

## Robust Design of Decentralized Power System Stabilizers using Meta-heuristic Optimization Techniques for Multimachine Systems

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**Abstract:** In this paper a classical lead-lag power system stabilizer is used for demonstration. The stabilizer parameters are selected in such a manner to damp the rotor oscillations. The problem of selecting the stabilizer parameters is converted to a simple optimization problem with an eigen value based objective function and it is proposed to employ simulated annealing and particle swarm optimization for solving the optimization problem. The objective function allows the selection of the stabilizer parameters to optimally place the closed-loop eigen values in the left hand side of the complex  $s$ -plane. The single machine connected to infinite bus system and 10-machine 39-bus system are considered for this study. The effectiveness of the stabilizer tuned using the best technique, in enhancing the stability of power system. Stability is confirmed through eigen value analysis and simulation results and suitable heuristic technique will be selected for the best performance of the system.

**Keywords:** Rotor oscillations, Power system stability, Robust control, Simulated Annealing, Particle swarm optimization, Multimachine system.

### 1 Introduction

During changes in operating conditions, oscillations of small magnitude and low frequency often persist for long period of time and in some cases even restrict the power transfer capability. Power system stabilizer (PSS) is designed to damp the low frequency oscillations of power system [1].

PSS is used to damp the generator rotor oscillations by controlling its excitation using auxiliary stabilizing signals. The widely used conventional power system stabilizer (CPSS) is designed using the theory of phase compensation and a lead-lag compensator [7].

An inter connected power system, depending on its size, has hundreds to thousands of modes of oscillation. In the analysis and control of system

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stability, two distinct types of system oscillations are usually recognized. The first type is associated with swinging of units in a generating station with respect to the rest of the power system. Such oscillations are referred to as local plant mode oscillations. The frequencies of these oscillations are typically in the range of 0.8 to 2.0 Hz. The second type of oscillations is associated with the swinging of many machines in one part of the system against machines in the other parts. These are referred to as inter area mode oscillations, and have frequencies in the range of 0.1 to 0.7 Hz. The basic function of PSS is to add damping to both types of system oscillations. Other modes of oscillations controlled by PSS are torsional modes and control modes which are associated with the excitation system and the field circuit [5].

The over all excitation control system is designed so as to maximize the damping of the local plant mode as well as inter area mode oscillations without compromising the stability of other modes and to enhance system transient stability. Rotor speed deviation is input to PSS which results damping torque.

## 2 Problem Formulation

In this section, the eigen value based objective function is used for robust selection of PSS parameters [2], and the optimization problem is solved with the help of simulated annealing and particle swarm optimization.

Consider the problem of determining the parameters of a stabilizer that relatively stabilizes a family of  $N$  plants,

$$\dot{X}(t) = A_k X(t) + B_k U(t), \quad k = 1, 2, 3, \dots, N - (1),$$

where,  $\dot{X}(t) \in R_n$  is the state vector and  $X(t) \in R_m$  is the control vector.

Very often, the closed-loop modes are specified to have some degree of relative stability. In this case closed-loop eigen values are constrained to lie to the left of a vertical line corresponding to a specified damping factor.

A necessary and sufficient condition for the set of plants in equation (1) to be stable is that the eigen values of the closed-loop system should lie in the left-hand side of a vertical line in the complex s-plane. This condition motivates the following approach for determining the parameters of the PSS.

Select the parameters of the PSS to minimize the following objective function:

$$J = \max \left\{ \operatorname{Re}(\lambda_{k,i}) + \beta \right\}, \quad k = 1, 2, 3, \dots, N, \quad i = 1, 2, \dots, n,$$

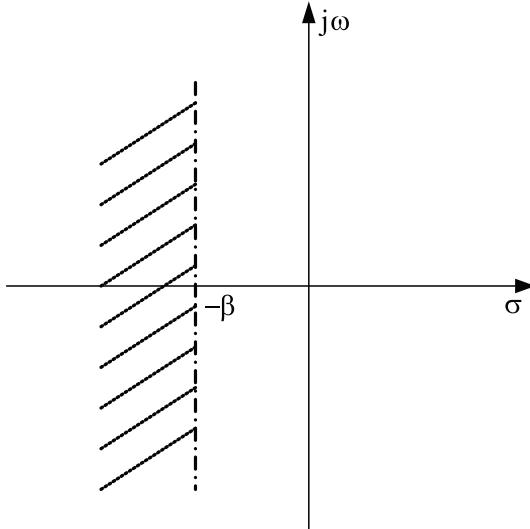
where  $\lambda_{k,i}$  is  $i$ -th closed loop eigen value of the  $k$ -th plant and  $\beta$  is relative stability factor.

The objective function is subject to a set of constraints based on the finite bounds for stabilizer parameters.

In this paper instead of  $N$  number of plants, single-machine-infinite-bus system is considered initially. Then the objective function can be modified as,

$$J = \max \{ \operatorname{Re}(\lambda_k) + \beta \} .$$

The relative stability is determined by the value of  $\beta$  as shown in Fig. 1.



**Fig. 1 – Region in the left-hand side of a vertical plane.**

Plainly, if a solution is found such that  $J < 0$ , then the resulting parameters simultaneously relatively-stabilize the collection of plants. The existence of a solution is verified numerically [2] by minimizing  $J$ .

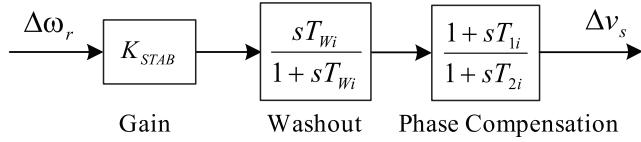
### 3 System Model

Damping torque is produced to overcome rotor oscillations. The action of a PSS is to extend the angular stability limits of a power system by providing supplemental damping through the generator excitation [3].

Controller is designed to compensate lag between exciter input and electrical torque.

$$\frac{\Delta T_{PSS}}{\Delta V_s} = K \angle -\theta .$$

The amount of damping introduced depends on the gain of PSS transfer function at that particular frequency of oscillation. The conventional lead-lag power system stabilizer is shown in Fig. 2.



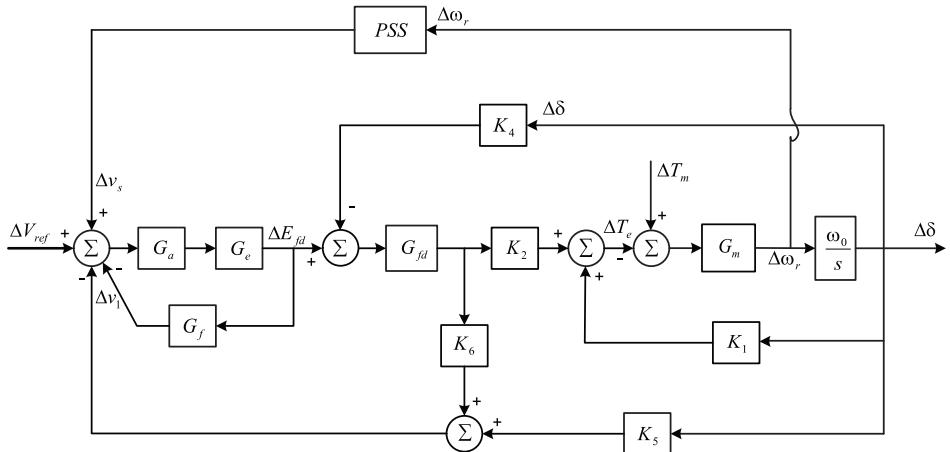
**Fig. 2 – Lead-Lag Power System Stabilizer.**

In this paper single machine connected to infinite bus system as well as 10-machine 39-bus system is considered [10]. The supplementary stabilizing signal considered is one proportional to speed [6]. A widely used conventional PSS is considered throughout the study.

The transfer function of PSS with single phase compensation block is

$$\frac{\Delta V_S}{\Delta \omega_r} = K_{STAB} \frac{sT_{Wi}}{1+sT_{Wi}} \frac{1+sT_{Li}}{1+sT_{2i}}.$$

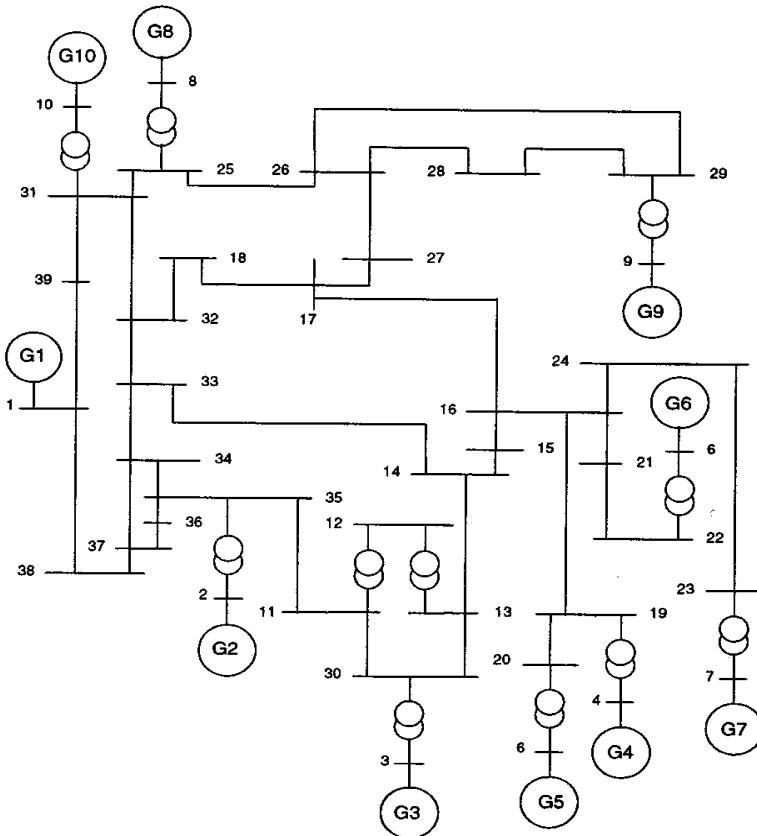
The first term is stabilizer gain. The second term is washout term with a time lag  $T_W$ . The third term is a lead compensation [3] to improve the phase lag through the system. The numerical values of  $T_W$ ,  $T_2$  and SMIB system data are given in Section 8.1. The remaining parameters namely  $K_{STAB}$  and  $T_1$  are assumed to be adjustable parameters. The optimization problem is selection of these PSS parameters easily and accurately. The optimization problem can be solved using the simulated annealing as well as particle swarm optimization. The SA algorithm is explained in Section 8.1 and PSO algorithm is explained in Section 8.1.



**Fig. 3 – Block diagram of AVR with PSS.**

For a given operating point, the power system is linearized around the operating point, the eigen values of the closed-loop system are computed, and the objective function is evaluated. It is worth mentioning that only the system electromechanical modes are incorporated in the objective function. The bounds on the parameters used in the SA are given in Section 8.1. The block diagram of AVR with PSS is shown in Fig. 3.

The subscript ‘ $i$ ’ in PSS transfer function indicates the values for  $i$ -th machine. The single line diagram of 10-machine 39-bus system is shown in Fig. 4 and the system data can be found in [2].



**Fig. 4 – Single line diagram of 10-Machine, 39-Bus System.**

#### 4 Simulated Annealing

Simulated Annealing [8] is an optimization technique that simulates the physical annealing process in the field of combinatorial optimization. Annealing is the physical process which involves heating up of a solid until it melts,

followed by slow cooling down by decreasing temperature. Thermal equilibrium at any temperature ( $T$ ) of solid is maintained as constant for a period of time which is given by Boltzmann distribution. It gives the probability of the solid being in an energy state  $i$  with energy  $E_i$  at temperature  $T$  as,

$$P_i = k e^{\left(\frac{-E_i}{T}\right)},$$

where  $k$  is Boltzmann's constant. The analogy between a physical annealing process and a combinatorial optimization problem is based on the following

Solutions obtained for an optimization problem are equivalent to the configurations of a physical system.

The cost of a solution is equivalent to the energy of a configuration.

A parameter  $C_p$  is introduced to control the temperature  $T$ .

## 5 Particle Swarm Optimization

Similar to evolutionary algorithms, the PSO technique conducts searches using a population of particles, corresponding to individuals. In a PSO system [11], particles change their positions by flying around in a multidimensional search space until a relatively changed position has been encountered, or until computational limitations are exceeded.

The following are the advantages of PSO [9] over other traditional optimization techniques.

PSO is a population based search algorithm. This property ensures it to be less susceptible to getting trapped on local minima.

PSO uses payoff (performance index or objective function) information to guide the search in the problem space.

PSO uses probabilistic transition rules and not deterministic rules. Hence, PSO is a kind of stochastic optimization algorithm that can search a complicated and uncertain area.

## 6 Simulation Results

In this part of the study, a single machine is connected to infinite bus through a transmission line, and operating at different loading conditions [4], is considered. The linearized model of this system, is considered along with voltage regulator and exciter [10]. The constants  $K_1$  to  $K_6$ , with the exception of  $K_3$  are dependent on the actual real power ( $P$ ) and reactive power ( $Q$ ) loading as well as the excitation levels in the machine. The constant  $K_3$  is a function of the ratio of impedances.

The operating points are selected based on the different loading conditions. The simultaneous damping enhancement of the system is demonstrated by considering five different loading conditions.

The operating points were selected randomly as follows:

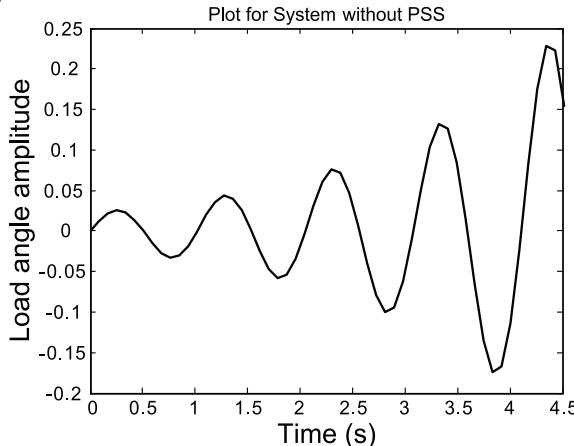
$$(P_o, Q_o) = (0.9, 0.3), (0.8, -0.1), (0.5, 0.5), (0.6, -0.2), (1.0, 0.6).$$

The eigen values are found by transferring the transfer function of the system into state space model.

The eigen values of the system for the five operating points with out PSS are:

1.  $0.4981 \pm 6.6288i, -33.6805, -17.3597,$
2.  $0.7513 \pm 7.3702i, -11.5526, -39.9942,$
3.  $0.0283 \pm 5.3580i, -25.0504 \pm 9.1822i,$
4.  $0.1936 \pm 6.9157i, -10.7786, -39.6528,$
5.  $0.5410 \pm 6.1171i, -21.6341, -29.4919.$

From the eigen values it is clear that the system is unstable due to their location in the right half of the  $s$ -plane. The response of the system without PSS is shown in Fig. 5.



**Fig. 5 – Response of the system without PSS.**

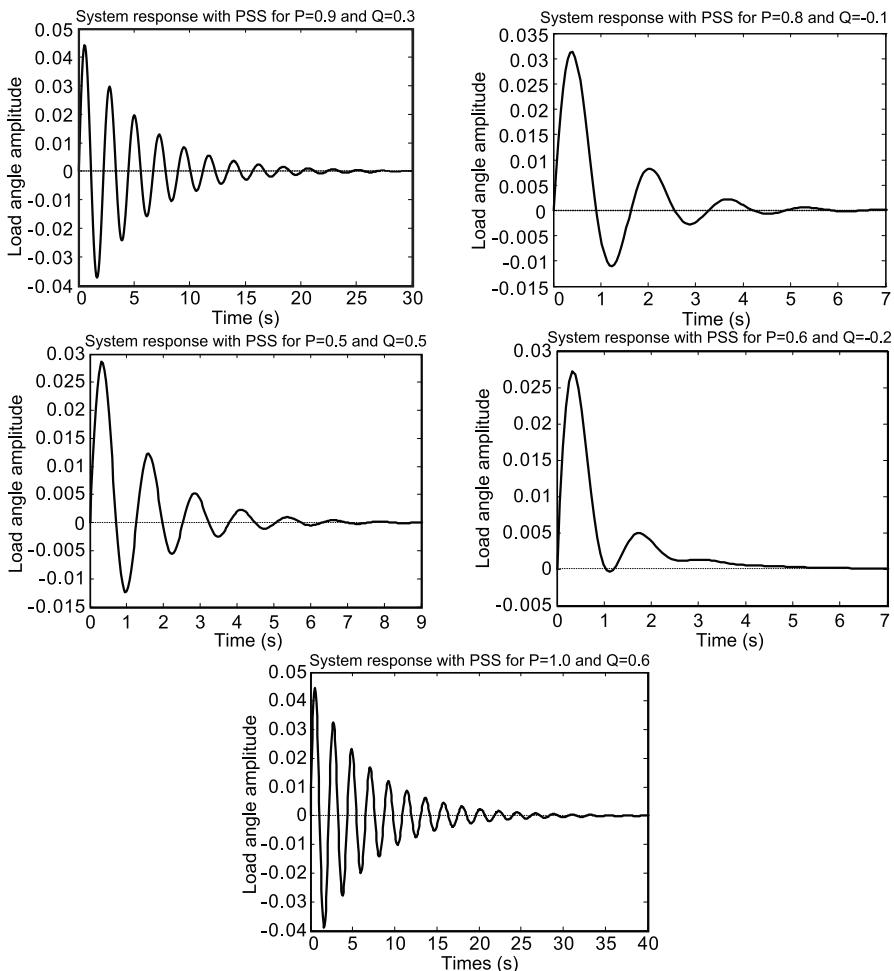
For the system to be stable the real part of eigen value should be located in the in the left hand side of s-plane.

## 6.1 Results for simulated annealing

Simulated annealing is used to optimize the objective function  $J$  for five different kind of operating conditions to shift the eigen values to the left vertical line.

The eigen values of the five systems with PSS are

1.  $-46.3181, -16.8295 \pm 43.2325i, -0.1883 \pm 2.8091i, -0.7077,$
2.  $-45.7209, -16.4755 \pm 28.1318i, -0.8262 \pm 3.8261i, -0.7372,$
3.  $-35.6015, -21.6986 \pm 13.9605i, -0.6626 \pm 4.9864i, -0.7376,$
4.  $-44.0326, -16.5329 \pm 18.3720i, -1.6023 \pm 4.3745i, -0.7585,$
5.  $-45.6105, -17.2183 \pm 40.4834i, -0.1521 \pm 2.8783i, -0.7102.$



**Fig. 6 – Response of the system with PSS using Simulated Annealing for SMIB.**

From Fig. 6 it is obvious that the rotor oscillations are damped and the above results shows that all the eigen values of the system were located in the left half of the  $s$ -plane to make the system stable.

## 6.2 Results for particle swarm optimization

The objective function  $J$  is optimized with the PSO to shift the electromechanical mode of each of the five systems to the left of the vertical line defined by  $\beta = -2.2$ .

The eigen values of the five systems, with PSS are:

1.  $-34.1670 \pm 5.0796i, -9.9855, -0.9887 \pm 7.7731i, -0.7644,$
2.  $-37.4972, -35.6751, -0.5533 \pm 9.3491i, -6.0033, -0.7792,$
3.  $-29.0306 \pm 7.0224i, -20.4992, -0.8755 \pm 5.6109i, -0.7500,$
4.  $-36.4206 \pm 0.9044i, -1.3020 \pm 9.2146i, -4.8165, -0.7998,$
5.  $-33.0536 \pm 5.8900i, -12.5555, -0.8183 \pm 6.9006i, -0.7620.$

All the eigen values of the system were located in the left half of the  $s$ -plane. So the system is stable.

Fig. 7 shows response of the system with power system stabilizer for various operating conditions whereas their parameters were tuned by using Particle Swarm Optimization (PSO). It is indicating simultaneous improvement in the response of the five systems.

The above two techniques can be compared as shown in **Table 1**.

**Table 1**  
*Comparison between Simulated annealing and Particle Swarm Optimization.*

Loading Condition ( Po , Qo )	Simulated Annealing		Particle Swarm Optimization	
	Settling Time in Sec.	Peak Amp.	Settling Time in Sec.	Peak Amp.
0.9, 0.3	20.8	0.0443	4.51	0.0236
0.8, -0.1	4.6	0.0313	6.32	0.0211
0.5, 0.5	5.58	0.0287	4.92	0.0275
0.6, -0.2	3.96	0.0272	5.06	0.0218
1.0, 0.6	4.09	0.0324	4.99	0.026

From the above comparison the simulated annealing is having higher settling time, higher peak amplitude and higher computational time than Particle swarm optimization. So tuning of PSS parameters by using PSO is more optimal than SA.

In this section 10-machine 39-bus system as shown in Fig. 4 is considered. The system will have PSSs in all the generators except  $G_1$  for illustration and comparison purpose. To demonstrate the effectiveness of the proposed PSSs over a wide range of operating conditions, the following disturbances are considered:

A three phase fault disturbance at bus 29 at the end of line 26-29. The fault is cleared by tripping the line 26-29 with successful reclosure after 1.0 seconds.

A three phase fault disturbance at bus 14 at the end of line 14-15. The fault is cleared by tripping the line 14-15.

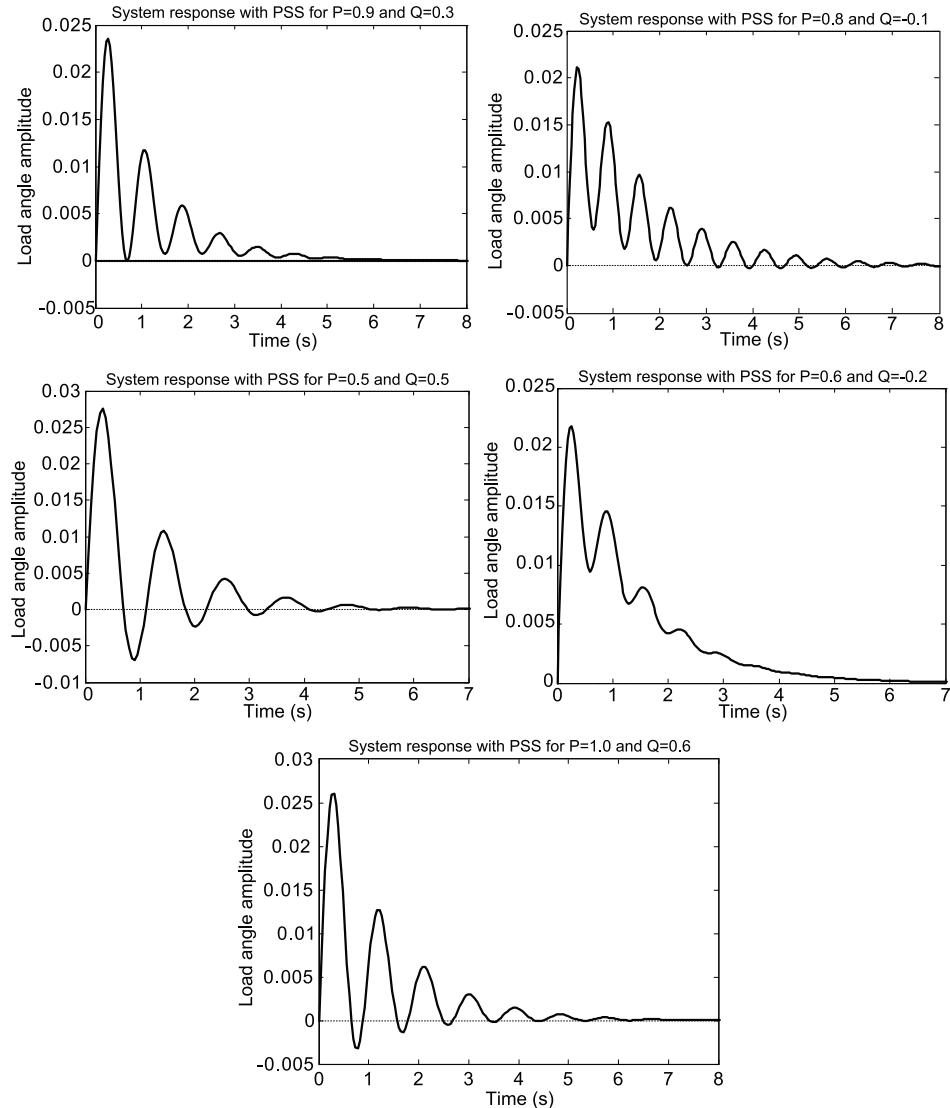
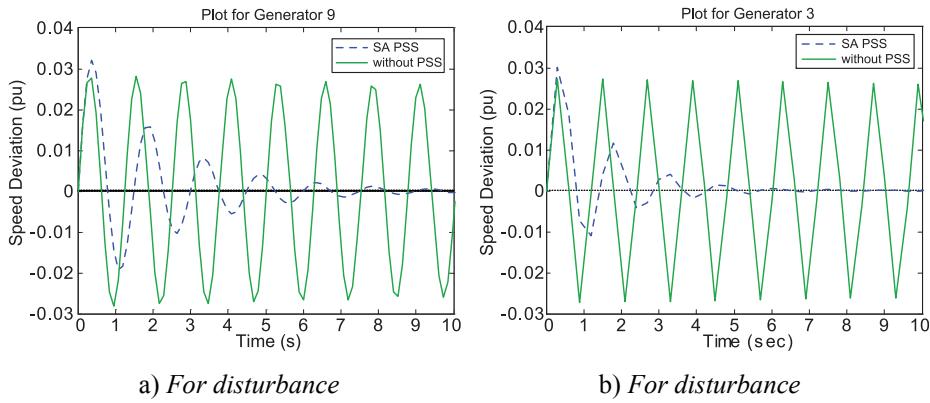


Fig. 7 – Response of the system with PSS using Particle Swarm Optimization for SMIB.

### 6.3 Results for Simulated Annealing

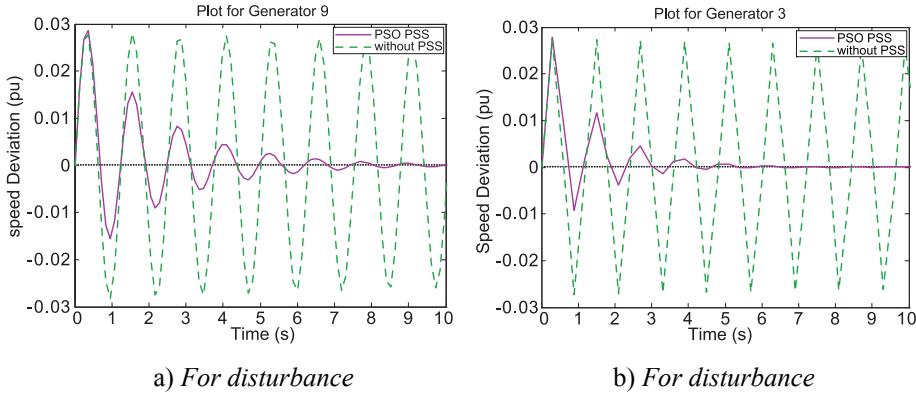
For the above disturbances, the speed deviation of  $G_9$  and  $G_3$ , as the nearest generator to the fault location is shown in Fig. 8. It is clear that the system response with the proposed SA is stable.



**Fig. 8 – Response of 10-machine 39-bus system using SA.**

## 6.4 Results for Particle Swarm Optimization:

For disturbances, the speed deviation of the generators  $G_9$  and  $G_3$  are shown in Fig. 9. Compare to Figs. 8 and 9; it is clear that the system response with the proposed PSO is having lesser oscillatory response than SA.



**Fig. 9 – Response of 10-machine 39-bus system using SA.**

## 7 Conclusion

The use of Simulated Annealing and PSO to design robust power system stabilizers for power systems working at various operating conditions are investigated in this paper for both SMIB system and 10-machine 39-bus system. The problem of selecting the PSS parameters, which simultaneously improve the damping at various operating conditions, is converted to an optimization problem with an eigen value-based objective function which is solved by both Simulated Annealing and PSO.

The objective function presented in this paper is allowing the robust selection of stabilizer parameters that will optimally place the closed-loop eigen values in the left hand side of a vertical line in complex  $s$ -plane. By comparing the above two meta-heuristic optimization techniques, it is found that Particle Swarm Optimization is better than Simulated Annealing in tuning the parameters of the Power system stabilizer, to reduce intra and inter area rotor oscillations over a wide range of operating conditions. The PSOPSS is having more practical significance than SAPSS.

## 8 Appendix

### 8.1 System Data

Single-Machine-Infinite-Bus System:

$$G_m = \frac{1}{2H_s + K_D} ; \quad G_a = \frac{1}{sT_A + 1} ; \quad G_e = \frac{1}{sT_E + 1} ;$$

$$G_f = \frac{sK_f}{1 + sT_F} ; \quad G_{fd} = \frac{K_3}{1 + sT'_{do}K_3} .$$

The system data are as follows:

Machine (p.u)

$$x_d = 1.7 ; \quad x'_d = 0.254 ; \quad x_q = 1.64 ; \quad \omega_0 = 120\pi \text{ rad/s} ;$$

$$T'_{do} = 5.9 \text{ s} ; \quad K_D = 0 ; \quad H = 2.37 \text{ s} .$$

Transmission Line (p.u)

$$r_e = 0.02 ; \quad x_e = 0.4 .$$

Exciter and Stabilizer

$$K_A = 400 ; \quad T_A = 0.05 \text{ s} ; \quad K_F = 0.025 ; \quad T_F = 1.0 \text{ s} ;$$

$$K_E = -0.17 ; \quad T_E = 0.95 \text{ s} ; \quad T_W = 10 \text{ s} ; \quad T_2 = 0.0227 \text{ s} .$$

Bounds for the stabilizer adjustable gain and time constants are [0.01, 10] and [0.03, 1.0], respectively.

## 8.2 Simulated Annealing Algorithm

Step (1): Set the initial value of  $C_{p0}$  and randomly generate an initial solution  $x_{\text{initial}}$  and calculate its objective function. Set this solution as the current solution as well as best solution, i.e.  $x_{\text{initial}} = x_{\text{current}} = x_{\text{best}}$ .

Step (2): Randomly generate an  $n_l$  of trial solutions in the neighborhood of the current solution.

Step (3): Check acceptance criterion of these trial solutions and calculate the acceptance ratio. If the acceptance ratio is close to 1 go to step 4, else set  $C_{p0} = \alpha C_{p0}$ ,  $\alpha > 1$ , and go back to step 2.

Step (4): Set the chain counter  $k_{\text{ch}} = 0$ .

Step (5): Generate a trial solution  $x_{\text{trial}}$ . If  $x_{\text{trial}}$  satisfies the acceptance criterion, set  $x_{\text{current}} = x_{\text{trial}}$ ,  $J(x_{\text{current}}) = J(x_{\text{trial}})$ , and go to step 6; else go to step 6.

Step (6): Check the equilibrium condition. If it is satisfied go to step 7, else go to step 5.

Step (7): Check the stopping criteria. If one of them is satisfied then stop; else set  $k_{\text{ch}} = k_{\text{ch}} + 1$  and  $C_p = \mu C_p$ ,  $\mu < 1$ , and go back to step 5.

## 8.3 Particle Swarm Optimization Algorithm

Step (1): Set the time counter  $t = 0$  and generate random  $n$  particles,  $\{X_j(0), j = 1, 2, \dots, n\}$ , where  $X_j(0) = [x_{j,1}(0), x_{j,2}(0), \dots, x_{j,m}(0)]$ .  $x_{j,k}(0)$  is generated by randomly selecting a value with uniform probability over the  $k$ -th optimized parameter search space  $[x_k^{\min}, x_k^{\max}]$ .

Generate randomly initial velocities of all particles,  $\{V_j(0), j = 1, 2, \dots, n\}$ , where  $V_j(0) = [v_{j,1}(0), v_{j,2}(0), \dots, v_{j,m}(0)]$ .  $v_{j,k}(0)$  is generated by randomly selecting a value with uniform probability over the  $k$ -th optimized parameter search space  $[-v_k^{\max}, v_k^{\max}]$ .

Each particle in the initial population is evaluated using the objective function,  $J$ . For each particle, set  $X_j^*(0) = X_j(0)$  and  $J_j^* = J_j$ ,  $j = 1, 2, \dots, n$ . Search for the best value of objective function  $J_{\text{best}}$ . Set the particle associated

with  $J_{\text{best}}$  as the global best,  $X^{**}(0)$ , with an objective function of  $J^{**}$ . Set the initial value of the inertia weight  $w(0)$ .

Step (2): Update the time counter  $t = t + 1$ .

Step (3): Update the inertia weight  $w(t) = \alpha \cdot w(t - 1)$ .

Step (4): Using the global best and individual best, the  $j$ -th particle velocity in the  $k$ th dimension is updated according to the following equation:

$$v_{j,k}(t) = w(t)v_{j,k}(t - 1) + \\ + c_1 r_1(x_{j,k}^*(t - 1) - x_{j,k}(t - 1) + c_2 r_2(x_{j,k}^{**}(t - 1) - x_{j,k}(t - 1)),$$

where  $c_1$  and  $c_2$  are positive constants and  $r_1$  and  $r_2$  are uniformly distributed random numbers in  $[0,1]$ .

Step (5): Based on the updated velocities, each particle changes its position according to the following equation:

$$x_{j,k}(t) = v_{j,k}(t) + x_{j,k}(t - 1).$$

Step (6): Each particle is evaluated according to the updated position.  $J_j < J_j^*$ ,  $j = 1, 2, \dots, n$ , then update individual best as  $X_j^*(t) = X_j(t)$  and  $J_j^* = J_j$ , and go to step 7; else go to step 7.

Step (7): Search for the minimum value  $J_{\min}$  among  $J_j^*$ , where  $\min$  is the index of the particle with minimum objective function value, i.e.,  $\min \in \{j; j = 1, 2, \dots, n\}$ . If  $J_{\min} < J^{**}$  then update global best as  $X^{**} = X_{\min}(t)$ , and  $J^{**} = J_{\min}$  and go to step 8; else go to step 8.

Step (8): If one of the stopping criteria is satisfied, then stop, or else go to step 2.

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