref})\left[(\Delta f_2 - K_{22} \operatorname{sgn}(\omega - \omega_{ref}) \right] + \dot{V}_1, \end{gathered} \tag{27}$$

$$\dot{V}_1 = -K_1 \left(\varphi_{rd} - \varphi_r \right) - K_2 (\omega - \omega_{ref})^2 < 0 , \tag{28}$$

the term

$$\left[(\varphi_{rd} - \varphi_r^*)(\Delta f_1 - K_{11} \operatorname{sgn}(\varphi_{rd} - \varphi_r^*) \right] + (\omega - \omega_{ref})\left[\Delta f_2 - K_{22} \operatorname{sgn}(\omega - \omega_{ref}) \right],$$

$\forall(\varphi_{rd} - \varphi_r^*)$, $\forall(\omega - \omega_{ref})$ and $\forall T_l$ then $\dot{V} < \dot{V}_1 < 0$.

All variation Δf_i can be absorbed by $K_{ii} \succ \Delta f_i$. The equation (29) is satisfied since $K_i \succ 0$ and $\left|\Delta f_i\right| \prec \beta_i \prec K_{ii}$.

The function given in (28) is globally negative-definite. Hence, using Lyapunov's theorem we conclude that:

$$\lim_{t \to \infty} (\varphi_{rd} - \varphi_r^*) = 0, \qquad \lim_{n \to \infty} (\omega - \omega_{ref}) = 0. \tag{29}$$

5 Flux Observer Design

The flux observer has been performed using standard problem structure where the controller is in fact the observer and the same optimization mechanism is used to achieve the synthesis [13, 19]. The inputs and outputs are as it is indicate by Fig. 2 and robustness is improved by tacking into account rotor resistance and speed variations. The design consists of finding $u = G_{obs} y$ to minimize, closed- loop H_∞ LPV norm from w to z according to the small gain theorem. The flux observer can be built up using polytopic representation of induction motor with mixed sensitivity structure and it is computed under LMI convex optimization using the LMI tool box. Described by $2^2 = 4$LTI corner the observer have 6 state.

In Fig. 2 $\mathbf{w}=\begin{bmatrix} V_{s\alpha} & V_{s\beta} & \eta_m \end{bmatrix}$ constitute the exogenous inputs, $\mathbf{z}=\begin{bmatrix} e_\alpha & e_\beta \end{bmatrix}$ the outputs, $\mathbf{y}=\begin{bmatrix} i_{s\alpha} & i_{s\beta} \end{bmatrix}$ the measurements and $\mathbf{u}^T=[\hat{\varphi}_{s\alpha}\ \hat{\varphi}_{s\beta}]$ the control input.

The tracking errors of rotor flux components are given as $e_\alpha=\varphi_{r\alpha}-\hat{\varphi}_{r\alpha}$ and $e_\beta=\varphi_{r\beta}-\hat{\varphi}_{r\beta}$. The robust quadratic stability and performance is achieved for $\gamma=0.0086$ using following shaping filter:

$$W=\operatorname{diag}\left(\frac{0.006}{s+7.3\times10^3},\frac{0.006}{s+7.3\times10^3}\right).$$

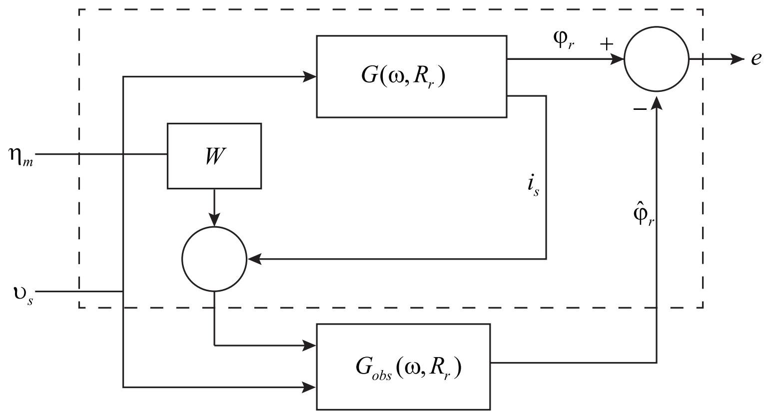

Fig. 2 – *Mixed sensitivity structure for flux observer.*

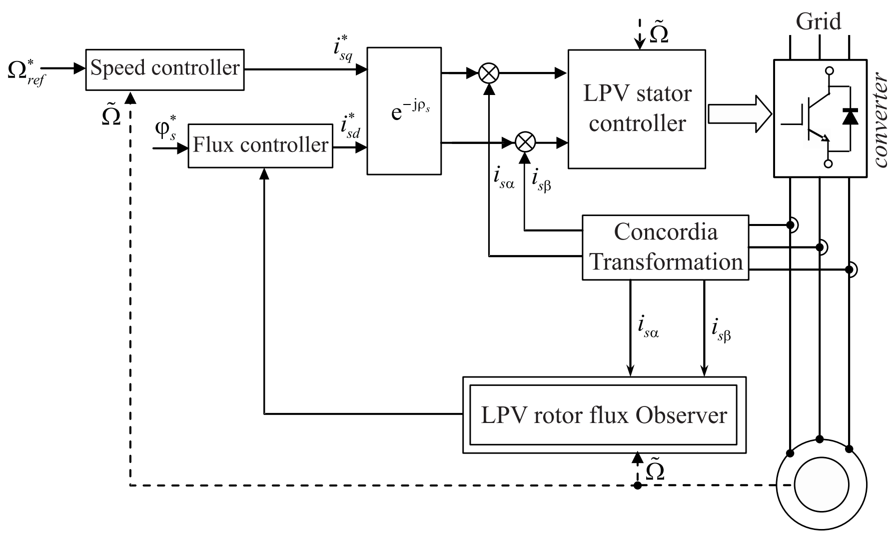

Fig. 3 – *General block diagram of the suggested* IM *control scheme.*

Fig. 3 gives a general description of the diagram block suggesting an induction motor control scheme. As it is shown we note that flux given by an LPV observer and speed are nonlinear feedback-controlled. The stator current components are transformed into (α,β) frame and then controlled by an LPV controller.

6 Simulation Results

The performances of controllers are investigated by simulation on induction motor which parameters values are given in appendix. Full non-linear simulations were carried out for the speed, flux step demand and for parameter variation see Figs. 4 and 5.

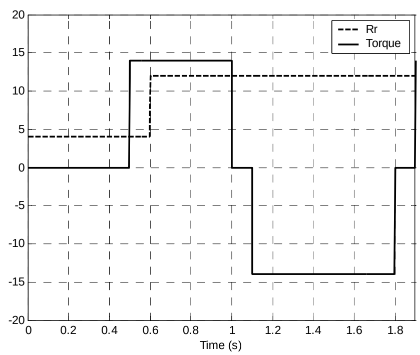

Fig. 4 – *Rotor resistance and load torque*

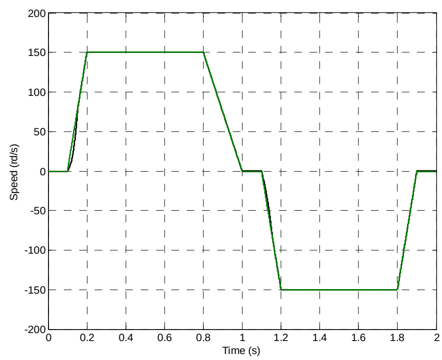

Fig. 5 – *Speed tracking.*

Fig. 6 represents response of the rotor speed following the specified reference. At 1.1 s a reversal speed test from157 rad/s to −157 rad/s was performed with loaded machine (14 Nm) at 0.5 s.

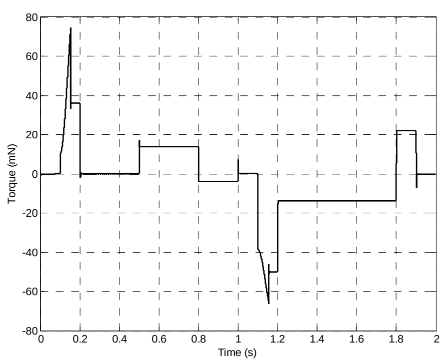

Fig. 6 – *Torque time variation.*

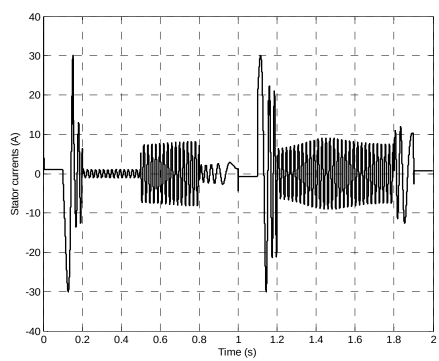

Fig. 7 – *Current trackin g response.*

Fig. 7 and Fig. 8 show good current reference tracking without any effect of parameter variation. We can note furthermore that the current peak stays within the admissible limits. However the tracking of the flux given in Fig. 9 depends entirely on the flux estimation accuracy.

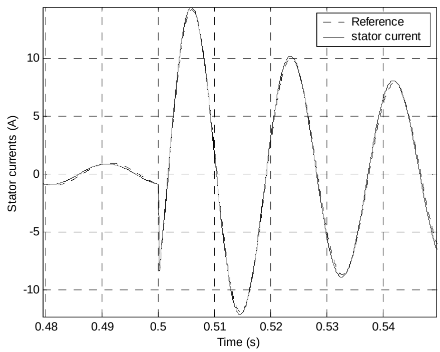

Fig. 8 – *Current error zoom.*

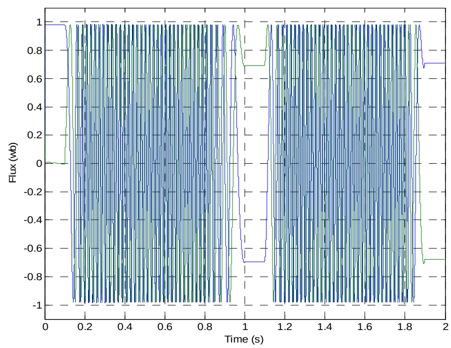

Fig. 9 – *Flux tracking reference.*

7 Conclusion

The main objectives of this paper are to show the high performances provieded by the robust non linear controller over the entire operating range. An LMI based approach has been proposed to design a quadratically stable flux observer and an output Lyapunov feedback regulator to track the stator current. In both cases we have obtained a scheduled time varying system (LPV) which ensures a finite L_2 attenuation for a given closed-loop transfer function which represent the design requirement. It is clearly turned out that with the use of the LPV techniques associated with Lyapunov feedback controller the robustness and stability of the whole drive was demonstrate. The main advantage of using LPV methods is that they provide a systematic way of designing an H_∞ flux observer for the induction motor assuming that the rotor speed and resistance are available. Stability of the flux estimator was demonstrated using small-gain based analysis. The simulation results demonstrate clearly high performances of the induction motor control according to the profile defined above.

8 Appendix

The machine parameters are as follows:

– Resistance of the rotor: $R_r = 4\,\Omega$,

– Resistance of the stator: $R_S = 8\,\Omega$,

– Inductance of the rotor: $L_r = 0.47\,\text{H}$,

– Inductance of the rotor: $L_S = 0.47\,\mathrm{H}$,
– Mutual inductance: $M = 0.44\,\mathrm{H}$,
– Inertia: $J = 0.04\,\mathrm{kgm}^2$,
– Number of poles pairs: $p = 2$.

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