

# Numerical Analysis of Relative Phase and Amplitude at the Interaction Two Solitons in Optical Fibers

Branimir Jakšić<sup>1</sup>, Mihajlo Stefanović<sup>2</sup>, Petar Spalević<sup>1</sup>,  
Ana Savić<sup>3</sup>, Radoslav Bogdanović<sup>4</sup>

**Abstract:** In this paper presented the analysis propagation solitons pair in optical fiber from the standpoint of relative amplitude and relative phase. Consider the influence of changes relative phase and amplitude in the interaction of two solitons in optical fibers. Shows the simulation (in the space-time domain) of movement solitons pairs in optical fiber with the change of these two parameters.

**Keywords:** Solitons, Relative phase, Relative amplitude.

## 1 Introduction

When passing through an optical fiber, the various components of the signal is spread at different rates. This effect affect the expansion and distortion of signals. In addition, there are various nonlinear effects that also contribute to the impulses distortion during propagation (damping impulses). This distortion is not a major problem at small distances, but the system for the transmission of signals at large distances, these effects are very detrimental for the correct transmission. In order to avoid, these effects can be to balance with appropriate form of impulses, solitons.

Soliton is the impulse that is characterized by the fact that his form does not change during transmission through a specific environment. Environment in which can survive soliton must be such that the nonlinearity and dispersion of the environment compensate each other [1, 2]. Single-mode fiber with negative group velocity dispersion is one such environment, provided that in the fiber extends momentum sufficient intensity to characteristics nonlinear fibers come to expression. From theory of propagation of optical impulses through the fiber is well known to affect his form of group velocity dispersion resulting from the material dispersion and waveguide dispersion. Negative group velocity dispersion

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<sup>1</sup>Faculty of Technical Science, Kneza Miloša 7, 38220 Kosovska Mitrovica, Serbia;  
E-mail: mihajlo.stefanovic@elfak.ni.ac.rs

<sup>2</sup>Faculty of Electronics, Aleksandra Medvedeva 14, 18000 Niš, Serbia

<sup>3</sup>College of Electrical Engineering, Vojvode Stepe 283, 1000 Belgrade, Serbia

<sup>4</sup>The Serbian Railways, 18000 Niš, Serbia

means that the higher frequency components in the spectrum of impulses transmissions faster than lower frequency components [3-5]. Nonlinear characteristics of the fibers will appear at high field intensities. Highest density power in the downtown core and decline to the shell. Where the largest field intensity, the refractive index is greatest. This behavior is known as Kerr effect. The result of this phenomenon is the dependence of the speed of propagation of the signal level. This leads to a current change phase depending on the signal level that is manifested with self-phase modulation impulses. When balanced the effects of negative group velocity dispersion and self-phase modulation due to Kerr effect the impulse given which does not change shape during propagation [6].

It is proven that self-phase modulation, as a nonlinear optical effect, leading to the occurrence frequency shift, called chirp. It will cause a raise or lower edge of the optical pulse. However, it is possible that this nonlinear effect used to reduce the dispersion in optical fibers. In this case, the tendency of the spread of pulses in time to confront with effect of compression, which is generated due to nonlinear interactions.

Balance of nonlinearity and dispersion results in:

- nonlinearity causes the breaking waves, and
- dispersion causes spreading of the wave package.

There are several types of mechanisms for generating optical solitons. The first successful experiment was a model of two-cavities laser published 1984th year. During 1987. year, Berg and Christiansen proposed the first theoretical model for the soliton laser, they use Maxwell-Bloch equations to describe the main cavity, the nonlinear Schrödinger equation to describe the cavity of fiber. We know that the effect of soliton propagation dependent on the group velocity dispersion and nonlinear effects, and is a major problem in the soliton laser to find a suitable method for the use of nonlinear materials. Spatial solitons can be transmitted by optical nonlinear medium which has the Kerr effect and then non-linear effects of medium can accurately compensate for linear diffraction effect. Also, we used magnetic-optical effects so that we can control the interaction between two different impulses, and photo-refractive effects that can hold a spatial soliton in the crystal.

Development of computer technology it is possible that the numerical methods to get several different types of equations that give soliton solutions [8]. In this paper we used soliton solutions of the Schrödinger equation.

## **2 Soliton Solution Schrödinger Equation**

If we neglect losses in the fiber, for solitons can be written Schrödinger equation in the form:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3} = j\gamma |A|^2 A, \quad (1)$$

where is  $A(z, t)$  complex envelope of field that describes the propagation of electric fields in  $z$  propagation direction at time  $t$  so that  $|A|^2$  corresponds to optical power, and  $\beta_1 = \frac{d\beta}{d\omega}$ ,  $\beta_2 = \frac{d^2\beta}{d\omega^2}$  and  $\beta_3 = \frac{d^3\beta}{d\omega^3}$  certain levels of nonlinearity and  $\gamma$  is the nonlinear parameter, which is:

$$\gamma = \frac{2\pi n_2}{\lambda A_{eff}}, \quad (2)$$

where  $n_2$  is nonlinear coefficient of refractive index and  $A_{eff}$  is effective area mode. Parameters  $\beta_2$  and  $\gamma$  describe the effects of dispersion and nonlinearity, respectively.

These two effects, nonlinearity and dispersion can compensate each other so that in the solution equation has no breaking waves, while wave packets indefinitely retain their form (no dispersion of the wave). Expansion wave, in fact, impossible to grow the top that would be broke, while, on the other hand, nonlinearity, leading to the development of peak waves, preventing the spread of waves – this two effects are in very fine balance.

This equation for the propagation of waves suitable for pulse of width a few picoseconds (ps) or longer.

Additional high-order nonlinearity  $\beta_3$  required to describe the properties of ultrashort pulses. While working wavelength is not equal to wavelength of zero-dispersion, we can consider that  $\beta_3 = 0$ .

We will use the normalized variables for time, distance transmission and amplitude of field, respectively:

$$\begin{aligned} \tau &= \frac{t - \beta_1 z}{T_0}, \\ \xi &= \frac{z}{L_D}, \\ U &= \frac{A}{\sqrt{P_0}}, \end{aligned} \quad (3)$$

where is  $T_0$  pulse width,  $P_0$  peak power of pulse and  $L_D$  dispersion length, which is defined by the formula:

$$L_D = \frac{T_0}{|\beta_2|}. \quad (5)$$

Now we can write the propagation equation in the form of:

$$j \frac{\partial U}{\partial \xi} + \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} + N^2 |U|^2 U = 0. \quad (6)$$

Equation (6) represents the nonlinear Schrödinger equation, in which the second member have a negative sign for positive dispersion. The parameter  $N$  is defined by the relation:

$$N^2 = \gamma P_0 L_D = \frac{\gamma P_0 T_0^2}{|\beta_2|}. \quad (7)$$

Schrödinger equation we use for describe the propagation of pulses in dispersion middle, with no loss in a nonlinear medium.

The so-called soliton solutions exist in the anomalous dispersion regime ( $\beta_2 < 0$ ), which meets the criteria:

$$N = \sqrt{\frac{L_D}{L_{NL}}}, \quad (8)$$

where  $L_{NL}$  nonlinear length.

Impulse that travels through the medium will have its original form (unaltered form) if it is  $N = 1$ . This form is called basic or fundamental solitons. If  $N \neq 1$ , pulse (wave) will change its shape during propagation, but these changes will be periodic, so that after an appropriate period the wave have a primary shape. Solitons, where the  $N = 2, 3, \dots$  called higher-order solitons.

Soliton solution for the case  $L_D = L_{NL}$  including its phase is:

$$U(\xi, \tau) = \text{sech}(t) e^{j \frac{\xi}{2}}. \quad (9)$$

Period soliton is:

$$z_0 = \frac{\pi}{2} L_D = \frac{\pi T_0^2}{2|\beta_2|}. \quad (10)$$

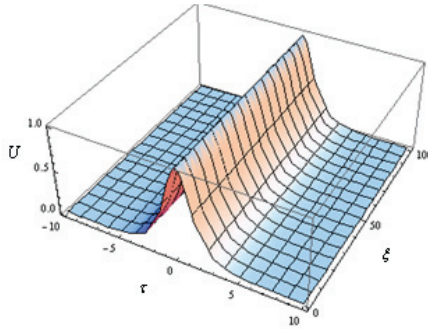
Only phases of basic solitons are cyclical changes with the period  $z = \pi L_D$ . Other soliton solutions higher order of interest for communication with optical fibers are those with the same shape of initial pulse as well as basic

$$U(0, \tau) = N \text{sech}(\tau). \quad (11)$$

These solutions are changing its shape periodically during propagation, but restored its original shape after the soliton period length

$$\xi = \frac{\pi}{2}. \quad (12)$$

The appearance of the basic soliton is shown in Fig. 1.



**Fig. 1** – *The appearance of the basic soliton.*

The time interval  $T_b$  between two adjacent bits or pulses determines the bit rate  $R$  of transmission communication system ( $R=1/T_b$ ). Therefore, it is necessary to determine how near can be two solitons and to avoid their mutual interaction. Amplitude solitons pair at the entrance of the fiber can be written in the following normalized form [8]:

$$U(0, \tau) = \text{sech}(\tau - \Delta T_0) + r \text{sech}[r(\tau + \Delta T_0)]e^{i\theta}, \quad (13)$$

where is  $r$  relative amplitude,  $\theta$  relative phase, and  $\Delta T_0$  the initial distance pulses.

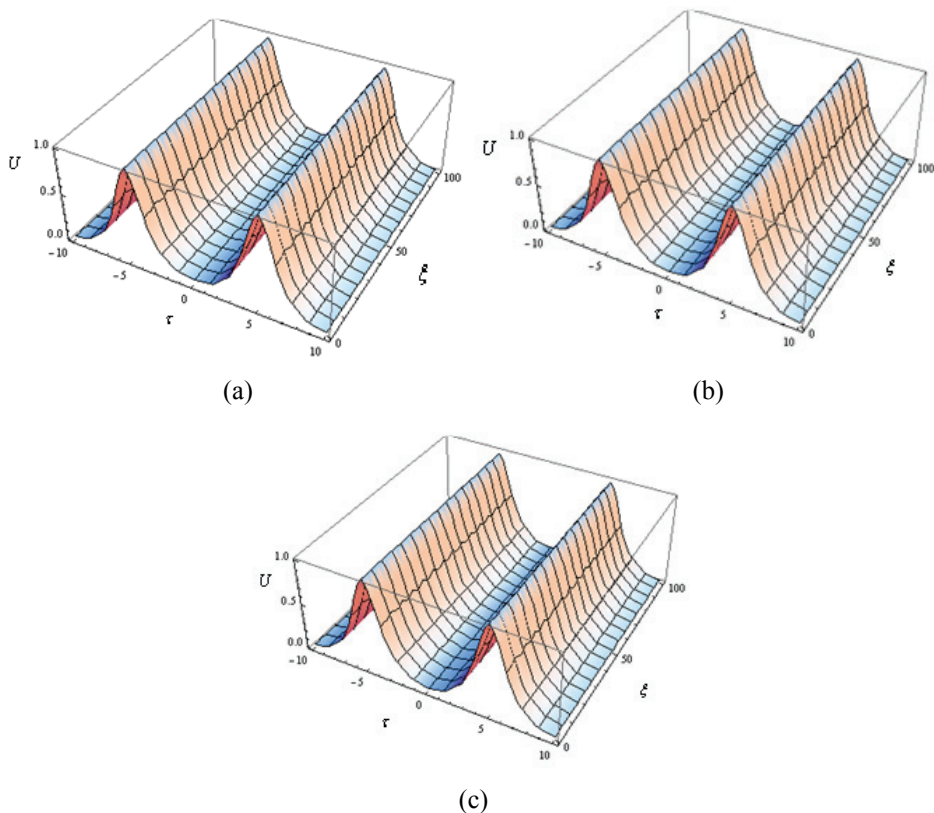
If we consider two solitons of equal amplitude and phase, they will periodically overlap in the following intervals of length:

$$\xi = \frac{\pi}{2} e^{\Delta T_0}. \quad (14)$$

This periodic overlap is not desirable for the system. One way to avoid head to head action between two adjacent solitons is increasing the mutual distance between the initial pulse to be  $\Delta T_0 \geq 4$ .

### 3 Relative Phase

Fig. 2 shows the propagation solitons pair at changing the relative phase, while the relative amplitudes fixed. Home distance pulse is  $\Delta T_0 = 4$ .

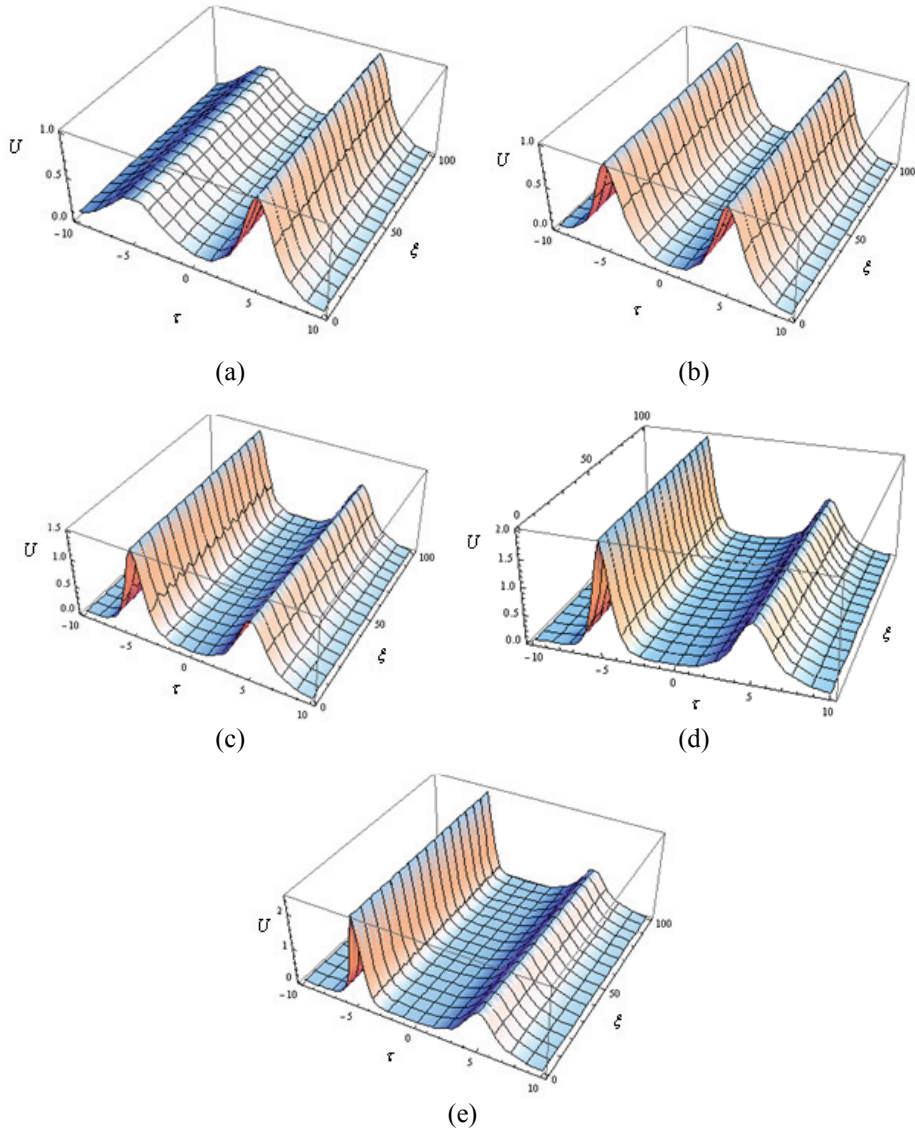


**Fig. 2** – Propagation solitons pair at changing the relative phase for: (a)  $\theta = 0$  ; (b)  $\theta = \pi / 2$  ; (c)  $\theta = \pi$  .

In Fig. 2 is clearly seen that at change the relative phase no overlapping of solitons, but to a lesser extent there is interaction. This interaction is reflected in the creation of so-called repulsive forces between the two solitons. When  $\theta$  increases and the repulsive force will increase, but not so much to spoil the ideal transmission.

#### 4 Relative Amplitude

Fig. 3 shows the propagation solitons pair for different values of relative amplitude, and at a constant phase. And in this case to the initial distance between solitons is  $\Delta T_0 = 4$  .



**Fig. 3** – Propagation solitons pair at changing the relative amplitude for: (a)  $r = 0.5$ ; (b)  $r = 1$ ; (c)  $r = 1.5$ ; (d)  $r = 2$ ; (e)  $r = 2.5$ .

As can be seen in Fig. 3, there is overlap and mutual interaction between solitons. For value of the relative amplitude of  $r=1$  there is no interaction between, so it's now possible transmission along optical fibers. In Fig. 2 we see that if the relative amplitude decreases or increases, compared to  $r = 1$ , a soliton

gradually loses its initial amplitude and shape, so that it gradually disappears, while the second increases amplitude.

This phenomenon undermines the ideal data transmission using solitons in optical fiber, so that when designing systems and soliton connection in optical fiber must take into account the relative amplitude and spacing between the solitons.

## **5 Conclusion**

Presented analysis and simulation of the propagation of solitons pair through the nonlinear dispersive single-mode optical fiber from the standpoint of changing the relative phase and relative amplitude of solitons pairs. It was found that for change the relative phase is not occurs overlap each other solitons, but there are some repulsive force between the solitons. Due to changes in the relative amplitudes leads to overlapping of solitons, ie solitons lose their shape.

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