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Capacity Evaluation for Maximal Ratio Combining over κ-μ Fading Channels

Mihajlo Stefanović[1], Stefan Panić[2], Jelena Anastasov[1], Aleksandra Cvetković[1], Zoran Popović[3]

Abstract: Analysis of channel capacity per unit bandwidth of L - branch maximal ratio combining (MRC) receiver operating over κ-μ fading channels for two adaptive transmission schemes is presented in this paper. Using the proposed system model, the optimal power and rate adaptation and constant transmit power policies are analyzed. New expressions for capacity evaluation are derived in terms of finite sums. The effects of diversity order and fading parameters on the channel capacity for given techniques have been considered and numericaly presented.

Keywords: κ-μ fading channels, Channel Capacity, Maximal Ratio Combining, Optimal power and rate adaptation policy, Constant transmit power policy.

1 Introduction

Since the demand for wireless communication services have been growing at a rapid pace in recent years, conserving, sharing and using bandwidth efficiently is of primary concern in future of wireless communication systems [1]. Therefore, channel capacity is one of the most important concerns in the design of wireless systems, as it determines the maximum attainable throughput of the system. It can be defined as the average transmitted data rate per unit bandwidth, for a specified average transmit power and specified level of received outage or bit-error rate (BER). Skilful combination of bandwidth efficient coding and modulation schemes can be used to achieve higher channel capacities per unit bandwidth.

However, mobile radio links are, due to the combination of randomly delayed, reflected, scattered, and diffracted signal components, subjected to severe multipath fading, which leads to serious degradation in the link signal-to-noise ratio (SNR). The multipath fading in wireless communications, is

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[1]Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia; E-mail: mihajlo.stefanovic@elfak.ni.ac.rs; anastasovjelena@gmail.com; caleksandra@gmail.com
[2]Faculty of Natural Science and Mathematics, University of Priština, Lole Ribara 29, 38000 Kosovska Mitrovica, Serbia; E-mail: stefanpnc@yahoo.com
[3]Technical Faculty in Čačak, Svetog Save 65, 32000 Čačak, Serbia; E-mail: pop@tfc.kg.ac.rs

modeled by several distributions including Weibull, Nakagami-*m,* Hoyt, Rayleigh, and Rice [2]. Considering important phenomena inherent to radio propagation, κ-μ fading model was recently proposed in [3] as a fading model which describes the short-term signal variation in the presence of line-of-sight (LOS) components. This distribution is more realistic than other special distributions, since its derivation is completely based on a non-homogeneous scattering environment. Also κ-μ model as general physical fading model includes Rayleigh, Ricean, and Nakagami-*m* fading models as special cases [3]. As its name implies, it is written in terms of two physical parameters, namely κ and μ. The parameter κ is related to the multipath clustering, whereas the parameter μ is the ratio between the total power of the dominant components and the total power of the scattered waves. For the case of $\kappa = 0$, the κ-μ distribution is equivalent to the Nakagami-*m* distribution. When $\mu = 1$, the κ-μ distribution becomes the Rician distribution with κ as the Ricean factor. Moreover, κ-μ distribution fully describes the characteristics of the fading signal in terms of measurable physical parameters.

Diversity combining is a powerful technique that can be used to combat fading in wireless systems resulting in improving link performance. The optimal diversity combining technique is maximum ratio combining (MRC) [2]. This combining technique involves co-phasing of the useful signal in all branches. It multiplicates the received signal in each branch by a weight factor that is proportional to the estimated ratio of the envelope and the power of that particular signal and then sums the received signals from all antennas. By co-phasing, all random phase fluctuations of the signal that emerged during the transmission are eliminated. For this process it is necessary to estimate the phase of the received signal, so this technique requires the entire amount of the channel state information of the received signal, and separate receiver chain for each branch of the diversity system, which increases the complexity of the system.

Another effective scheme that can be used to overcome fading influence is adaptive transmission. Adaptation of certain parameters during signal transmission can help better utilization of the channel capacity. Real-time balancing of the link budget through adaptive variation of the transmitted power level, symbol transmission rate, constellation size, coding rate, or any combination of these parameters is basic principle of adaptive transmission. These transmissions provide a much higher channel capacities per unit bandwidth by taking advantage of favorable propagation conditions: transmitting at high speeds under favorable channel conditions and responding to channel degradation through a smooth reduction of their data throughput. A reliable feedback path between that estimator and the transmitter and accurate channel estimation at the receiver is required for achieving good performances of adaptive transmission. Widely accepted adaptation policies include optimal power and rate adaptation (OPRA), constant power with optimal rate adaptation (ORA), channel inversion with fixed rate (CIFR), and truncated channel inversion with fixed rate policy (TIFR) [4]. The performance of adaptation schemes is further improved by combining them with space diversity.

Capacity of Rayleigh fading channels using four adaptation policies and different diversity schemes is analyzed in [1]. Usage of maximal ratio diversity combining (MRC) in capacity analysis of fading channels is also obtained in [5]. There are many published analysis for capacity of Rayleigh, Nakagami-*m*, Ricean fading channels under different adaptive transmissions and diversity techniques [6-7]. In [8] channel capacity of maximal ratio combining over exponentially correlated Nakagami-*m* fading channels for optimal power and rate adaptation policy is analyzed. Nevertheless, to the best of the authors' knowledge, a thorough analytical study of the κ-μ fading channel capacity, e.g., under the OPRA and ORA adaptation policies, has not been previously considered in the open technical literature.

Capacities per unit bandwidth of κ-μ fading channels under the optimal power and rate adaptation and constant transmit power adaptation policies are observed in this paper. The MRC is picked to counteract the negative effects of multipath fading. The derived expressions for proposed system model are quite general. Based on analytical analysis, some numerical results are provided pointing out the effects of the various system's parameters on the capacity per unit bandwidth.

2 κ-μ Fading Channels: Maximal Ratio Diversity Reception

It is shown in [9], that the sum of κ-μ squares is κ-μ square as well (but with different parameters), which is an ideal choice for MRC analysis. The expression for the probability density function (pdf) at the output of MRC micro-diversity system follows [9]

$$\begin{aligned} p_\gamma^{MRC}(\gamma) = \frac{L\mu}{k^{(L\mu-1)/2}\,\mathrm{e}^{L\mu k}}\left(\frac{1+k}{L\Omega}\right)^{(L\mu+1)/2} \cdot \\ \cdot \gamma^{(L\mu-1)/2}\,\mathrm{e}^{-\mu(1+k)\gamma/\Omega}\, I_{L\mu-1}\left(2\mu L\sqrt{\frac{(1+k)k\gamma}{L\Omega}}\right). \end{aligned} \tag{1}$$

In the pervious equation, In(x) denotes the n-th order modified Bessel function of first kind [10, eq.8.445], κ and μ are well-known κ-μ fading parameters, while L denotes the number of diversity branches at MRC combiner.

3 Adaptation Policies with MRC Diversity Reception

During our analysis it is assumed that the variation in the combined output SNR over κ-μ fading channels γ is tracked perfectly by the receiver and that variation of γ is sent back to the transmitter via an error-fee feedback path. Comparing to the rate of channel variation, the time delay in this feedback is negligible. These assumptions allow the transmitter to adopt its power and rate to the actual channel state. Channel capacity of the fading channel with received SNR distribution, $p_\gamma(\gamma)$, and optimal power and rate adaptation is given by [11]:

$$<C>_{pra} = B\int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma)\,\mathrm{d}\gamma\,, \tag{2}$$

where B (Hz) denotes the channel bandwidth and γ_0 is the cut-off level SNR bellow which transmission of data is suspended. This cut-off level must satisfy:

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right) p_\gamma(\gamma)\,\mathrm{d}\gamma = 1\,. \tag{3}$$

To achieve the capacity in (2), the transmitter has to adapt its power and rate to the actual channel state; when γ is large, high power levels and rates are allocated for good channel conditions and lower power levels and rates for unfavourable channel conditions when γ is small. Substituting (1) into (3), we found that the cut-off level must satisfy:

$$\begin{aligned}&\sum_{i=0}^{\infty} \frac{(kL\mu)^i}{\mathrm{e}^{L\mu k}\,\Gamma(i+L\mu)\,i!}\left(\frac{1}{\gamma_0}\Lambda^{(c)}\left(L\mu+i, \frac{\mu(1+k)\gamma_0}{\Omega}\right)-\right.\\ &\left.-\frac{\mu(1+k)}{\Omega}\Lambda^{(c)}\left(L\mu+i-1, \frac{\mu(1+k)\gamma_0}{\Omega}\right)\right)-1=0,\end{aligned} \tag{4}$$

with $\Lambda^{(c)}(\alpha, x)$ denoting higher incomplete Gamma function [10, (8.350-2[7])]. Substituting (1) in (2), considering (4), we obtain the capacity per unit bandwidth, $\langle C\rangle_{pra}/B$, as:

$$\frac{\langle C\rangle_{pra}}{B} = \sum_{i=0}^{\infty} \frac{L\mu}{k^{(L\mu-1)/2}\,\mathrm{e}^{L\mu k}}\left(\frac{1+k}{L\Omega}\right)^{(L\mu+1)/2} \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right)\gamma^{L\mu+i-1}\,\mathrm{e}^{-\mu(1+k)\gamma/\Omega}\,\mathrm{d}\gamma\,. \tag{5}$$

Now, by making change of variables, $t=\frac{\mu(1+k)\gamma}{\Omega}$ and $\mathrm{d}t=\frac{\mu(1+k)}{\Omega}\mathrm{d}\gamma$, $\langle C\rangle_{pra}/B$ is obtained as:

$$\frac{\langle C\rangle_{pra}}{B}=\sum_{i=0}^{\infty}\frac{(L\mu k)^{i}}{\Gamma(i+L\mu)\,i!\,\mathrm{e}^{L\mu k}}\left(\int_{0}^{\infty}\log_2\left(\frac{t\Omega}{\mu(1+k)\gamma_0}\right)t^{L\mu+i-1}\,\mathrm{e}^{-t}\,\mathrm{d}t-\right.$$
$$\left.-\int_{0}^{\gamma_0\mu(1+k)/\Omega}\log_2\left(\frac{t\Omega}{\mu(1+k)\gamma_0}\right)t^{L\mu+i-1}\,\mathrm{e}^{-t}\,\mathrm{d}t\right)= \tag{6}$$
$$=\sum_{i=0}^{\infty}\frac{(L\mu k)^{i}}{\Gamma(i+L\mu)\,i!\,\mathrm{e}^{L\mu k}}(I_1-I_2).$$

By denoting $f_1(t)=\log_2\left(\frac{t\Omega}{\mu(1+k)\gamma_0}\right)t^{L\mu+i-1}$, integral I_1 can be solved by applying Gauss-Laguerre quadrature formulae:

$$I_1=\int_{0}^{\infty}f_1(t)e^{-t}\,\mathrm{d}t\cong\sum_{k=1}^{R}A_k f_1(t_k). \tag{7}$$

In the pervious equation A_k and t_k, $k=1,2,\ldots,R$, are respectively weights and nodes of Laguerre polynomials [12, p. 875-924].

Similarly, integral I_2 can be solved by applying Gauss-Legendre quadrature formulae:

$$I_2=\left(\frac{\gamma_0\mu(1+k)}{2\Omega}\right)^{L\mu+i}\int_{-1}^{1}f_2(u)\,\mathrm{d}u\cong\left(\frac{\gamma_0\mu(1+k)}{2\Omega}\right)^{L\mu+i}\sum_{k=1}^{R}B_k f_2(u_k), \tag{8}$$

where B_k and u_k, $k=1,2,\ldots,R$, are respectively weights and nodes of Legendre polynomials. Since no data is sent when $\gamma<\gamma_0$, the optimal policy suffers a probability of outage P_{out} equal to the probability of no transmission, given by:

$$P_{out}=\int_{0}^{\gamma_0}p_\gamma(\gamma)\,\mathrm{d}\gamma=1-\int_{\gamma_0}^{\infty}p_\gamma(\gamma)\,\mathrm{d}\gamma. \tag{9}$$

After simple mathematical manipulations (9) can be written in the form of

$$P_{out}=\sum_{i=0}^{\infty}\frac{(kL\mu)^{i}}{\mathrm{e}^{L\mu k}\,\Gamma(i+L\mu)\,i!}\Lambda\left(L\mu+i,\frac{\mu(1+k)\gamma_0}{\Omega}\right). \tag{10}$$

When the transmitting power of the system is constant and optimal rate adaptation (ORA) technique is used at the transmitter, the channel capacity can be given as:

$$\langle C\rangle_{ora} = B\int_0^\infty \log_2(1+\gamma)\, p_\gamma(\gamma)\,\mathrm{d}\gamma\,. \tag{11}$$

Substituting (1) into (11) and making change of variables as in (6), the expression (11) becomes:

$$\frac{\langle C\rangle_{ora}}{B} = \sum_{i=0}^{\infty} \frac{(L\mu k)^i}{\Gamma(i+L\mu)\, i!\, \mathrm{e}^{L\mu k}} \int_0^\infty \log_2\left(1+\frac{t\Omega}{\mu(1+k)}\right) t^{L\mu+i-1} e^{-t}\,\mathrm{d}t\,. \tag{12}$$

Further, the above expression can be presented as

$$\frac{\langle C\rangle_{ora}}{B} = \sum_{i=0}^{\infty} \frac{(L\mu k)^i}{\Gamma(i+L\mu)\, i!\, \mathrm{e}^{L\mu k}} \int_0^\infty f_3(t)\, e^{-t}\,\mathrm{d}t \cong \sum_{i=0}^{\infty} \frac{(L\mu k)^i}{\Gamma(i+L\mu)\, i!\, \mathrm{e}^{L\mu k}} \sum_{k=1}^{R} A_k f_3(t_k)\,, \tag{13}$$

where A_k and t_k, $k = 1,2,\ldots,R$, are respectively weights and nodes of Laguerre polynomials [12, p. 875-924].

4 Numerical Results

Fig. 1 shows capacity per unit bandwidth of κ-μ fading channels, for power and rate adaptation policy, for various values of fading correlation and severity. Here we can see that as the diversity order increases, capacity per unit bandwidth curves improve for this policy.

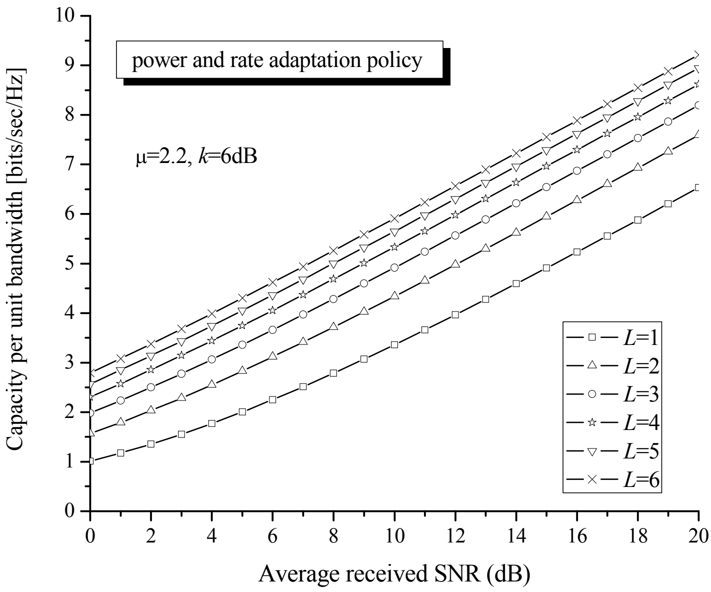

Fig. 1 – *Power and rate adaptation policy capacity per unit bandwidth over* κ-μ *fading channels, for various values of diversity over.*

Fig. 2 shows the calculated channel capacity per unit bandwidth as a function of average received SNR for two different adaptation policies. From this figure we see that the optimal power and rate adaptation yields a noticable increase in capacity over constant transmit power adaptation and this increase also exist when fading channel condition is changed ($\mu = 0.5$, $\mu = 2.5$). We can conclude that, the constant transmit power policy suffers larger capacity penalty relative to optimal power and rate adaptation policy.

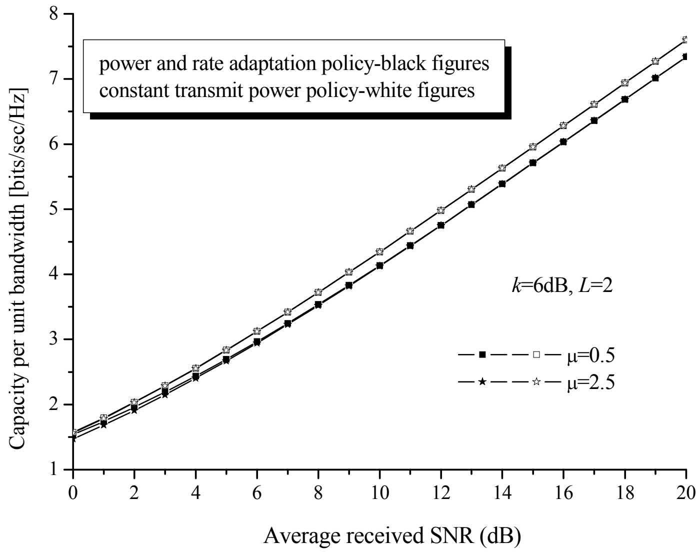

Fig. 2 – *Capacity per unit bandwidth for* κ-μ *fading channels with MRC diversity under different adaptation policies.*

5 Conclusion

The channel capacity per unit bandwidth for two different adaptation policies over *κ-μ* fading channels with MRC diversity is discussed in this paper. Finite sums-form expressions were derived for the optimal power and rate adaptation and constant transmit power policies for MRC diversity reception case. Capitalizing on them, analyticaly obtained results are graphicaly presented in order to show the effects of various system's parameters. The power and rate adaptation policy provides higher capacity as compared to the other policy.

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