

## Two Approaches for Log-Compression Parameter Estimation: Comparative Study\*

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**Abstract:** Standard ultrasound devices perform nonlinear compression reducing dynamic range of the signal. In order to reconstruct original signal it is necessary to find out statistics of the signal before and signal after compression. There are two techniques for compression parameter estimation or, that is equivalent, compressed signal reconstruction advised in literature. In the paper we perform comparison of these techniques both for computer generated signals and ultrasound images.

**Keywords:** Speckles, Ultrasound, Log-compression.

### 1 Introduction

The advantage of ultrasound technique among other diagnostic tools is its noninvasive, portable and low-cost nature [1]. On the other hand its most evident drawback is low signal to noise ratio (SNR). It makes both ultrasound diagnostics and further ultrasound signal processing difficult.

Better SNR is possible to achieve considering noise statistics; although total signal (signal plus noise) statistics is changed by log-compression (in Post Processing block in Fig. 1). This operation reduces dynamic range of the signal to match the lower dynamic range of the video output device (display).

Ultrasound devices with additional "raw data" output are still not in wide use. Beside clinician mode, which is corrupted by operator manual tuning such as contrast adjustment, these devices provide research mode for quantitative analysis.

When the compression parameter and total signal statistics is determined it is possible to suppress noise in ultrasound image and therefore obtain better SNR.

The aim of this paper is to determine one of the parameters of the log-compression and to reconstruct original signal before compression. Two approaches considered in [2, 3] are analyzed and results of both methods are later compared.

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## 2 Speckle Statistics

Fig. 1 shows ultrasound block scheme. Transducer converts electrical signals into sound field that is transmitted in the human body. Sound waves will be reflected, refracted and diffused in the tissue. Human tissue, according to ultrasound wave propagation, consists of two types of scatterers: structural and diffuse scatterers [4]. Structural scatterers are regularly distributed in the tissue and they represent its anatomic structure. Diffuse scatterers are micro structures in the tissue which are even or less of ultrasound wavelength.

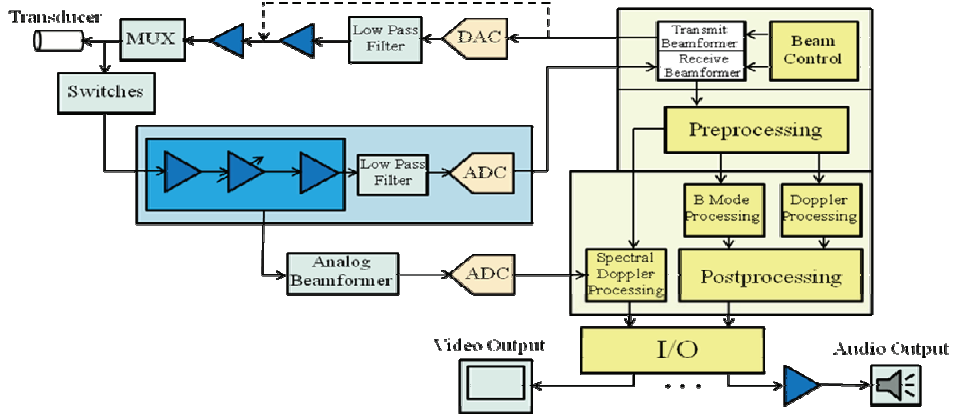


Fig. 1 – Ultrasound device scheme.

After scattering on diffuse reflectors, sound wave is represented with sum of components with uniformly distributed phases [5],  $\theta_i$  :

$$\underline{X} = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} \xi_i e^{j\theta_i}, \quad (1)$$

or:

$$\underline{X} = X_r + jX_i = \sqrt{X_r^2 + X_i^2} e^{j\arctan(X_i/X_r)}, \quad (2)$$

where  $\xi_i$  and  $\theta_i$  are respectively amplitude and phase of the  $i$ -th component;  $X_r$  and  $X_i$  are respectively real and imaginary part of the received signal. For  $N \rightarrow \infty$ , with respect to central limit theorem, the joint density function for real and imaginary parts,  $p_{X_r, X_i}(X_r, X_i)$ , approaches normal distribution. Amplitude distribution of the received signal, according to (2), will be Rayleigh:

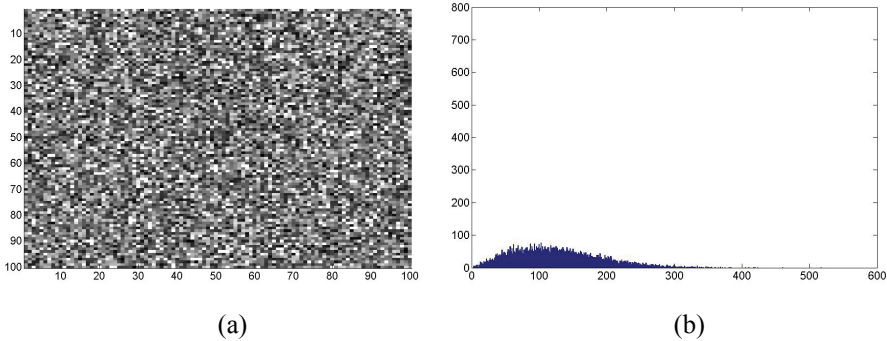
$$p_X(X) = \frac{X}{\sigma^2} e^{-\frac{X^2}{2\sigma^2}}. \quad (3)$$

Note this is valid only for diffuse scatterers. The presence of structural component causes homodyned K-distribution (for low scatterer density) and Rice distribution (higher scatterer density) [5].

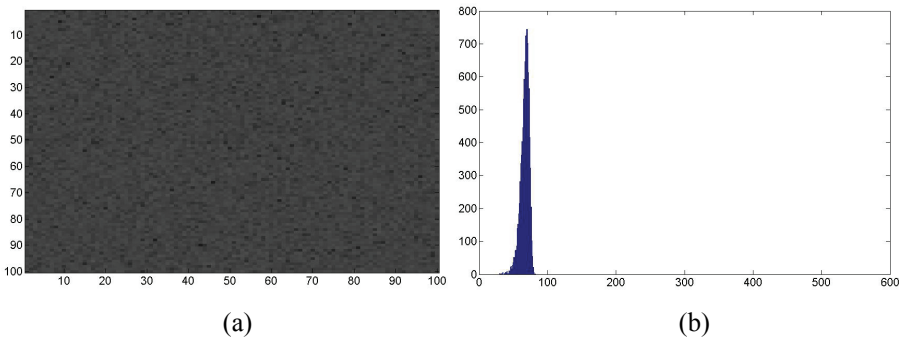
Compression is one of post processing techniques in ultrasound devices (Fig. 1) and can be modeled as follows:

$$Y = D \ln X_e + G, \tag{4}$$

where  $D$  and  $G$  are parameters of log-compression.  $Y$  is output signal (pixel value) with lower dynamic range than in the input signal,  $X_e$ . Here  $X_e$  is the envelope of the signal  $X$ . From (4) it is obvious how to reconstruct the original signal from B – scan images when the parameters of log-compression are available. As  $G$  is additive parameter, the shape of distribution of the output signal is determined only by parameter  $D$ . The influence of compression on signal statistics is illustrated in Fig. 2. There is computer generated image with Rayleigh distribution (Fig. 2) and the same image after log-compression (Fig. 3). Comparing histograms of these 2D signals it is clear how the signal distribution is affected by compression.



**Fig. 2** – (a) Computer generated signal with Rayleigh distribution; (b) Its histogram.



**Fig. 3** – (a) Signal after log-compression. (b) Its histogram.

### 3 Approach 1

As amplitudes of the received signal were Rayleigh distributed, the intensity of the received signal, defined as  $I = X^2$ , will be distributed accordingly:

$$p_I(I) = \frac{p_X(X^{-1})}{\left| \frac{dI}{dX} \right|}. \quad (5)$$

From (5) the density function of intensity,  $I$ , is given by:

$$p_I(I) = \frac{1}{2\sigma^2} e^{-\frac{I}{2\sigma^2}}. \quad (6)$$

This is exponential distribution. For moments of exponential distribution is proved to be:

$$\langle I^n \rangle = (2\sigma^2)^n \Gamma(n+1), \quad (7)$$

where  $\Gamma(\cdot)$  denotes Gamma function. In [2] it is proposed to calculate normalized moments:

$$r = \frac{\langle I^n \rangle}{\langle I \rangle^n} = \Gamma(n+1). \quad (8)$$

This parameter is used for estimation of parameter of compression .

Algorithm for  $D$  parameter estimation, proposed in [2], compares theoretically calculated values for  $r$  as in (8) and those measured from the image. Error vector representing the difference between these two set of values is then minimized using Levenberg-Marquart algorithm. As the final result there is estimation of compression parameter,  $\hat{D}$ .

### 4 Approach 2

Algorithm proposed in [3] is based on statistics for output signal  $Y$  given in (4). According to (4) and (5), the density function of the signal  $Y$  is given by:

$$p_Y(Y) = \frac{2}{D} e^{-\theta - e^{-\theta}}, \quad (9)$$

where  $\theta = \ln(2\sigma^2) - 2\frac{Y-G}{D}$ . Function (9) corresponds to double exponential distribution and its standard deviation is calculated as follows:

$$\sigma_Y = \frac{\pi D}{\sqrt{24}}. \quad (10)$$

Estimation for compression parameter is calculated from the image as:

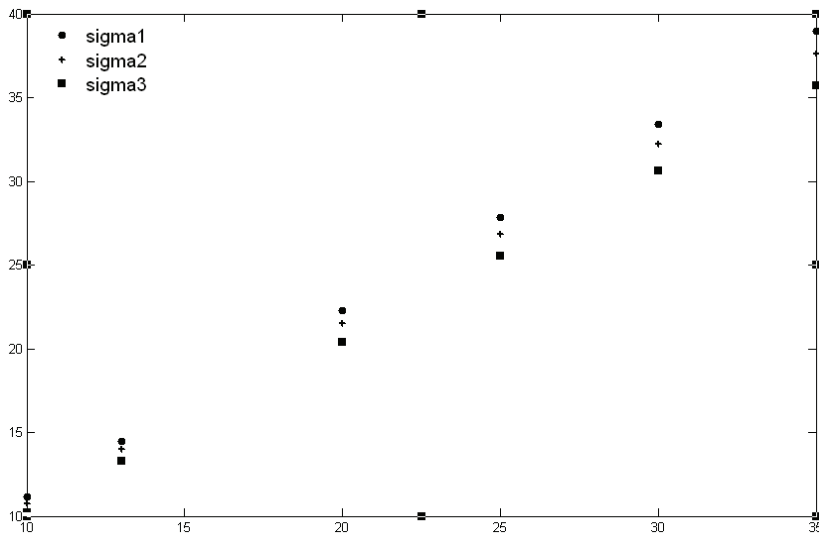
$$D = \frac{\sqrt{24}\sigma_Y}{\pi}. \quad (11)$$

where  $\sigma_Y$  is the standard deviation of the homogeneous regions in the output image.

## 5 Results

Both approaches are tested on computer generated images and then with real B-scan images.

Fig. 4 shows dependence of estimation  $\hat{D}$  upon real values for compression parameter,  $D$ . Images of size  $100 \times 100$  pixels are divided in three sets corresponding to parameter of the Rayleigh distribution,  $\sigma$ .



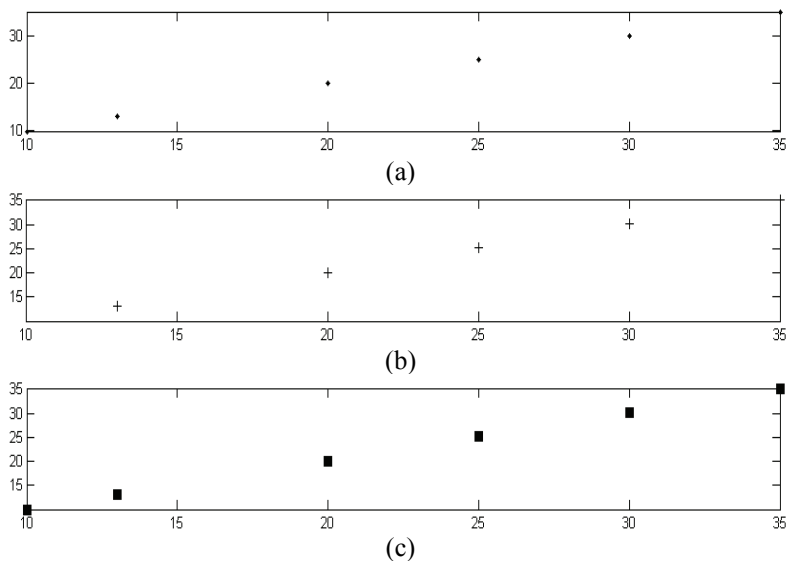
**Fig. 4** – Approach 1. Dependence of estimated parameter  $\hat{D}$  upon used value for  $D$  for three values of  $\sigma$ :  $\sigma = 20$  (sigma1),  $\sigma = 50$  (sigma2) and  $\sigma = 80$  (sigma3).

The same conditions are considered in the approach number two and the results are represented in Fig. 5.

Fig. 6 shows the effect of compression of real ultrasound images. There is image before compression and log-compressed image.  $D$  is calculated using approach proposed in [3] and its estimation is  $\hat{D} = 34.9189$ . Value of parameter

$G$  was chosen to make all pixels in image lie in the range  $0 \div 255$ . The choice of  $G$ , as it is said before, does not affect the shape of the distribution of the output signal.

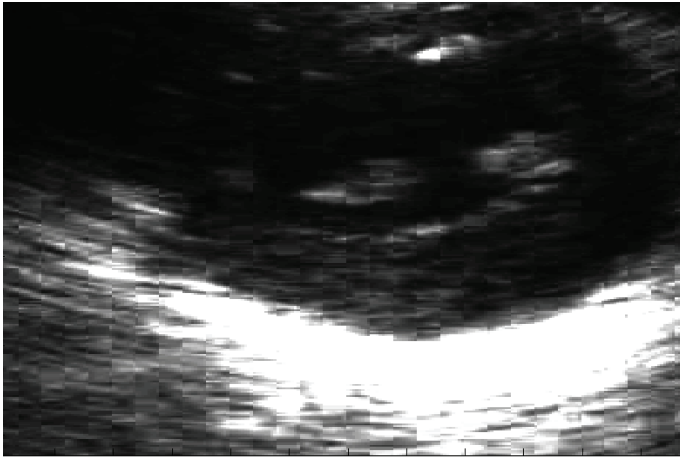
When calculating  $D$  only regions in image without coherent component of the signal are considered, because the statistics assumed in (6) is valid for diffuse scatterers only.



**Fig. 5** – Approach 2. *Dependence of estimated parameter upon used value for  $D$  for three values of  $\sigma$  : (a)  $\sigma = 20$  (sigma1), (b)  $\sigma = 50$  (sigma2); (c)  $\sigma = 80$  (sigma3).*



**Fig. 6** – (a) *Log-compressed image.*



**Fig. 6** – (b) *Reconstructed signal.*

## 6 Conclusion

Comparing results of both approaches (Figs. 4 and 5), it is clear that approach number two provides better results. This could be explained as it is based on statistics for compressed signal, while approach number one is based on statistics for uncompressed signal. Approach number one, further, compares theoretically calculated normalized moments with those determined directly from the image. Moments calculated in image should fit Gamma function (see (8)), which is sensitive on the range of observation for  $n$ . This range should be narrow in order to prevent Levenberg-Marquart algorithm from oscillating, but not too small to make precise approximation of Gamma function.

In Fig. 6, ultrasound images before and after compression show the significance of the decompression in the sense of computer vision.

## 7 References

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