Application of a Fast and Elitist Multi-Objective Genetic Algorithm to Reactive Power Dispatch

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Abstract: This paper presents an Elitist Non-Dominated Sorting Genetic Algorithm version II (NSGA-II), for solving the Reactive Power Dispatch (RPD) problem. The optimal RPD problem is a nonlinear constrained multi-objective optimization problem where the real power loss and the bus voltage deviations are to be minimized. Since the problem is treated as a true multi-objective optimization problem, different trade-off solutions are provided. The decision maker has an option to choose a solution among the different trade-off solutions provided in the pareto-optimal front. The standard IEEE 30-bus test system is used and the results show the effectiveness of NSGA-II and confirm its potential to solve the multi-objective RPD problem. The results obtained by NSGA-II are compared and validated with conventional weighted sum method using Real-coded Genetic Algorithm (RGA) and NSGA.

Keywords: Reactive Power Dispatch, Multi-objective, Non-dominated sorting genetic algorithm, Optimization.

1 Introduction

The purpose of optimal reactive power dispatch (RPD) is to minimize the network real power loss and improve voltage profiles by regulating generator bus voltages, switching on/off static var compensators and changing transformer tap-settings. The reactive power dispatch is a complex problem in a large scale power system. In addition, the system losses can be minimized via redistribution of reactive power in the system. Therefore, the problem of the reactive power dispatch can be optimized to improve the voltage profile and minimize the system losses as well [1].

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The optimal RPD is a constrained, nonlinear, discrete optimization problem. The traditional approach is to formulate this problem as a single-objective optimization problem with constraints. In this approach, the objective may consist of a single term [3-7] or it may consist of multiple terms [8, 9]. One may use dynamic programming or genetic algorithm to solve. However, this approach only results in a single optimal solution where tradeoffs between different components of the objective function must be fixed in advance of solution. In order to provide a means to assess tradeoffs between two conflicting objectives, one may formulate the RPD problem as a multi-objective problem. In this case, solution requires a multi-objective algorithm such as NSGA-II. Srinivas and Deb developed NSGA in which a ranking selection method emphasizes current non-dominated solutions and a niching method maintains diversity in the population [10]. Multi-objective evolutionary algorithms including NSGA that use non-dominated sorting and sharing have been criticized mainly for i) computational complexity, ii) non-elitism approach and iii) the need for specifying a sharing parameter. Deb et al. alleviated these difficulties in NSGA-II [11, 12].

In comparison with single-objective optimization techniques, the Pareto-based multi-objective optimization methods have a number of advantages in solving constrained optimization problems [12]. First, the Pareto-optimal solution set contains trade-off solutions, including solutions that violate constraints so that a solution with a permissible constraint violation can still be considered if there is a substantial gain in the objective function value. Second, decomposing the original single-objective function into multiple, conflicting objectives gives more flexibility in exploring the solution space. As described in [13], this flexibility enables identification of local optima, transformed by the multi-objective approach into tradeoff solutions along the Pareto front that would otherwise not be found by the single-objective approach.

The goal of this paper is to formulate the RPD problem as a multi-objective optimization and illustrate its solution using Pareto based multi-objective optimization NSGA-II. Two different multi-objective problem formulations are provided.

The paper is organized as follows: Section 2 describes the two different multi-objective RPD problem formulations. Section 3 describes NSGA-II implementation to the RPD problem. Section 4 describes obtaining the true Pareto-front. Section 5 provides test results, and section 6 concludes.

2   Reactive Power Dispatch Problem Formulation

The optimal RPD problem is to optimize the steady state performance of a power system in terms of one or more objective functions while satisfying

several equality and inequality constraints. Generally the problem can be formulated as follows.

A. *Problem Formulation*

Objective Functions:

i) Real Power Loss (P_L)

This objective is to minimize the real power loss in transmission lines that can be expressed as

$$P_L = \sum_{k=1}^{nl} g_k [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)], \tag{1}$$

where:

nl is the number of transmission lines,

g_k is the conductance of the k-th line,

$V_i \angle \delta_i$ and $V_j \angle \delta_j$ are the voltages at end buses i and j of the k-th line respectively.

ii) Voltage Deviation (VD)

This objective is to minimize the deviations in voltage magnitudes at load buses that can be expressed as

$$VD = \sum_{k=1}^{NL} |V_k - 1.0|, \tag{2}$$

where:

NL is the number of Load bus.

Equality Constraints:

$$0 = P_i - V_i \sum_{j \in N_i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}), \quad i \in N_{B-1}$$

$$0 = Q_i - V_i \sum_{j \in N_i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}), \quad i \in N_{PQ}, \tag{3}$$

where:

V_i is voltage magnitude at i-th bus,

G_{ij}, B_{ij} are mutual conductance and susceptance between bus i and bus j,

θ_{ij} is voltage angle difference between bus i and bus j,

N_{B-1} is the total number of buses excluding slack bus,

N_{PQ} is the set of PQ buses,

N_i is the number of buses.

Inequality Constraints:

In the control variables, the generator bus voltages (AVR operating values) are taken as continuous variable, where as the transformer tap settings and shunt susceptance values are taken as discrete values. The load bus voltages and reactive power generation Q_g are taken as state variables.

Continuous control variable:

$$V_{G_i}^{\min} \leq V_{G_i} \leq V_{G_i}^{\max}; \quad i \in N_G \tag{4}$$

Discrete Control variables:

$$T_k^{\min} \leq T_k \leq T_k^{\max}; \quad k \in N_T,$$
$$B_{sh_i}^{\min} \leq B_{sh_i} \leq B_{sh_i}^{\max}; \quad i \in N_{sh}. \tag{5}$$

State Variables:

$$V_{PQ_i}^{\min} \leq V_{PQ_i} \leq V_{PQ_i}^{\max}; \quad i \in N_{PQ},$$
$$Q_{G_i}^{\min} \leq Q_{G_i} \leq Q_{G_i}^{\max}; \quad i \in N_G. \tag{6}$$

The objective function of the target power system is calculated using load flow calculation with the above mentioned equality and inequality constraints.

3   NSGA-II Implementation to RPD Problem

Classical optimization methods can at best find one solution in one simulation run, thereby making those methods inconvenient to solve multi-objective optimization problems. Evolutionary Algorithms, on the other hand, can find multiple optimal solutions in one single simulation run due to their population approach. The NSGA-II algorithm, procedure, and the fitness function evaluation are described in this section. These descriptions are applicable to the problem formulation.

The NSGA-II algorithm is described in [11,12]. This paper makes use of NSGA-II algorithm to solve the RPD problem because it has been demonstrated to be among the most efficient algorithms for multi-objective optimization on a number of benchmark problems [11]. In addition, it has been shown that NSGA-II outperforms two other contemporary multi-objective evolutionary algorithms: Pareto-archived evolution strategy (PAES) [14] and strength-Pareto evolutionary algorithm (SPEA) [15] in terms of finding a diverse set of solutions and in converging near the true Pareto-optimal set.
NSGA-II Algorithm for RPD:

Step 1: Identify the control variables like generator voltage limits and transformer tap setting limits for RPD problem

Step 2: Select the parameters like number of population, maximum number of generation, crossover and mutation probabilities.

Step 3: Generate initial population

Step 4: Evaluation of objective functions (i.e., real power loss and bus voltage deviation) for initial population

Step 5: Set the generation count

Step 6: Perform SBX crossover and polynomial mutation for the set of individuals

Step 7: Perform non-dominated sorting. (i.e., sorting the population according to each objective function value in ascending order of magnitude)

Step 8: Calculate crowding distance between the solutions. For each objective function, the boundary solutions (solutions with smallest and largest function values) are assigned an infinite distance value. All other intermediate solutions are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. This calculation is continued with other objective functions. The overall crowding-distance value is calculated as the sum of individual distance values corresponding to each objective. Each objective function is normalized before calculating the crowding distance.

Step 9: Perform selection based on tournament selection thereby a higher fitness is assigned to individuals located on a sparsely populated part of the front.

Step 10: Increment the generation count and repeat the steps from 6 to 9 until the count reaches the maximum number of generation.

4   Obtaining the Reference Pareto Front

An alternative approach, weighted sum method with (single-objective) GA, is described here to solve the RPD problem. It is used to validate the results obtained using NSGA-II.

A Pareto-optimal set is defined as a set of solutions that are not dominated by any feasible member of the search space; they are optimal solutions of the multi-objective optimization problem. To evaluate the performance of NSGA-II, a reference Pareto optimal set is needed. One of the classical methods is Weighted Sum method [12, pp. 50]. Using this method only one solution may be found along the Pareto front in a single simulation run. The real coded GA

with SBX crossover and polynomial mutation is used to obtain the true Pareto-front. The major difference between NSGA-II and GA lies in the selection process. In NSGA-II, non-dominated sorting is used in the selection process, whereas in the single-objective GA, tournament selection [12, pp. 89] is used. The sum of weights is always 1. The weight value for the first objective function is $w1$, and the weight value for second objective function is $w2$, and $w2 = (1-w1)$. The weight $w1$ is linearly varied from 0 to 1 to produce the Pareto front. Both the objective function and constraints are normalized.

5   Test Results

The implementation was done using MATLAB version 7.4, on a IBM PC with Pentium dual core processor having 1.86GHz speed and 1 GB RAM. In this section, subsections A to C apply to problem formulation and Section D gives results for each problem formulation separately.

A. *Test System Description*

This paper uses the standard IEEE 30 bus system as test system. The representation of the test system and the detailed data are given in [2]. The system has 6 generators and 4 transformers and, therefore, the number of the optimized variables is 10 in this problem with a base MVA of 100. The lower voltage magnitude limits at all buses are 0.95 pu and the upper limits are 1.1 pu for generator buses 1, 2, 5, 8, 11, and 13, and 1.05 pu for the remaining buses. The lower and upper limits of the transformer tappings are 0.9 and 1.1 pu respectively.

B. *Control Parameters for RPD problem*

The following control parameters are considered in the RPD problem.

a) Automatic Voltage Regulator (AVR) operating values (continuous variable)

b) On-load Tap Changing (OLTC) transformer taps position (discrete variable)

The above control parameters are treated in load flow calculation as follows:

a) AVR operating values are treated as voltage specification values.

b) OLTC tap positions are treated as tap ratio to each tap position.

C. *Parameters for the NSGA-II Algorithm*

The results are sensitive to algorithm parameters, typical of heuristic techniques. Hence, it is required to perform repeated simulations to find suitable values for the parameters. The best parameters for the NSGA-II, selected through 10 test simulation runs, are given in **Table 1**.

Table 1
Best Parameters For RPD.

NSGA-II (Parameters)	Parameter values (type)
Population size, N_p	50
Number of iteration	500
P_c, Crossover probability	0.8
P_m, Mutation probability	0.2
Cross over index (η_c)	5,(Simulated Binary Crossover)
Mutation index (η_m)	18, (Polynomial mutation)

D. *Results and Discussion*

The problem was handled as a multi-objective optimization problem where both power loss and voltage deviations were optimized simultaneously with the proposed approach. The diversity of the Pareto optimal set over the trade-off surface is shown in Fig. 2. It is worth mentioning that the Pareto optimal set has 50 non-dominated solutions generated by a single run.

(a) *Identification of Reference Pareto-front*

The different Pareto-fronts obtained using GA, NSGA-II and NSGA are given in Figs. 1, 2 and 3. It shows the trade-off between the Active Power Loss (Obj fn1) and Voltage Deviation (Obj Fn 2). The constrained minimum solution for the single objective optimization problem is also illustrated in Fig. 1.
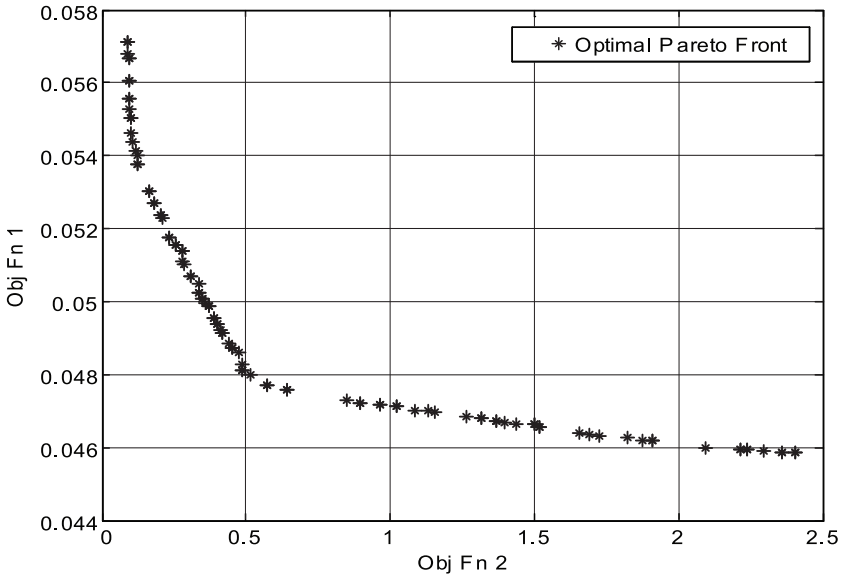

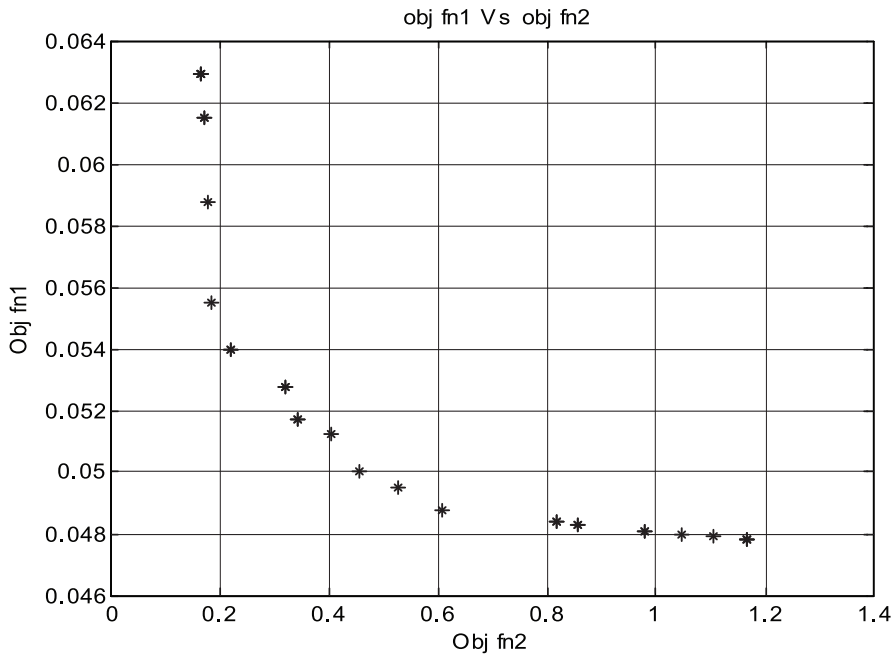**Fig. 1** – *The representation of reference pareto-front using single-objective real-coded* GA.

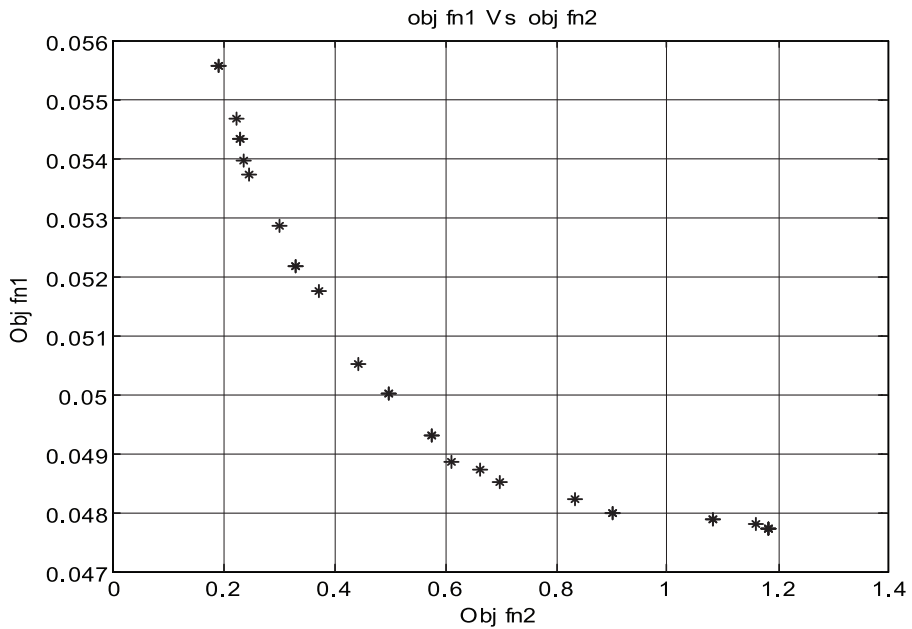**Fig. 2 –** *The Pareto-Optimal Set of Multi-objective*
Optimization problem with NSGA-II

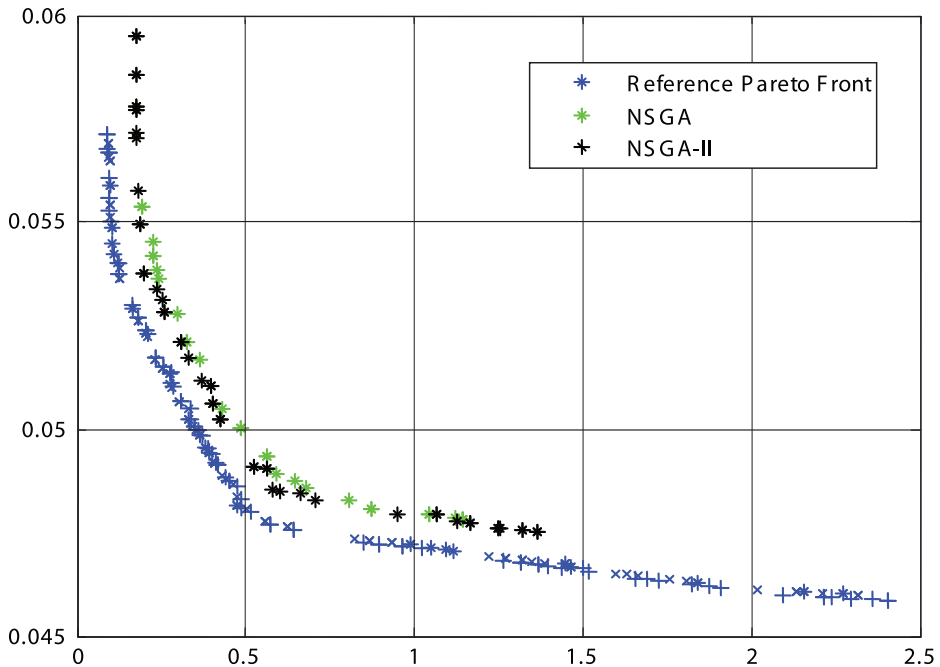**Fig. 3 –** *The Pareto-Optimal Set of Multi-objective Optimization problem with* NSGA.

Fig. 4 – *Comparison between* GA, NSGA-II *and* NSGA.

In Real GA technique, the weight $w1$ is linearly varied from 0 to 1 with an increment of 0.05 for each trial. A population size of 50 and maximum iteration of 500 are used. About 50 points are obtained to construct the Pareto front through GA and the execution time is 14,557 seconds (4.04 hours). In contrast, NSGA-II and NSGA produces the Pareto-optimal front in a single simulation run and the execution time is only 2357 seconds for NSGA-II and 7569 seconds for NSGA. The NSGA-II is computationally more efficient than GA and NSGA.

(b)   *Trade-off analysis*

Table 2 gives a comparison between the results of NSGA-II with NSGA. It is clear that the results of the NSGA-II is almost identical to that of individual optimization. It can be concluded that the NSGA-II is capable of exploring more efficient and non-inferior solutions. This demonstrates that performance of NSGA-II is superior than that of NSGA in terms of maintaining diversity among the solutions in the pareto-front.

127

Table 2
*Test Results of Best P$_1$ and Best VD using NSGA-II
and NSGA for IEEE 30 bus system.*

Decision Variables and Objectives	NSGA-II		NSGA	
	Best P_L	Best VD	Best P_L	Best VD
V_{G1}	1.0999	1.0365	1.0996	1.0432
V_{G2}	1.0913	0.9868	1.0923	1.0260
V_{G5}	1.0710	1.0081	1.0698	1.0111
V_{G8}	1.0737	1.0529	1.0750	1.0660
V_{G11}	1.0989	1.0271	1.0960	0.9818
V_{G13}	1.0985	1.0330	1.0938	1.0452
T_{6-9}	0.9702	0.9389	0.9754	0.9394
T_{6-10}	1.0043	0.9257	1.0173	0.9475
T_{4-12}	1.0491	0.9686	1.0456	0.9893
T_{27-28}	0.9807	0.9416	0.9847	0.9400
Power Loss (MW)	**4.7791**	**5.5556**	**4.7891**	**5.4665**
Voltage Deviation (pu)	**1.1592**	**0.1916**	**1.0822**	**0.2257**
Computation Time (s)	**2357**		**7569**	

For the given 30 bus system, the compromising active power loss is about 4.7791MW with the corresponding voltage deviation about 0.1916pu. Whereas, in NSGA the presence of active power loss and voltage deviation is in the order of about 4.7891MW and 0.2257pu for the same population and iteration size.

The extreme solutions are obtained by solving the problem as a single objective problem where the objective is either (a) Active power loss or (b) Voltage deviation. Results of these two cases are given in **Table 3**.

Table 3
Comparisons of Best solutions.

Objectives	NSGA-II	NSGA	Single Objective
P_L(MW)	4.7791	4.7891	4.6566
VD (pu)	0.1916	0.2257	0.0980

The number of iterations can be increased furthermore for both NSGA-II and NSGA with the cost of computation time. But a very little improvement was observed in the aspect of solution quality. Through the fronts obtained above in Fig. 4, it can be concluded that the front obtained with NSGA-II and NSGA are not true pareto-optimal front since the solutions are nearer to reference pareto-front. And may be the reference pareto-front can be a true optimal pareto-front (with optimal set of solutions). For large scale test system, the performance of NSGA-II may be better with increased number of population and iteration size.

According to the No Free Lunch (NFL) theorem all black-box algorithms perform equally well when compared over the entire set of optimization problems. An important problem related to NFL is finding a test problem for which a given algorithm is better than another given algorithm. This implies that the NSGA-II algorithm will perform better for this kind of power system optimization problem.

6   Conclusion

Multi-objective optimization enables evaluation of tradeoff solutions between different problem objectives through identification of the Pareto front. The NSGA-II algorithm provides that this can be done with very high computational efficiency. This paper describes the implementation of NSGA-II algorithm to the RPD problem. The problem has been formulated as Multi-objective optimization problem with competing real power loss and bus voltage deviations as objectives. The results show that the NSGA-II algorithm is efficient for solving multi-objective RPD problem where multiple pareto-optimal solutions can be found in one simulation run. In addition, the non-dominated solutions obtained are well distributed and have satisfactory diversity characteristics. The approach is quite flexible so that other formulations using different objectives and/or a larger number of objectives are possible.

7 Acknowledgment

8 Appendix

8.1 NSGA-II Algorithm

The NSGA-II algorithm [11, 12] uses non-dominated sorting for fitness assignments. One individual is said to dominate another if i) its solution is no worse than the other in all objectives and ii) its solution is strictly better than the other in at least one objective. All individuals not dominated by any other individuals are assigned front number 1. Individuals dominated only by the individuals in front number 1 are assigned front number 2, and so on. The simulated binary crossover and polynomial mutation generate new offspring, and tournament is then used to select the population for next generation.

Simulated Binary Crossover (SBX)

The SBX operator works with two parent solutions and creates two offspring. The difference between offspring and parent depends on crossover index η_c. It has two properties: (a) the difference between corresponding decision variables of the created offspring is proportional to the difference between corresponding decision variables of the parent solutions; (b) offspring having decision variables nearer to those of the parent solutions are more likely to be selected. The procedure for finding the offspring solutions $x_j^{(1,t+1)}$ and $x_i^{(2,t+1)}$ from parent solutions $x_i^{(1,t)}$ and $x_i^{(2,t)}$ is given below: A spread factor β_i is defined as the ratio of the absolute difference in children values to that of the parents:

$$\beta_i = \left| (x_i^{(2,t+1)} - x_i^{(1,t+1)}) / (x_i^{(2,t)} - x_i^{(1,t)}) \right|. \tag{7}$$

First a random number u_i between 0 and 1 is created. Thereafter, from a specified probability distribution function, the ordinate β_{qi} is found so that the area under the probability curve from 0 to β_{qi} is equal to the chosen random number u_i:

$$P(\beta_i) = \begin{cases} 0.5(\eta_c + 1)\beta_i^{\eta_c}, & \text{if } \beta_i \leq 1; \\ 0.5(\eta_c + 1)\dfrac{1}{\beta_i^{\eta_c+2}}, & \text{otherwise.} \end{cases} \tag{8}$$

Using equation 10, calculate β_{qi} by equating the area under the probability curve equal to u_i as follows:

$$\beta_{qi} = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}}, & \text{if } u_i \leq 0.5; \\ \left(\dfrac{1}{2(1-u_i)}\right)^{\frac{1}{\eta_c+1}}, & \text{otherwise.} \end{cases} \tag{9}$$

In the above equations (8,9), the distribution index η_c is any positive real number. After obtaining β_{qi}, the children solutions are calculated as follows:

$$\begin{aligned} x_i^{(1,t+1)} &= 0.5\left[(1+\beta_{qi})x_i^{(1,t)} + (1-\beta_{qi})x_i^{(2,t)} \right], \\ x_i^{(2,t+1)} &= 0.5\left[(1-\beta_q i)x_i^{(1,t)} + (1+\beta_{qi})x_i^{(2,t)} \right]. \end{aligned} \tag{10}$$

Polynomial mutation

The probability of creating a solution near to the parent is higher than the probability of creating one distant from it. The shape of the probability distribution is directly controlled by an external parameter η_m and the distribution remains unchanged throughout the iterations. Like in the SBX operator, the probability distribution can also be a polynomial function, instead of a normal distribution [12]:

$$y_i^{(1,t+1)} = x_i^{(1,t+1)} + (x_i^{(U)} - x_i^{(L)})\delta_i, \tag{11}$$

where, the parameter δ is calculated from the polynomial probability distribution

$$P(\delta_i) = 0.5(\eta_m + 1)(1 - |\delta|)^{\eta_m},$$

$$\delta_i = \begin{cases} (2r_i)^{\frac{1}{\eta_m-1}} - 1, & \text{if } r_i < 0.5; \\ 1 - [2(1-r_i)]^{\frac{1}{\eta_m+1}}, & \text{if } r_i \geq 0.5. \end{cases} \tag{12}$$

For handling the bounded decision variables, the mutation operator is modified for two regions, i.e. $[x_j^{(L)}, x_i]$ and $[x_i, x_i^{(U)}]$.

Selection

Selection is made using tournament between two individuals. The individual with the lowest front number is selected if the two individuals are from different fronts. The individual with the highest crowding distance [12, pp.247] is selected if they are from the same front, i.e., a higher fitness is assigned to individuals located on a sparsely populated part of the front. In every iteration, the N existing individuals (parents) generate N new individuals (offspring). Both parents and offspring compete with each other for inclusion in the next iteration.

9   References

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