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Energy Tunneling Through Narrow Waveguide Channel and Design of Small Antennas*

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Abstract: In this paper we investigate the conditions for energy tunneling through narrow channel obtained by reducing the height of rectangular waveguide. Tunneling of the energy occurs at the frequency for which the effective dielectric permittivity of the channel becomes equal to zero, so it can be treated as an ENZ (epsilon-near-zero) metamaterial. We investigated how geometry of the channel and dielectric permittivity affect the transmission coefficient and field density in the channel. Adding slots in the channel, which are placed orthogonally to the wave propagation, we designed a small antenna with directivity of 5.44 dBi at the frequency of 3 GHz.

Keywords: ENZ metamaterial, Zeroth-order resonance, Fabry-Perot resonance, High-directivity antenna.

1 Introduction

During the last decade there has been a great interest in metamaterials whose relative dielectric permittivity ε_r is close to zero (epsilon-near-zero, ENZ metamaterials). This kind of metamaterials can be designed using standard techniques, like split-ring resonators [1], however dispersion characteristic of rectangular waveguide near the cut-off frequency can also be used to mimic the behaviour of ENZ metamaterial [2]. There can be a number of applications for this structure, from cloaking devices [3], confining energy beyond diffraction limit, and waveguide coupling through very narrow channel without energy loss, to designing small, and high-directivity antennas.

2 Theoretical Analysis

Rectangular waveguide with width a and height b (where $a > b$) can support propagation of TE and TM modes. The mode cut-off frequency is determined by expression:

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$$f_{c(m,n)} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}, \tag{1}$$

where ε_r is relative dielectric permittivity of the material filling the waveguide.

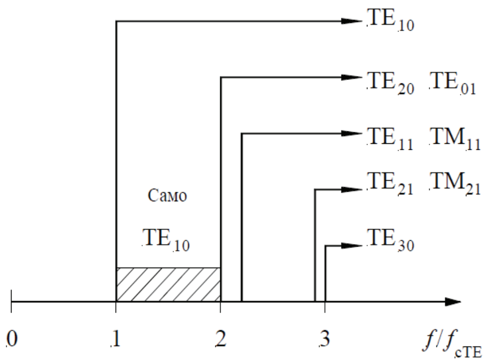


Fig. 1 – *The sequence of modes in rectangular waveguide with height b = a/2.*

It can be seen clearly from Fig. 1 that dominant mode in rectangular waveguide is TE_{10}, while the next mode is TE_{20}. Frequency range between cut-off frequencies of those two modes is marked as a pass band of rectangular waveguide. It is important to emphasize that cut-off frequencies of TE_{10} and TE_{20} modes do not depend on b since $n = 0$. But, if we reduce the height of rectangular waveguide b, the cut-off frequencies for higher modes for which $n \neq 0$, will shift toward higher frequencies.

If we start to reduce the height of rectangular waveguide continuously (Fig. 2), it is intuitively obvious that at one point the reflection coefficient will considerably grow and stop propagation of energy through the very narrow channel.

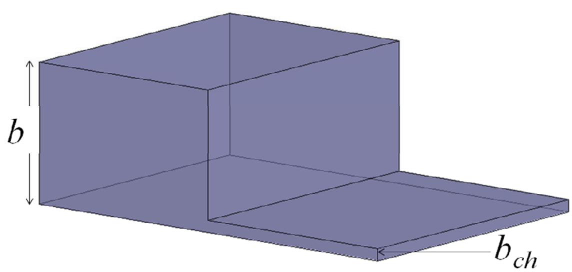


Fig. 2 – *The reduction of waveguide height b.*

If instead of configuration in Fig. 2 we use the structure shown in Fig. 3 with Π-channel, transmission will be possible in the narrow frequency band as long as dielectric permittivity of material filling the channel is smaller than in input waveguides. Later on, the explanation of this phenomena will be provided.

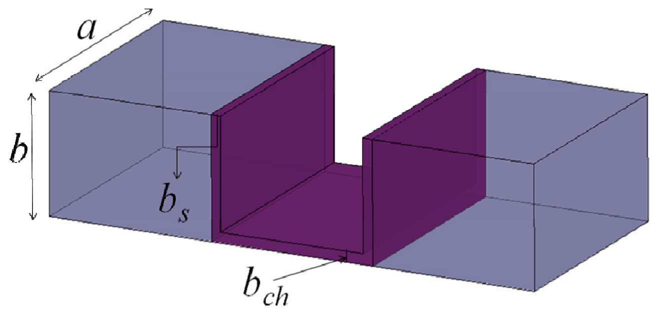


Fig. 3 – *Waveguide with narrow Π-channel* (b_{ch}<< b).

According to [2], it is possible to consider propagation of TE_{10} mode in a narrow rectangular waveguide as propagation of TEM mode in a parallel plate waveguide with effective permittivity ε_{reff}:

$$\beta_{TE10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} = \beta_{TEM} = \frac{2\pi f \sqrt{\varepsilon_{reff}}}{c}, \tag{2}$$

where

$$k = \frac{2\pi f \sqrt{\mu_r \varepsilon_r}}{c}.$$

The effective permittivity of the channel can be derived as:

$$\varepsilon_{reff} = \varepsilon_r - \frac{c^2}{4 f^2 a^2}. \tag{3}$$

Here c is the speed of light in vacuum, and ε_r is dielectric permittivity of the channel. It can be easily seen that ε_{reff} equals to zero at the cut-off frequency of the channel: $f_{c(10)} = c/2a\sqrt{\varepsilon_r}$. This explains why we can consider this structure as an ENZ metamaterial around frequency $f_{c(10)}$.

Structure in Fig. 3 consists of two rectangular waveguides connected by narrow channel. Dielectric permittivity of material in the input waveguides should be greater than in the channel, in order to provide transmission of the tunnelling frequency within the pass band of input waveguides. Taking into account that ε_{reff} in the channel is near zero close to cut-off frequency, which is not equal to zero, we come to conclusion that wave vector β is also closed to zero and wavelength approaches infinity. This kind of behaviour is characteristic for balanced left-handed (LH) metamaterials that have the zeroth-order frequency (ZOR) [6] at the transition between left- and right-handed regions. At that frequency β is equal to zero. Energy transfer through the channel at ZOR frequency is obtained with relatively small losses. Also, field is constant along the channel and field density is a very large and proportional to waveguide/channel height ratio.

3 Simulation Results for ENZ Channel

We considered firstly, the structure from Fig. 2 with waveguide width and height a = 101.6 mm and $b = a/2$ = 50.4 mm. Since dielectric permittivity in the input waveguides is $\varepsilon_r = 2$, cut-off frequency for TE_{10} mode is 1.044 GHz, and for TE_{20} mode 2.088 GHz. Values for the channel height are chosen to be: $b_{ch1} = b$, $b_{ch2} = b/2$=25.4 mm, $b_{ch3} = b/8$ = 6.35 mm and $b_{ch4} = b/64$ = 0.8mm. Simulation results for transmission and reflection coefficients are shown in Figs. 4a and 4b respectively.

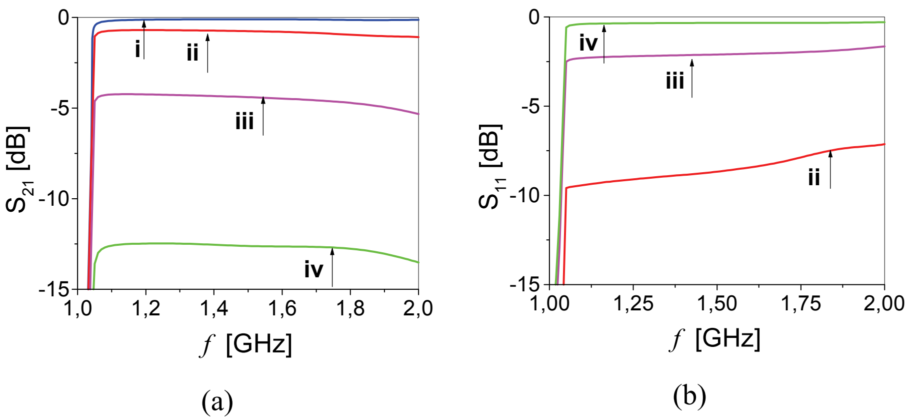


Fig. 4 – *Transmission* (a) *and reflection coefficients* (b) *for different values of* b_{ch}: (i) $b_{ch1} = b$ = 50.8 *mm*, (ii) b_{ch2} = =25.4 *mm*, (iii) b_{ch3} = 6.35 *mm*; (iv) b_{ch4} = 0.8 *mm*.

As can be seen from Fig. 4, even if the height of waveguide is reduced at some point to the half of its original size, transmission is still possible. Considerable reduction of transmission coefficient occurs when the channel height is less then $b_{ch3} = b/8$ as a consequence of large reflection at the discontinuity.

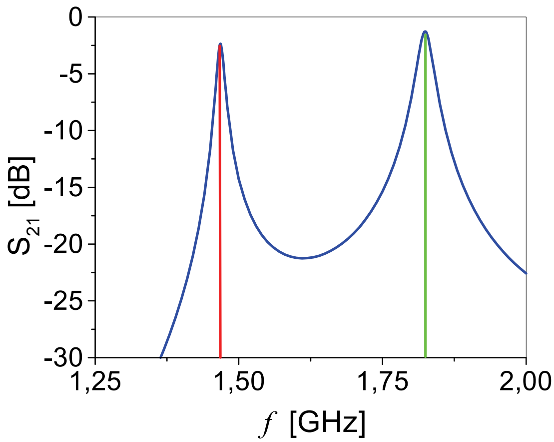


Fig. 5 – *Transmission coefficient for* Π*-channel for* $b_s = b_{ch} = 0.8$ *mm.*

To compensate the influence of discontinuity at the channel/waveguide junctions, we added two transition layers with thickness $b_s = b_{ch}$, which form Π-channel, as it is shown in Fig. 3 (values of a and b are the same as before). Dielectric permittivity of the channel and transition area is $\varepsilon_{rch} = 1$ (air), and of input waveguides $\varepsilon_r = 2$. Transmission coefficient for Π-channel with $b_s = b_{ch} = 0.8$ mm is shown in Fig. 5. It can be seen that the first transmission peak appears at 1.476 GHz and represents the zeroth-order resonance, ZOR. That is actually the frequency at which the energy tunnelling is appeared, since the effective permittivity is equal to zero. The second transmission peak is due to Fabry-Perot resonance and its position strongly depend on the channel length, which is not the case for ZOR, as long as $b_{ch} \ll a$ [7].

The field distribution in ENZ channel and real part of Poynting vector are shown in Figs. 6a and 6b respectively.

In approximation of perfectly conducting metallic walls and lossless dielectrics, field density in the channel is increased with reduction of channel height and transmission is perfect. In reality, field density in channel is restricted by break down voltage in dielectric and transmission is lowered due to finite conductivity of metallic walls and dielectric losses. Simulated results for $b_{ch1} = b/16 = 3.18$ mm and $b_{ch2} = b/64 = 0.8$ mm are shown in Fig. 7. Metal used is copper with $\sigma = 58$ MS/m. Detailed discussion on losses in ENZ channel is given in our previous work [8].

As it was pointed out before, the second transmission peak is due to Fabry-Perot resonance and is highly dependent on the channel length, that is not the case with the zeroth-order resonance (ZOR) frequency at which tunneling of energy is occurred. Change of Fabry-Perot resonance for various lengths of a narrow channel ($b_{ch} = b/64 = 0.8$ mm, $L_1 = 95.25$ mm, $L_2 = 127$ mm and $L_3 = 190.5$ mm) can be seen in Fig. 8. This property can be used to manipulate the second transmission peak in order to remove it or leave it within the pass band of the waveguide.

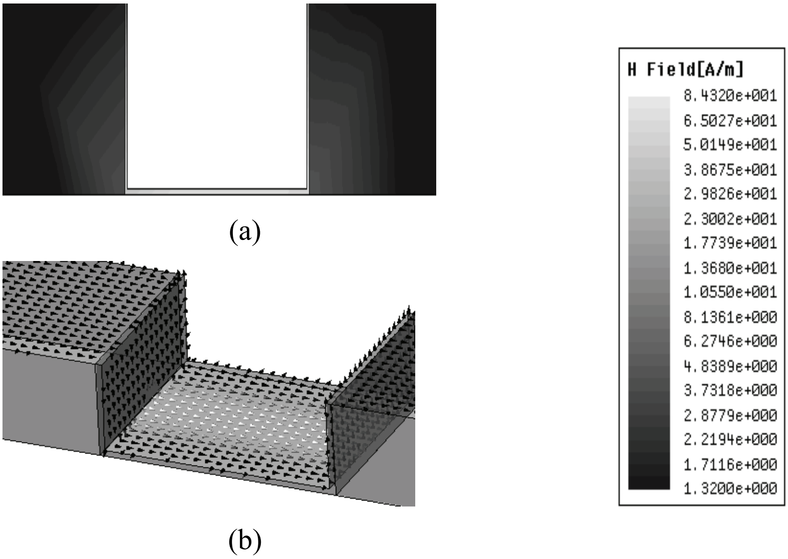


Fig. 6 – (a) *Field distribution in a narrow channel shows H-field density inside the channel enhanced by factor* b/b_{ch} *in comparison to field in waveguide sections; field density is constant along the channel*; (b) *Real part of Poynting vector shows energy flow through the channel; energy is concentrated in the middle of the channel and gradually descending toward edges.*

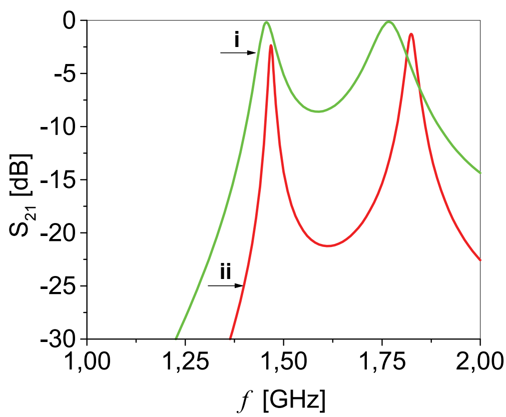


Fig. 7 – *Transmission coefficient for different values of channel height* b_{ch}: (i) $b_{ch1} = 3.18$ *mm*, (ii) $b_{ch2} = 0.8$ *mm.*

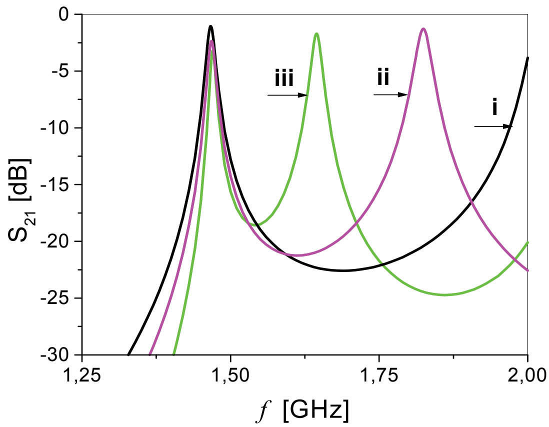


Fig. 8 – *Shifting the second transmission peak* (*Fabry-Perot resonance*) *due to variation of channel length*: (i) L_1 = 95.25 *mm*, (ii) L_2 = 127 *mm*, (iii) L_3 = 190.5 *mm*.

4 Antenna Design and Simulation Results

Using ENZ channel, simple antenna can be designed by placing one or few slots on the top or bottom side of ENZ channel along or normal to the direction of propagation. If the slots are oriented along the propagation direction they should be placed at a certain distance from the middle of the channel in order to get noticeable radiation. Much higher radiation can be achieved if the slots are perpendicular to the direction of wave propagation. Simulated radiation pattern for two slots placed symmetrically around the middle of the channel and perpendicular to the direction of propagation are given below. Channel dimension is 50×50 mm, slots dimensions are 1×50 mm, and the distance between them is 3 mm.

The main drawback of this antenna is that it operates in very narrow frequency range, as can be seen in Fig. 9b. The level of radiation here is shown through the value of $\rho = |S_{11}|^2 - |S_{21}|^2$ since designed antenna is a progressive wave antenna.

Radiation patterns with respect to φ and θ are given in Figs. 10a and 10b. It can be seen that this antenna has following 3dB-beamwidths: θ_{H3dB} = 139.7° and θ_{E3dB} = 84.3° that gives very high directivity of 5.44 dBi.

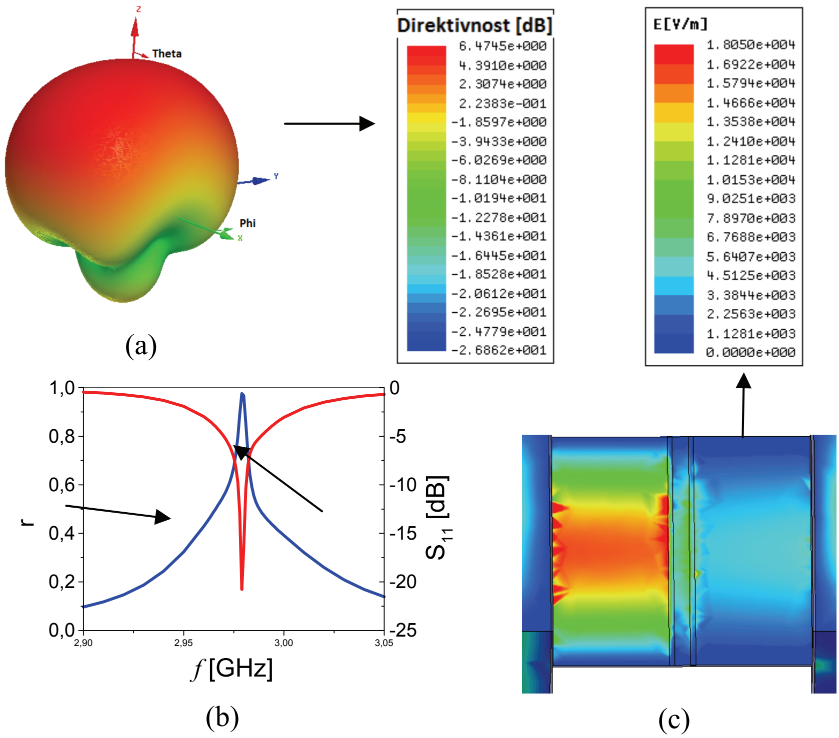
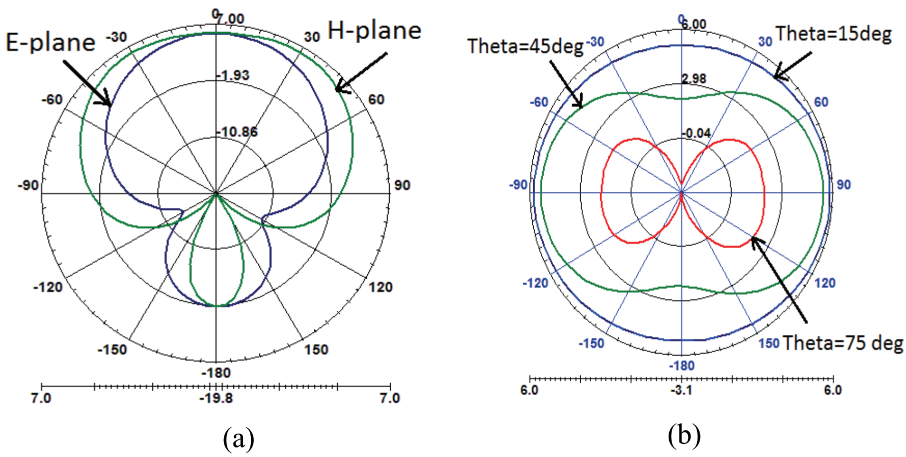


Fig. 9 – (a) 3D *radiation pattern*; (b) *Radiated power and reflection coefficient*; (c) *E-field in configuration with two radiating slots in the middle of the channel.*



Sl. 10 – *Radiation pattern for*: (a) φ = 0° (*E-plane*) *and* φ = 90° (*H-plane*);
(b) θ = 15°, θ = 45° *and* θ = 75°.

6 Conclusion

In this paper we investigate how geometry parameters of the ENZ channel influence the energy tunneling through the channel. If the channel height is getting smaller, the energy confining is increased, as well as the reflection and losses, while the frequency range in which tunneling occurs becomes narrower. Changing the channel length affects only the position of Fabry-Perot transmission peak, while there is no influence to ZOR resonance. ENZ channel is used in design of a small, high-directivity antenna which consists of two vertical slots, perpendicular to wave propagation.

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