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# Relative Measurement Error Analysis in the Process of the Nakagami-*m* Fading Parameter Estimation

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**Abstract:** An approach to the relative measurement error analysis in the process of the Nakagami-m fading signal moments estimation will be presented in this paper. Relative error expressions will be also derived for the cases when MRC (Maximal Ratio Combining) diversity technique is performed at the receiver. Capitalizing on them, results will be graphically presented and discussed to show the influence of various parameters, such as diversity order and fading severity on the relative measurement error bounds.

**Keywords:** Relative measurement error analysis, Nakagami-*m* fading parameter estimation.

## 1 Intorduction

Many emerging applications of wireless communications require transmitters positioning in deep urban and moderate indoor environments. The problem is that in these environments the transmitted signals do not always reach the receiver in a direct line-of-sight (LoS) transmission. There are usually multiple obstacles surrounding the transmission path, causing reflection, diffraction and scattering. The individual signals superimpose at the receiver antenna. Since most of these obstacles are close to the receiver, their path difference is mostly below 150 m [1]. Measurement campaigns have confirmed that most multipath components have an excess delay of below 500 ns. Since this delay is less than a chip length, the line-of-sight and multipath components are highly correlated, leading to constructive or destructive interference, depending on the actual excess delay. The movement of the transmitters, receivers and some obstacles form a dynamically varying multipath environment where constructive and destructive interference alternate over time. This characteristic constitutes a multipath fading channel. The resulting fading process degrades the receiver performance. While receivers are

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becoming increasingly sensitive in order to detect strongly attenuated signals, the detection threshold has to be lowered with the reduced signal power [2].

The long-term signal variation is described by lognormal distribution, whereas the short-term signal variation is described by several other distributions such as Hoyt, Rayleigh, Rice, Nakagami-*m*, and Weibull. Nakagami-*m* fading describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves. It provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications [3].

Diversity reception, based on using multiple antennas at the receiver (space diversity), is very efficient methods used for improving system's quality of service (QoS), so it provides efficient solution for reduction of signal level fluctuations in fading channels. Multiple received copies of signal could be combined on various ways [4, 5]. The optimal combining technique is maximum ratio combining (MRC). This combining technique involves co-phasing of the useful signal in all branches, multiplication of the received signal in each branch by a weight factor that is proportional to the estimated ratio of the envelope and the power of that particular signal and the summing of the received signals from all antennas [6].

Simulation, Nakagami-*m* parameter estimation and measurement has been present in the literature [7, 8]. However, to the best of author's knowledge, no expression study of relative error measurement in the process of the Nakaga mi-*m* fading parameter estimation based on higher order moments, for the cases with and without diversity reception, has been present in the literature yet.

## 2 System Model

In measurement, the total differential is used in estimating the error $\Delta y$ of a function $y$ based on the errors $\Delta x_1, \Delta x_2, \ldots$ of the parameters $x_1, x_2, \ldots$. Assuming that the interval is short enough for the change to be approximately linear [9]:

$$\Delta y = y'(x)\Delta x\,, \tag{1}$$

and that all variables are independent, then for all variables,

$$\Delta y = \sum_{i=1}^{n} \frac{\mathrm{d} f(x_i)}{\mathrm{d} x_i}\Delta x_i, \qquad y = f(x_1, x_2, \ldots, x_n)\,. \tag{2}$$

This is because the derivative $f_x$ with respect to the particular parameter $x$ gives the sensitivity of the function $f$ to a change in $x$, in particular the error $\Delta x$. As they are assumed to be independent, the analysis describes the worst-case scenario. The absolute values of the component errors are used, because after simple computation, the derivative may have a negative sign. One commonly distinguishes between the relative error and the absolute error. The absolute error is the magnitude of the difference between the exact value and the approximation. The relative error is the absolute error divided by the magnitude of the exact value. From previous relation it can be easily seen, that relative error can be written in the form of [10]:

$$g_y = \frac{\Delta y}{y} = \mathrm{d}(\ln(f(x_1, x_2, \ldots, x_n))) . \tag{3}$$

The probability density function (PDF) for a Nakagami-*m* distributed channel can be expressed as [3]:

$$p(r) = \frac{2r^{2m-1}m^m}{\Gamma(m)\Omega^m}\exp\left(-\frac{mr^2}{\Omega}\right), \tag{4}$$

where the channel amplitude $R \geq 0$, $\Omega = E(R^2)$ is average fading signal power, $E()$ is the expectation operator, and $\Gamma()$ is gamma function. Above, $m$ is the Nakagami fading parameter which determines the severity of the fading, namely $m$ is the inverse of the normalized variance of $R^2$:

$$m = \frac{E(R^2)^2}{Var(R^2)}, \tag{5}$$

where $Var(R^2)$ is the variance of $R^2$. The value for *m* ranges between $1/2$ and $\infty$. When $m \to \infty$, the channel converges a static channel. As special cases, Nakagami-*m* includes Rayleigh distribution when $m = 1$, and one-sided Gaussian distribution for $m = 1/2$. This basically means that, if $m < 1$, the Nakagami-*m* distributed fading is more severe than Rayleigh fading, and for values of $m > 1$, the fading circumstances are less severe. For the values of $m > 1$, the Nakagami-*m* distribution closely approximates the Rician distribution, and the parameters $m$ and the Rician factor $K$ (which determines the severity of the Rician fading) can be mapped via parameter $m$.

From previous is evident that for a reasonable estimation and measurement of Nakagami-*m* fading parameters is necessary to determine its moments. For the $n$-th order moment of the Nakagami random variable $r$ we have:

$$\overline{r^n} = \frac{1}{\Gamma(m)}\left(\sqrt{\frac{\Omega}{m}}\right)^n \Gamma\left(\frac{2m+n}{2}\right). \tag{6}$$

Let us now derive the relative error that is obtained in the process of the $n$-th moment determination. Considering (2) we obtain:

$$\ln \overline{r^n} = \frac{n}{2}\ln\Omega - \frac{n}{2}\ln m - \ln\Gamma(m) + \ln\Gamma\left(\frac{2m+n}{2}\right). \tag{7}$$

After differencing (7):

$$\frac{\mathrm{d}\overline{r^n}}{\overline{r^n}} = \frac{\mathrm{d}m}{m}\left| m\Psi_0\left(\frac{2m+n}{2}\right) - m\Psi_0(m) - \frac{n}{2}\right| + \frac{\mathrm{d}\Omega}{\Omega}\left|\frac{n}{2}\right| +$$
$$+\frac{\mathrm{d}n}{n}\left|\frac{n\ln\Omega}{2} - \frac{n\ln m}{2} + \frac{n\ln\Psi_0\left(\frac{2m+n}{2}\right)}{2}\right| = \tag{8}$$
$$= \frac{\mathrm{d}m}{m}\delta_{1m} + \frac{\mathrm{d}\Omega}{\Omega}\delta_{1\Omega} + \frac{\mathrm{d}n}{n}\delta_{1n}$$

relative error expression can be given in the form:

$$\frac{\Delta\overline{r^n}}{\overline{r^n}} = \frac{\Delta m}{m}\delta_{1m} + \frac{\Delta\Omega}{\Omega}\delta_{1\Omega} + \frac{\Delta n}{n}\delta_{1n}, \tag{9}$$

with $\Psi_0(x)$ denoting the digamma function [11].

Let us now consider diversity reception influence on the relative error in the process of the $n$-th moment determination. The resulting pdf of the instantaneous envelope $r$, at the output of the MRC combiner is given in the literature in the form of

$$p_{MRC}(r) = \frac{2r^{2mL-1}m^{mL}}{\Gamma(mL)\Omega^{mL}}\exp\left(-\frac{mr^2}{\Omega}\right), \tag{10}$$

while the $n$-th order moment of the random variable $r$ at the output of the combiner is given in the form of:

$$\overline{r^n{}_{MRC}} = \frac{1}{\Gamma(mL)}\left(\sqrt{\frac{\Omega}{m}}\right)^n \Gamma\left(\frac{2mL+n}{2}\right). \tag{11}$$

Based on (2) we have:

$$\ln \overline{r^n{}_{MRC}} = \frac{n}{2}\ln\Omega - \frac{n}{2}\ln m - \ln\Gamma(mL) + \ln\Gamma\left(\frac{2mL+n}{2}\right), \tag{12}$$

after differencing (12):

$$\frac{\mathrm{d}\overline{r^n}_{MRC}}{\overline{\overline{r^n}}_{MRC}} = \frac{\mathrm{d}m}{m}\left| mL\Psi_0\left(\frac{2mL+n}{2}\right) - mL\Psi_0\left(mL\right) - \frac{n}{2} \right| + \frac{\mathrm{d}\Omega}{\Omega}\left|\frac{n}{2}\right|$$
$$+\frac{\mathrm{d}n}{n}\left| \frac{n\ln\Omega}{2} - \frac{n\ln m}{2} + \frac{n\ln\Psi_0\left(\frac{2mL+n}{2}\right)}{2} \right| + \tag{13}$$
$$+\frac{\mathrm{d}L}{L}\left| \Psi_0\left(\frac{2mL+n}{2}\right)mL - mL\Psi_0\left(mL\right) \right| =$$
$$= \frac{\mathrm{d}m}{m}\delta_{1mMRC} + \frac{\mathrm{d}\Omega}{\Omega}\delta_{1\Omega MRC} + \frac{\mathrm{d}n}{n}\delta_{1nMRC} + \frac{\mathrm{d}L}{L}\delta_{1LMRC}$$

relative error expression can be given in the form:

$$\frac{\Delta\overline{r^n}_{MRC}}{\overline{\overline{r^n}}_{MRC}} = \frac{\Delta m}{m}\delta_{1mMRC} + \frac{\Delta\Omega}{\Omega}\delta_{1\Omega MRC} + \frac{\Delta n}{n}\delta_{1nMRC} + \frac{\Delta L}{L}\delta_{1LMRC}. \tag{14}$$

## 3 Relative Error Comparation

Let us now compare the expressions for the relative error in the process of the $n$-th moment determination from (9) and (14), in order to show how the diversity effect improves system performances. Since we know for sure what is the diversity order at the reception (number of antennas at the receiver, $\Delta L = 0$), and the moment order which is estimating, relative error expressions can be writen in the form of:

$$\frac{\Delta n}{n}\delta_{1n} \to 0; \quad \frac{\Delta n}{n}\delta_{1nMRC} \to 0;$$
$$\frac{\Delta L}{L}\delta_{1LMRC} \to 0;$$
$$\frac{\Delta\overline{r^n}}{\overline{\overline{r^n}}} \approx \frac{\Delta m}{m}\delta_{1m} + \frac{\Delta\Omega}{\Omega}\delta_{1\Omega}; \tag{15}$$
$$\frac{\Delta\overline{r^n}_{MRC}}{\overline{\overline{r^n}}_{MRC}} \approx \frac{\Delta m}{m}\delta_{1mMRC} + \frac{\Delta\Omega}{\Omega}\delta_{1\Omega MRC}.$$

Let us now observe ratio $\delta_{1m}/\delta_{1m\mathrm{MRC}}$. Knowing the basic properties of digamma function it is evident that for every values of parameters $m$ and $L$ and for each moment this ratio is higher than 1, which means $\delta_{1m} > \delta_{1m\mathrm{MRC}}$.

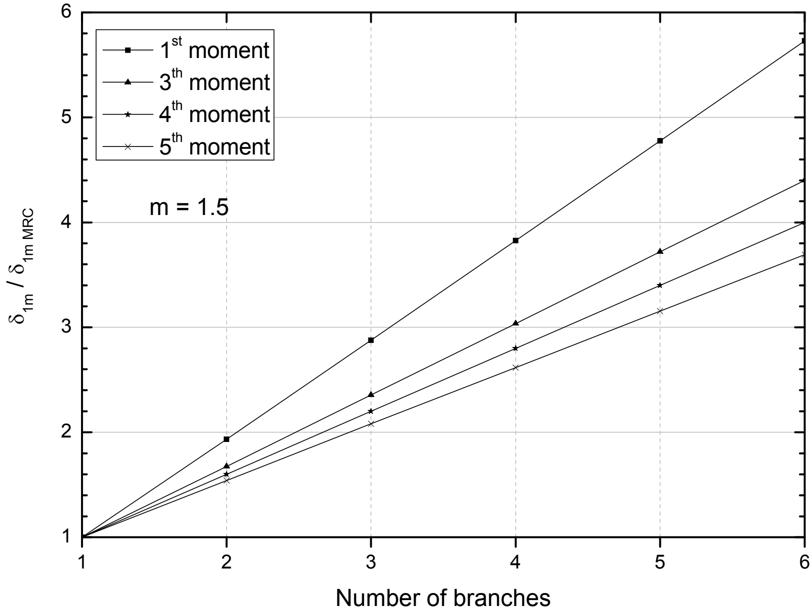


**Fig. 1** – $\delta_{1m}/\delta_{1m\text{MRC}}$ *ratio in the function of number of diversity reception branches* $L$ *, for various moment orders.*

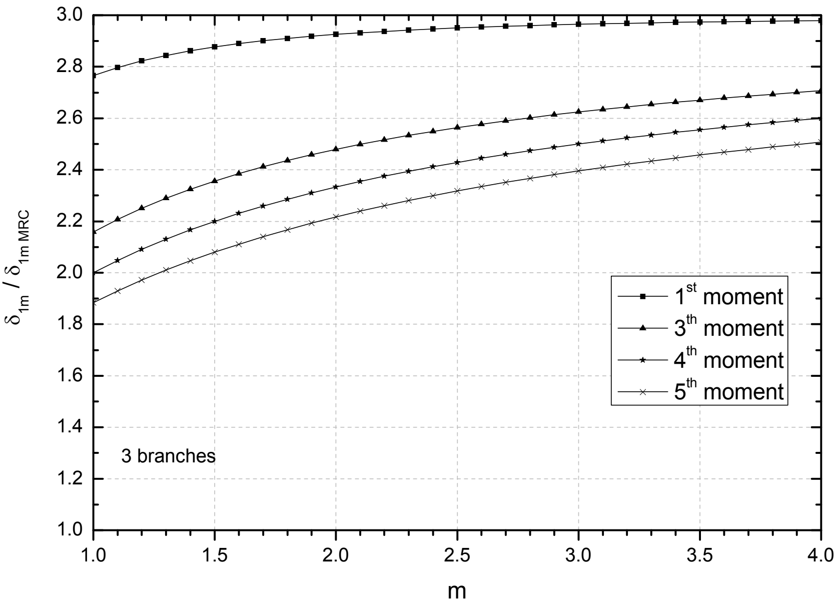


**Fig. 2** – $\delta_{1m}/\delta_{1m\text{MRC}}$ *ratio in the function of fading severity parameter* $m$ *, for various moment orders.*

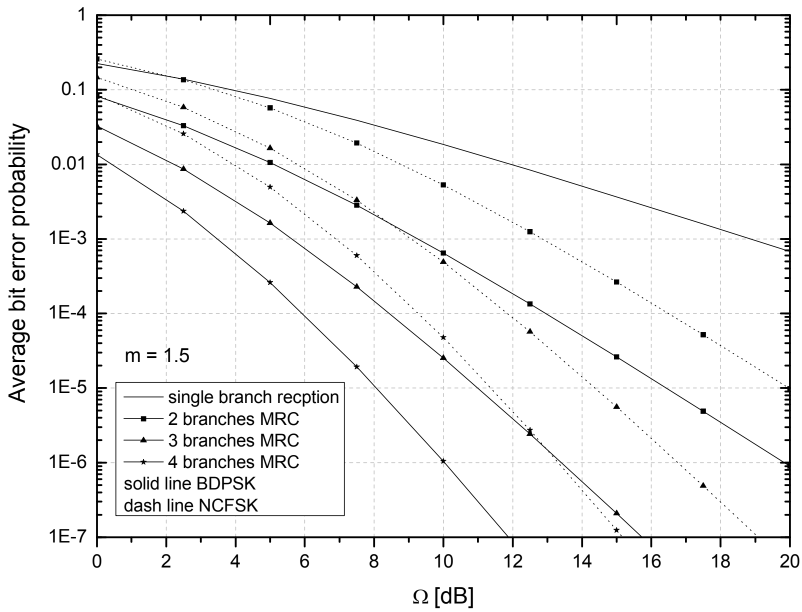


**Fig. 3** – *Average bit error probability in the function of average fading signal power* $\Omega$.

In Figs. 1 and 2 this ratio is presented in the function of fading severity parameter $m$ and number of branches $L$, for various order of moments. It can be seen, that this ratio grows as number of diversity branches at the reception grows, which means that relative error in the process of the moment determination decreases (better performances are obtained) as diversity order at the reception increases. Also is evident that smaller relative error is obtained when higher order moments are estimated in the presence of less severe fading (higher $m$ values).

Among the different performance criteria that can be used to characterize the random variable $r$ there is the average bit error probability (ABER). The average bit error probability (ABER) at the receiver output can be derived for noncoherent and binary signalling according to following expressions:

$$P_e = \int_0^\infty p_\xi(t) \frac{1}{2} e^{-gt} \, \mathrm{d}t \,, \tag{16}$$

where $g$ denotes modulation constant, i.e., $g = 1$ for BDPSK modulation scheme and $g = 1/2$ for NCFSK. Substituting (4) and (10) in (16) ABEP is shown on Fig. 3 for the case without diversity and cases when diversity reception is applied. A comparison of curves shows better performance of BDPSK modulation scheme against NCFSK modulation scheme. Also is more evident how ABER increases at both figures when the number of multiple correlated co-channel interferers increase due to growth of interference domination. From Fig. 3 can be seen how for the same average fading signal power average bit error probability behavior improves as the diversity order (number of branches) increases. For example, considering the case of four diversity branches, we can see that at the ABEP = $10^{-2}$ improvement is about 5 dB of $\Omega$.

Considering previously exposed analysis, the conclusion arises that lower relative error in the process of the Nakagami-$m$ moment determination is achieved when MRC diversity is present at the reception. Better performances are obtained for higher number of diversity branches, namely:

$$\left.\begin{array}{c} \delta_{1m} \geq \delta_{1mMRC}, \\ \dfrac{\Delta\Omega}{\Omega}\delta_{1\Omega} \geq \dfrac{\Delta\Omega}{\Omega}\delta_{1\Omega MRC} \end{array}\right\} \Rightarrow \frac{\Delta\overline{r^n}}{\overline{\overline{r^n}}} \geq \frac{\Delta\overline{r^n{}_{MRC}}}{\overline{\overline{r^n{}_{MRC}}}}. \tag{17}$$

Also better estimation is obtained for the cases of higher order moments and more severe fading.

## 4 Conclusion

In this paper an approach to the relative error analysis in the process of the $n$-th moment estimation is presented. Relative error expressions are obtained for the cases with and without MRC diversity combining at the receiver. Obtained results are graphically presented and discussed in order to show the influence of various parameters, such as diversity order and fading severity on the relative measurement error bounds. It is shown that better fading parameter estimation is achieved in the cases of higher diversity level, higher order moments and more severe fading.

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