

# Integrated Fuzzy Logic Based Intelligent Control of Three Tank System

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**Abstract:** An attempt has been made in this paper to analyze the efficiency of an intelligent fuzzy controller (IFLC) on three tank level system. Analysis of the effects studied through computer simulation using Matlab/Simulink toolbox show that the application of IFLC appears to be encouraging in the sense that it is robust in disturbance rejection under various conditions.

**Keywords:** Integrated fuzzy logic control, Interacting system, Non interacting system.

## 1 Introduction

The traditional control, which includes the classical feedback control, modern control theory and large-scale control system theory, has encountered many difficulties in its applications. The design and analysis of traditional control systems are based on their precise mathematical models, which are usually very difficult to achieve owing to the complexity, nonlinearity, time-varying and incomplete characteristics of the existing practical systems. One of the most effective ways to solve the problem is to use the technique of intelligent control system, or hybrid methodology of the traditional and intelligent control techniques.

The block diagram of classical feedback control system (FBC) is shown in Fig. 1. The feedback controller cannot anticipate and prevent errors, it can only initiate corrective action after an error has already developed [1]. It cannot give close control when there is a large delay in the process. So, one of the remedy for the problem is intelligent fuzzy control system [2]. Unlike a feedback control system, an intelligent fuzzy control system was developed using expert

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knowledge and experience gained about the process. The block diagram of integrated intelligent fuzzy logic controller is shown in Fig. 2. The conventional feedback controller is not replaced by the intelligent fuzzy controller. The intelligent fuzzy controller design consists of three stages: Fuzzification stage, Decision making logic and Defuzzification stage.

In this paper an attempt has been made to analyze the efficiency of an integrated intelligent fuzzy control using three tank level control system and the effects are studied through computer simulation using Matlab/Simulink toolbox [3,7,8]. The simulation results of IFLC are compared with classical control method.

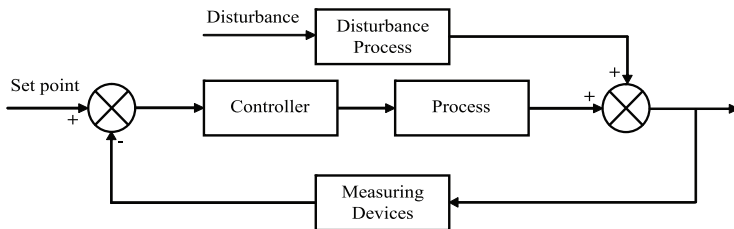


Fig. 1 – Block diagram of Feedback Control System.

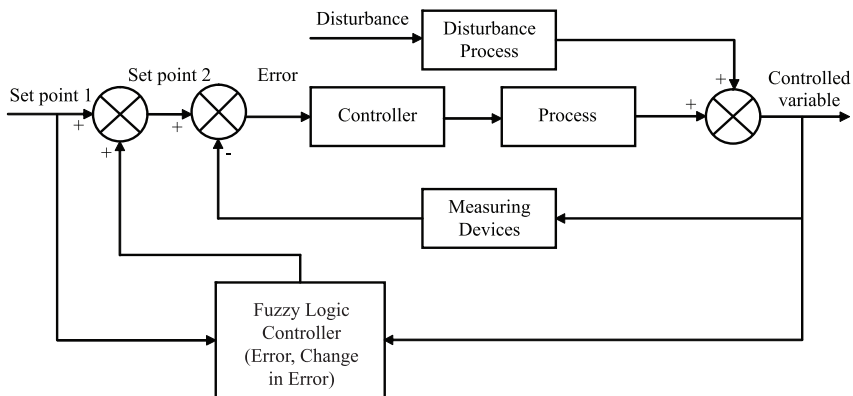
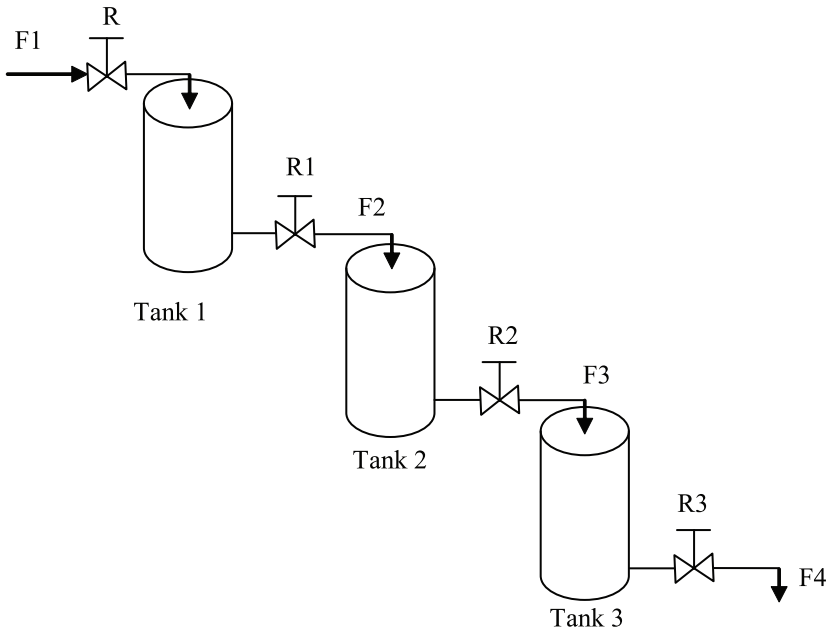


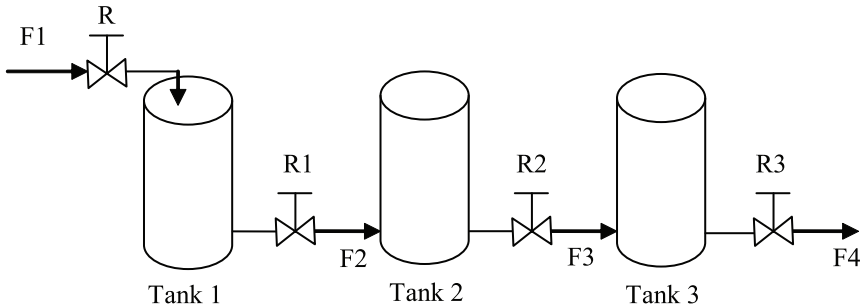
Fig. 2 – Block diagram of Integrated Intelligent Fuzzy Control System.

## 2 Mathematical Modeling of Three Tank System

Figs. 3, 4a, 4b and 4c show general model of noninteracting and interacting connection of a three tank system.



**Fig. 3 – Three Tank Noninteracting System.**



**Fig. 4a – Three Tank Interacting System: CASE-I.**

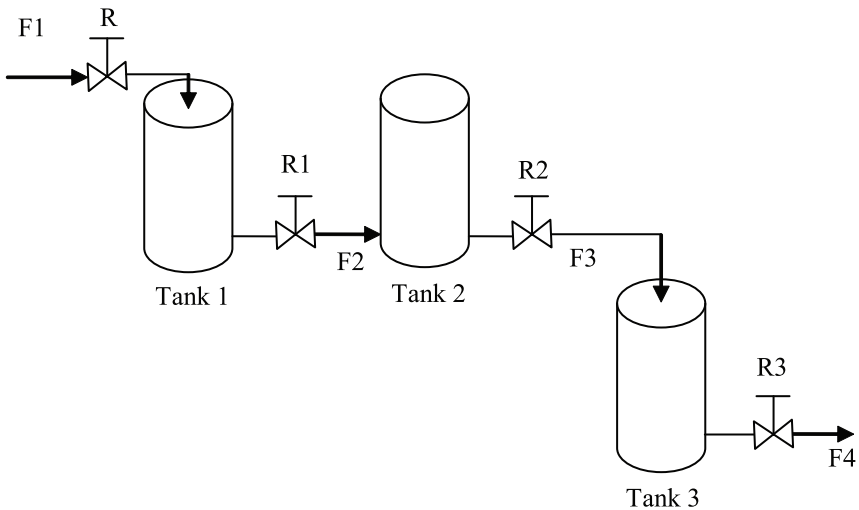
## 2.1 Noninteracting system

The basic model equation of noninteracting three tank system is given by

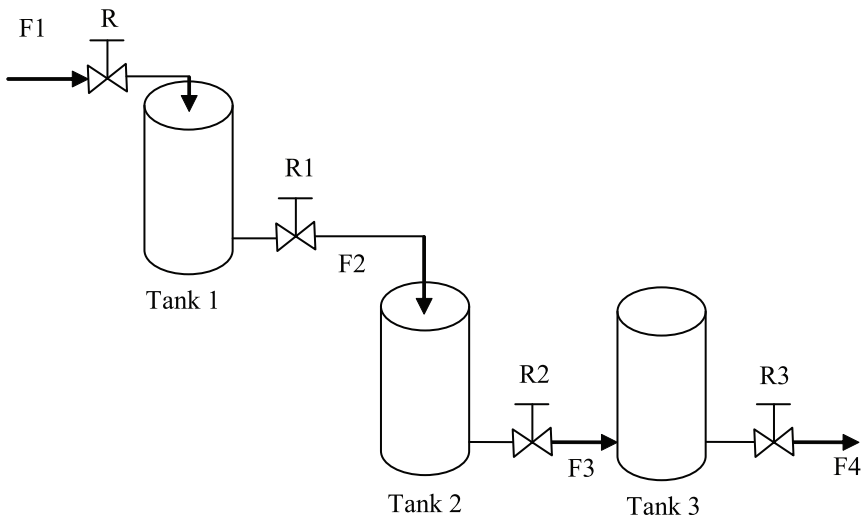
**Tank1:**

$$F_1(t) - F_2(t) = A_1 dh_1 / dt , \quad (1)$$

where  $F_1(t)$  is tank1 inflowing liquid ( $m^3/s$ ),  $F_2(t)$  tank1 outflowing liquid ( $m^3/s$ ),  $A_1$  area of the tank1 ( $m^2$ ) and  $h_1$  liquid level in tank1 (m).



**Fig. 4b** – Three Tank Interacting System: CASE –II.



**Fig. 4c** – Three Tank Interacting System: CASE –III.

**Tank 2:**

$$F_2(t) - F_3(t) = A_2 \frac{dh_2}{dt}, \quad (2)$$

where  $F_2(t)$  tank2 inflowing liquid ( $\text{m}^3/\text{s}$ ),  $F_3(t)$  tank2 outflowing liquid ( $\text{m}^3/\text{s}$ ),  $A_2$  area of the tank2 ( $\text{m}^2$ ) and  $h_2$  liquid level in tank2 (m).

**Tank 3:**

$$F_3(t) - F_4(t) = A_3 \, d h_3 / dt, \quad (3)$$

where  $F_3(t)$  tank3 inflowing liquid ( $\text{m}^3/\text{s}$ ),  $F_4(t)$  tank3 outflowing liquid ( $\text{m}^3/\text{s}$ ),  $A_3$  area of the tank3 ( $\text{m}^2$ ) and  $h_3$  liquid level in tank3 (m).

$$F_2(t) = h_1 / R_1, \quad (4)$$

$$F_3(t) = h_2 / R_2, \quad (5)$$

$$F_4(t) = h_3 / R_3, \quad (6)$$

where  $R_1$ ,  $R_2$  and  $R_3$  linear resistance of tank1, tank 2 and tank3 ( $\text{m}/(\text{m}^3/\text{s})$ )

The overall transfer function of noninteracting three tank system is given by

$$H_3(s) / F_1(s) = R_3 / (\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1). \quad (7)$$

By considering  $A_1 = A_2 = 1 \text{ m}^2$ ;  $A_3 = 0.5 \text{ m}^2$  and  $R_1 = 2$  ( $\text{m}/(\text{m}^3/\text{s})$ );  $R_2 = 2$  ( $\text{m}/(\text{m}^3/\text{s})$ );  $R_3 = 4$  ( $\text{m}/(\text{m}^3/\text{s})$ ).

**Process I:**

$$H_3(s) / F_1(s) = 4 / (2s + 1)(2s + 1)(2s + 1). \quad (8)$$

## 2.2 Interacting system

**Case (i):** Tank1, Tank2 and Tank3 are connected in series in Fig. 4a. The basic model equations of the interacting system is given by:

**Tank1:**

$$F_1(t) - F_2(t) = A_1 \, d h_1 / dt, \quad (9)$$

**Tank 2:**

$$F_2(t) - F_3(t) = A_2 \, d h_2 / dt, \quad (10)$$

**Tank 3:**

$$F_3(t) - F_4(t) = A_3 \, d h_3 / dt, \quad (11)$$

$$F_2(t) = (h_1 - h_2) / R_1, \quad (12)$$

$$F_3(t) = (h_2 - h_3) / R_2, \quad (13)$$

$$F_4(t) = h_3 / R_3, \quad (14)$$

By considering  $A_1 = A_2 = 1 \text{ m}^2$ ;  $A_3 = 0.5 \text{ m}^2$  and  $R_1 = 2$  ( $\text{m}/(\text{m}^3/\text{s})$ );  $R_2 = 2$  ( $\text{m}/(\text{m}^3/\text{s})$ );  $R_3 = 2$  ( $\text{m}/(\text{m}^3/\text{s})$ ).

The overall transfer function of the interacting three tank system is given by **Process II:**

$$\begin{aligned} H_3(s) / F_1(s) &= R_1 R_2 R_3 / [(A_1 R_1 s + 1)(A_2 R_1 R_2 s + R_2 + R_1) - R_2] \cdot \\ &\quad \cdot (A_2 R_2 R_3 s + R_2 + R_3) - R_1 R_3 (A_1 R_1 s + 1), \quad (15) \\ H_3(s) / F_1(s) &= 4 / (8s^3 + 12s^2 + 6s + 1). \end{aligned}$$

**Case (ii):**

Tank1 and Tank 2 interacting system, Tank3 noninteracting system is shown in Fig. 4b.

By considering  $A_1 = A_2 = 1\text{m}^2$ ;  $A_3 = 0.5\text{m}^2$  and  $R_1 = 2 \text{ (m/(m}^3\text{/s))}$ ;  $R_2 = 2 \text{ (m/(m}^3\text{/s))}$ ;  $R_3 = 2 \text{ (m/(m}^3\text{/s))}$ , the overall transfer function of the system is derived as **Process III:**

$$\begin{aligned} H_3(s) / F_1(s) &= R_1 R_2 R_3 / [(A_1 R_1 s + 1)(A_2 R_1 R_2 s + R_2 + R_1) - R_2] (A_3 R_3 s + 1), \quad (16) \\ H_3(s) / F_1(s) &= 8 / (16s^3 + 32s^2 + 16s + 2). \end{aligned}$$

**Case (iii):**

Tank1 noninteracting connection, Tank2 and Tank3 interacting connection is shown in Fig. 4c.

By considering  $A_1 = A_2 = 1\text{m}^2$ ;  $A_3 = 0.5\text{m}^2$  and  $R_1 = 2 \text{ (m/(m}^3\text{/s))}$ ;  $R_2 = 2 \text{ (m/(m}^3\text{/s))}$ ;  $R_3 = 2 \text{ (m/(m}^3\text{/s))}$ , the overall transfer function of the system is derived by **Process IV:**

$$\begin{aligned} H_3(s) / F_1(s) &= R_1 R_2 R_3 / [(A_2 R_2 s + 1)(A_3 R_2 R_3 s + R_2 + R_3) - R_3] (A_1 R_1 s + 1), \quad (17) \\ H_3(s) / F_1(s) &= 8 / (16s^3 + 32s^2 + 12s). \end{aligned}$$

In this section the mathematical model of three tank system under noninteracting and interacting conditions have been derived. The response of these transfer functions was analyzed using Matlab/Simulink tool box.

### 3 Design of PID Controller for Three Tank System

A typical block diagram of feedback control system is shown in Fig. 1. The output of the process is measured and its value is compared with the current set point to generate the error signal. The controller acts upon this error to generate a corrective action. The controller output and the error can be related by the following ways:

- (i) the controller output is proportional to the error;
- (ii) the controller output proportional to the integral of the error;
- (iii) the controller output proportional the derivative of the error.

In general, the controller output is related the linear combination of all the three actions. This action is called Proportional–Integral–Derivative (PID) control action. The design of controller consists of the following steps:

- (i) selection of the mode of the controller such as P, PI, PD, and PID;
- (ii) specification of the value of parameters associated with the selected controller.

In this paper Zeigler-Nichols (Z-N) tuning method [4] is used to find the controller parameters. The controller parameter for different arrangements of three tank system is provided in the **Table 1**. The servo and regulatory responses of the noninteracting and interacting three tank system is obtained and analyzed under P, PI, and PID controllers.

**Table 1**  
*Ziegler–Nichols Setting.*

		Non Interacting System	Interacting system: CASE I	Interacting system: CASE II	Interacting system: CASE III
P Controller	Kc	1	5.875	1.875	1.5
PI Controller	Kc	0.9	5.2875	1.6875	1.35
	Ti	5.8	3.75	5	6
PID Controller	Kc	1.2	7.05	2.25	1.8
	Ti	3.5	2.25	3	3.6
	Td	0.875	0.565	0.75	0.9

#### 4 Design of Integrated Fuzzy Logic Controller

The conventional PID controller cannot anticipate and prevent errors as it is insensitive to modeling errors. The feedback control is the basic technique to compensate the load disturbance entering the system. Feedback control has the potential to eliminate the effects with several drawbacks such as:

- it rejects load disturbance after it enters into the system,
- it cannot give good control when large delay is present.

In an attempt to minimize such drawbacks, an intelligent fuzzy logic based controller is augmented to the existing feedback controller and the effects are studied through computer simulation. The block diagram of the integrated intelligent fuzzy logic control system is shown in Fig. 2. The main advantage of this configuration is that it can improve the performance of the existing system without modifying the hardware components. This type of control system can be applied to all kind of processes. The development of fuzzy logic control consists of the following steps:

1. specify the range of controlled variable and manipulated variables;
2. divide these ranges into fuzzy sets and attach linguistic labels which can be used to describe them;
3. determine the rules (rule base), which relate the manipulated variable and controlled variable, to specify control action;
4. application of a suitable defuzzification method.

The number of necessary fuzzy sets and their ranges were designed based upon the experience gained on the process. The standard fuzzy set consists of three stages: Fuzzification, Decision- Making Logic and Defuzzification [5].

### 4.1 Fuzzification stage

This stage converts a crisp number into a fuzzy value within a universe of discourse. The triangular membership functions with seven linguistic values for error and change in error is used and is shown in Figs. 5a and 5b. The linguistic values are NB(Negative Big), NM(Negative Medium), NS(Negative Small), ZO(Zero), PS(Positive Small), PM(Positive Medium), PB(Positive Big).

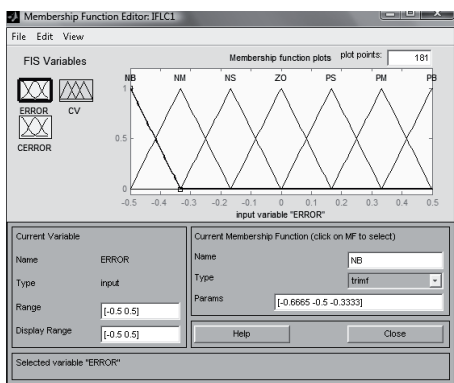


Fig. 5a – Membership functions for Error.

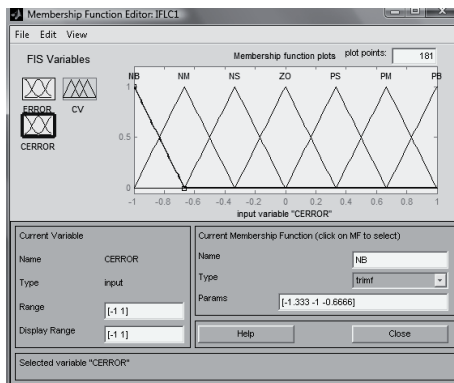


Fig. 5b – Membership function for Change in Error.

### 4.2 Decision Making Stage

This stage consists of fuzzy control rules which decide how the fuzzy logic control works. This stage is the core of the fuzzy control and is constructed from expert knowledge and experience. Based on the knowledge gained by analyzing the feedback control system decision making logic is given in Table 2 where 49 rules are used. The fuzzy logic control rule will be of the following type:

**IF (condition) AND (condition) THEN (action).**



The rules can be interpreted as follows and then similarly other rules can be interpreted in the same way.

IF error is NB AND change in error is NB THEN Control action is NB.

IF error is NB AND change in error is NM THEN Control action is NB.

**Table 2**

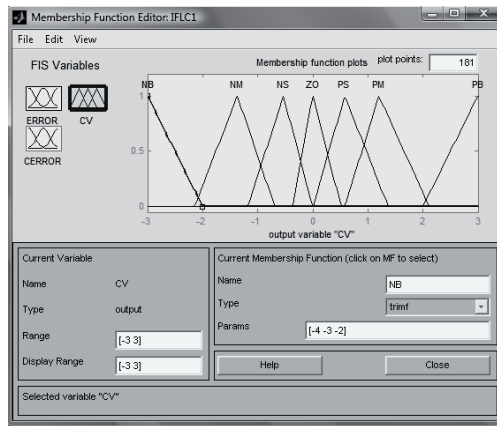
*Integrated Fuzzy Logic Decision Making Logic.*

CE \ E	E	NB	NM	NS	ZO	PS	PM	PB
	CO							
NB	NB	NB	NB	NB	NM	NS	NS	ZO
NM	NB	NB	NB	NM	NS	NS	ZO	PS
NS	NB	NM	NS	NS	NS	ZO	PS	PM
ZO	NM	NM	NS	NS	ZO	PS	PM	PM
PS	NM	NS	ZO	PS	PS	PM	PM	PB
PM	NS	ZO	PS	PS	PM	PB	PB	PB
PB	ZO	PS	PS	PM	PB	PB	PB	PB

E: Error; CE: Change in Error; CO: Controller Output.

### 4.3 Defuzzification Stage

It converts fuzzy value into crisp value. In this study centre of area (COA) method [6] is used. The triangular shaped membership function with seven linguistic values is used and it is shown in Fig. 5c. The range of error, change in error and the controller output are made on the basis of practical experience.



**Fig. 5c** – Membership function for Output.

## 5 Simulation Results

The study of feedback control system and integrated intelligent fuzzy control system is carried out on the three tank system under noninteracting and interacting conditions. The comparison of responses carried out both qualitatively and quantitatively and the results were presented in **Table 3**. The control objective of the three tank system is to control the level of Tank3 under various connections like noninteracting and interacting. The value of controller parameters for the noninteracting and interacting three tank system is obtained using Ziegler–Nichols method and is shown in **Table 1**.

**Table 3**  
*Comparison of performance of Three Tank Systems.*

NONINTERACTING SYSTEM				
Figure	Control Scheme	IAE	Peak Overshoot	Settling Time
Fig. 6	P	42.67	1.25	52
	PI	17.68	1.6	92
	PID	7.28	1.5	25
Fig. 7	IFLC	1.25	1.15	15
Fig. 8	P	42.67	1.2	42
	PI	17.68	1.6	100
	PID	7.28	1.4	32
Fig. 9	P	27.66	1.4	60
	PI	11.64	1.6	90
	PID	7.99	1.5	40
Fig. 10	P	10.7	1.6	72
	PI	9.3	1.9	120
	PID	6.5	1.7	42
Fig. 11	IFLC	1.5	1.1	30
Fig. 12	IFLC	1.75	1.01	25
Fig. 13	IFLC	1.55	1.12	40

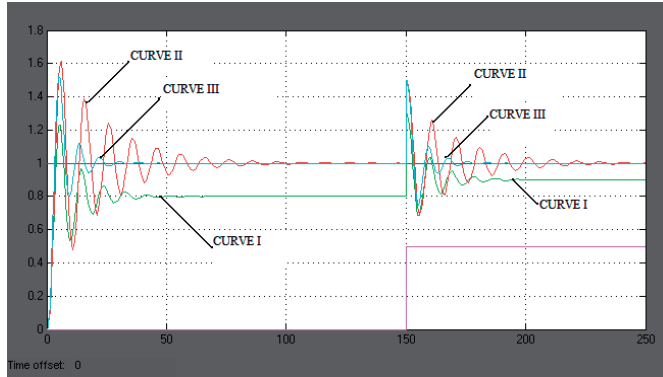
The servo response of the system to a unit step change in of the set point is shown in Fig. 6. The regulatory response of the system for unit step change in the load is also shown in the Fig. 6. From the responses it is clear that feedback controller takes corrective action after the load effects created on the system.

### ***Responses under integrated fuzzy logic controller***

The servo and regulatory responses of the system is shown in Fig. 7. The fuzzy logic augmented control system has considerably reduced the effect of load disturbance in the process variable as compared to the responses of feedback control system.

## 5.1 Response of noninteracting three tank system

### *Responses under classical feedback control system*

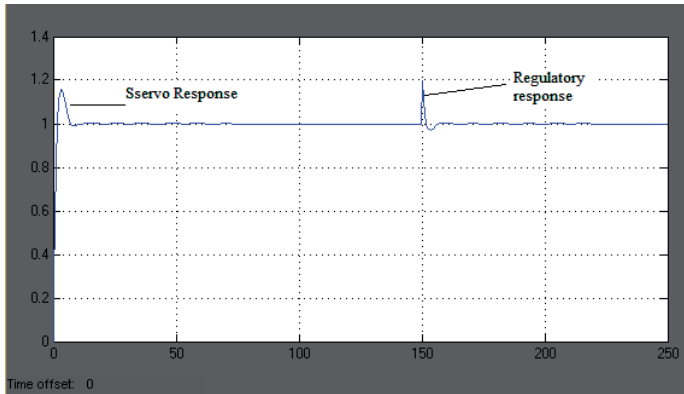


**Fig. 6 – Responses of Three tank Noninteracting system**

Curve I: Proportional Controller

Curve II: PI Controller

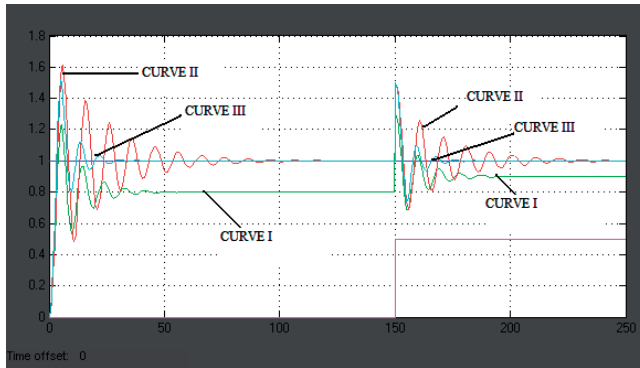
Curve III: PID Controller.



**Fig. 7 – Response of Three tank Noninteracting system under Integrated Fuzzy Logic Control.**

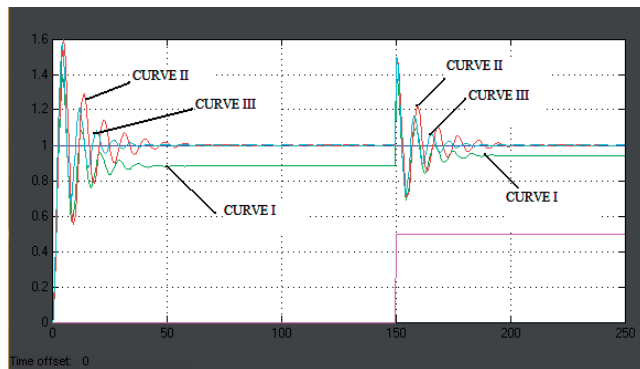
The servo and regulatory responses of feedback control system for case (i), case (ii) and case (iii) of interacting systems are shown in Figs. 8, 9 and 10. The servo and regulatory responses of integrated fuzzy logic control system are shown in Figs. 11, 12 and 13 of selected cases. The quantitative and qualitative analysis shows that integrated fuzzy logic based control is robust in load disturbance rejection under various conditions. The quantitative comparison of the responses of the selected system is presented in terms of peak time, maximum peak overshoot, integral of absolute value of error (IAE) and settling time.

## 5.2 Response of the interacting three tank system



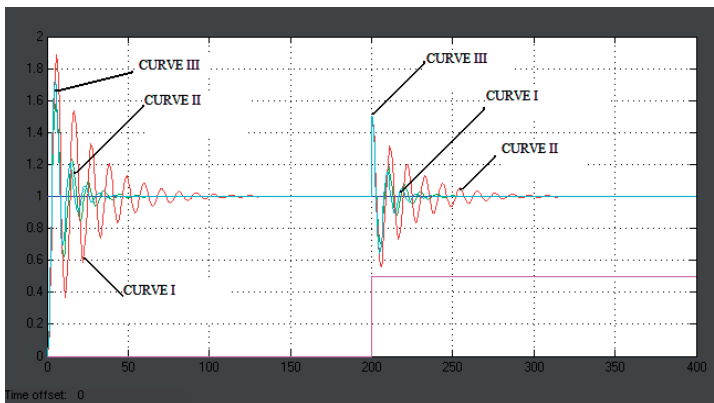
**Fig. 8** – Responses of Three tank Interacting system (CASE-I)

Curve I: Proportional Controller; Curve II: PI Controller; Curve III: PID Controller.



**Fig. 9** – Responses of Three tank Interacting system (CASE-II)

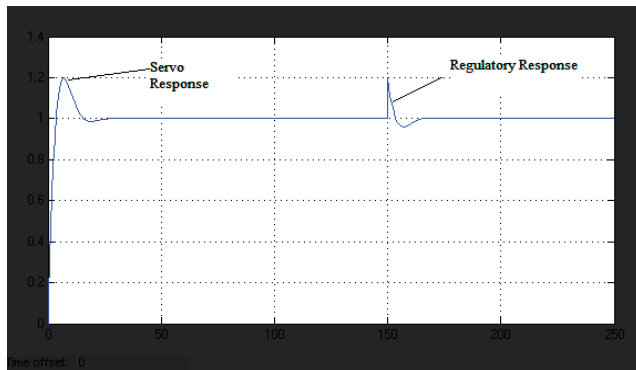
Curve I: Proportional Controller; Curve II: PI Controller; Curve III: PID Controller.



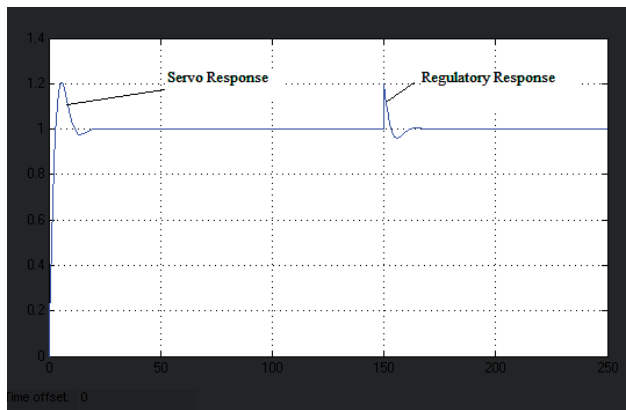
**Fig. 10** – Responses of Three tank Interacting system (CASE-III)

Curve I: Proportional Controller; Curve II: PI Controller; Curve III: PID Controller.

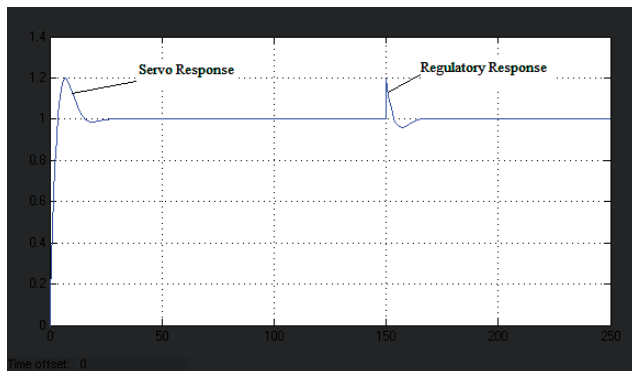
## Integrated Fuzzy Logic Based Intelligent Control of Three Tank System



**Fig. 11** – Response of Three tank Interacting system (CASE-I) under Integrated Fuzzy Logic Control.



**Fig. 12** – Response of Three tank Interacting system (CASE-II) under Integrated Fuzzy Logic Control.



**Fig. 13** – Response of Three tank Interacting system (CASE-III) under Integrated Fuzzy Logic Control.

## **6 Conclusion**

In this paper the disturbance rejection control under integrated intelligent fuzzy logic control was applied on the three tank system and the results obtained were compared with those obtained using classical feedback control (PID) method. The superiority of the integrated fuzzy logic control was analyzed through computer simulation using Matlab/Simulink software appears encouraging.

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