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Exact Linearization of an Induction Machine with Rotoric Flux Orientation

Abderrahim Bentaallah[1], Abdelkader Meroufel[1], Abdelber Bendaoud[1], Ahmed Massoum[1], Mohamed Karim Fellah[1]

Abstract: In this paper, we present the linearization control of an asynchronous machine. It allows decoupling and linearization of the system without including flux orientation. This non-linear control (NLC) applied to the asynchronous machine breaks up the system into two linear and independent mono systems. The speed and the Id current controls are carried out by traditional regulators PI. A qualitative analysis of the evolution of the principal variables describing the behaviour of the global system (IM-control) and its robustness is developed by several tests of digital simulation in the final stage. Numerous tests have been performed under Simulink/Matlab to show the control system performances.

Key words: Induction machine, Non-linear control, Field oriented control.

1 Indroduction

In recent years, asynchronous motors are more commonly used in the control processes that require different speeds and positions. The application of techniques of modern automation in electric machines control provides obtaining very high performances. The research in respective field is focused on the application of these techniques in the machine control.

In this paper, the application of the control by *Linearization Feedback* to an asynchronous machine [1–3] is the matter of our interest.

This technique enables us to linearize and decouple the system by using the differential geometry, whereupon the control by pole placement is applied to system. Finally, the control structure is tested by simulation on the linearized model of machine.

2 State Feedback Exact Linearization

Let us consider the class of the nonlinear dynamic system in the form:

[1]Intelligent laboratory Control & Electrical Power Systems, University of Sidi-Bel-Abbès, Algeria,
E-mails: ba_asmo@yahoo.fr; babdelber@univ-sba.dz

$$\begin{aligned}\dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_1 &= h_1(x) \\ &\vdots \\ y_m &= h_m(x),\end{aligned} \tag{1}$$

where $x \in R^n$, $h_1(x),\ldots,h_m(x)$ and $f(x)$, $g_1(x),\ldots, g_m(x)$ are differentiable vector functions of suitable size. Further, co-ordinates transformation and a nonlinear state feedback which linearize the system are to be found.

Thus let us consider a static nonlinear state feedback of the form:

$$\boldsymbol{u} = \boldsymbol{\alpha}(x) + \boldsymbol{\beta}(x)\boldsymbol{v}\,, \tag{2}$$

where $\boldsymbol{\beta}(x) = [\beta_{ij}(x)]$; for $i = 1,\ldots,m$ and $j = 1,\ldots,m$, is non singular and $\alpha(x) = [\alpha_1(x),\ldots,\alpha_m(x)]^{\mathrm{T}}$.

The exact linearization of system (1) with output $h_i(x)$ has to provide the solution to nonlinear state feedback (2) and the co-ordinates transformation: $\boldsymbol{z} = \boldsymbol{\Phi}(x) = [\Phi_1(x),\ldots,\Phi_n(x)]$ this put the closed loop system in the canonical form of Brunowsky [4, 5],

$$\begin{aligned}\dot{\boldsymbol{z}} &= \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{v} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{z}\end{aligned} \tag{3}$$

where $\boldsymbol{v}$ is the new control vector.

With $\boldsymbol{A} = \operatorname{diag}(\boldsymbol{A})$, $\boldsymbol{B} = \operatorname{diag}(\boldsymbol{B})$, and $\boldsymbol{C} = \operatorname{diag}(\boldsymbol{C})$:

$$\boldsymbol{A_i} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}; \qquad \boldsymbol{B_i} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}; \qquad \boldsymbol{C_i} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{4}$$

In relation to the equations of state (1) we define the vector relative degree $\{r_1,\ldots,r_m\}$. The system given by (1) has a vector relative degree $\{r_1,\ldots,r_m\}$ in a point x_0 if and only if:

1. The product

$$L_{gj} L_f^k h_i(x) = 0\,. \tag{5}$$

for $1 \le i \le m$, $1 \le j \le m$ and for $k < r_1 - 1$. $L_f h(x)$ is the Lie derivative of the function $h(x)$ according to the vector *f*.

2. The decoupling matrix

$$\boldsymbol{A}(x) = \left[L_{gi} L_f^{r_j - 1} h_j(x) \right]_{(i,j)} \tag{6}$$

is non-singular at the point x_0 for $1 \le i \le m$ and $1 \le j \le m$. The system is exactly linarizable if and only if $r_1 + \cdots + r_m = n$, i.e. after diffeomorphism and looping the system will consist of m linear and decoupled subsystems.

3 Nonlinear Model of the Asynchronous Machine

The machine model, in the selected reference frame *d-q* in such manner that rotor flux has a null component according to the *q* axis ($\Phi_{dr} = \Phi_r$, $\Phi_{qr} = 0$), is given by the following states equations:

$$\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{G}\boldsymbol{U} \tag{7}$$

with:

$$\boldsymbol{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_{ds} \\ i_{qs} \\ \Phi_{ds} \\ \Omega \end{bmatrix},\ \boldsymbol{U} = \begin{bmatrix} U_{ds} \\ U_{qs} \end{bmatrix},\ \boldsymbol{F}(\boldsymbol{X}) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix},\ \boldsymbol{G} = \left[g_1(x) \quad g_2(x) \right], \tag{8}$$

$$g_1(x) = \left[1/(\sigma L_s) \quad 0 \quad 0 \quad 0 \right]^{\mathrm{T}},\ g_2(x) = \left[0 \quad 1/(\sigma L_s) \quad 0 \quad 0 \right]^{\mathrm{T}},$$

$$\begin{aligned}
\dot{x}_1 &= -\left(\frac{R_s}{\sigma L_s} + \frac{M^2}{L_r^2} \frac{R_r}{\sigma L_s} \right) x_1 + \frac{1}{\sigma L_S} \frac{M}{L_r^2} R_r x_3 + R_r \frac{x_2^2}{x_3} + x_2 x_4 + \frac{1}{\sigma L_s} u_{ds} \\
\dot{x}_2 &= -\left(\frac{R_s}{\sigma L_s} + \frac{M^2}{L_r^2} \frac{R_r}{\sigma L_s} \right) x_2 - \frac{M}{\sigma L_S L_r} x_3 x_4 - \frac{M}{L_r} R_r \frac{x_1 x_2}{x_3} + x_1 x_4 + \frac{1}{\sigma L_s} u_{qs} \\
\dot{x}_3 &= \frac{R_r}{L_r} M x_1 - \frac{R_r}{L_r} x_3 \\
\dot{x}_4 &= \frac{1}{J} \frac{M}{L_r} x_2 x_3 - \frac{C_r}{J} - \frac{f_r}{j} x_4,
\end{aligned} \tag{9}$$

and

$$
\begin{gathered}
f_1(x) = -\left(\frac{R_s}{\sigma L_s} + \frac{M^2}{L_r^2}\cdot\frac{R_r}{\sigma L_s}\right)x_1 + \frac{1}{\sigma L_S}\cdot\frac{M}{L_r^2}R_r x_3 + R_r\frac{x_2^2}{x_3} + x_2 x_4 \\
f_2(x) = -\left(\frac{R_s}{\sigma.L_s} + \frac{M^2}{L_r^2}\frac{R_r}{\sigma.L_s}\right)x_2 - \frac{M}{\sigma L_S L_r}x_3 x_4 - \frac{M}{L_r}R_r\frac{x_1 x_2}{x_3} + x_1 x_4 \\
f_3(x) = \frac{R_r}{L_r}Mx_1 - \frac{R_r}{L_r}x_3 \\
f_4(x) = \frac{1}{J}\cdot\frac{M}{L_r}x_2 x_3 - \frac{C_r}{J} - \frac{f_r}{j}x_4,
\end{gathered} \tag{10}
$$

with

$$
\sigma = 1 - \frac{M^2}{L_s L_r},\ \lambda = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2},\ K = \frac{M}{\sigma L_s L_r}.
$$

The electromagnetic torque developed by machine is given by:

$$
C_{elm} = \frac{3}{2}\frac{M}{L_r}\Phi_{dr} I_{qs}. \tag{11}
$$

3.1 The output choice

The output choice is related to the objectives of control. As outputs x_3 (d-axis rotor flux components) and x_4 (speed) are chosen [6, 7]:

$$
\boldsymbol{Y}(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}. \tag{12}
$$

3.2 Input/Output linearization

The determination of the relative degree allows the nonlinear system to check the admittance of an input/output linearization:

a) Degree relating to the output $Y_1(x)$:

$$
\begin{gathered}
\dot{Y}_1(x) = \dot{h}_1(x) = L_f h_1(x) \\
\ddot{Y}_1(x) = \ddot{h}_1(x) = L_f^2 h_1(x) + L_g L_f h_1(x) u_{ds}.
\end{gathered} \tag{13}
$$

The relative degree associated to $Y_1(x)$ is $r_1 = 2$.

b) Degree relating to the output $Y_2(x)$:

$$\dot{Y}_2(x) = \dot{h}_2(x) = L_f h_2(x)$$
$$\ddot{Y}_2(x) = \ddot{h}_2(x) = L_f^2 h_2(x) + L_g L_f h_2(x) u_{qs}. \tag{14}$$

The relative degree associated to $Y_2(x)$ is $r_2 = 2$.

With

$$L_f h_1(x) = \dot{h}_1(x)$$
$$L_f h_1(x) = \frac{R_r}{L_r} M x_1 - \frac{R_r}{L_r} x_3$$
$$L_f^2 h_1(x) = \frac{R_r}{L_r}\left(M\left(-\lambda x_1 + \frac{MR_r}{\sigma L_s L_r} x_3 + R_r \frac{x_2^2}{x_3} + x_2 x_4 + \frac{1}{\sigma L_s} u_{ds} \right) - \frac{R_r}{L_r} M x_1 - \frac{R_r}{L_r} x_3 \right)$$
$$L_f h_2(x) = \dot{h}_2(x)$$
$$L_f h_2(x) = \frac{1}{J}\frac{M}{L_r} x_2 x_3 - \frac{C_r}{J}$$
$$L_f^2 h_2(x) = \frac{M}{JL_r}\left(-\lambda x_2 - \frac{M}{\sigma L_s L_r} x_3 x_4 - \frac{MR_r}{L_r}\frac{x_1 x_2}{x_3} + x_1 x_4 + \frac{u_{qs}}{\sigma L_s} \right) x_3 + x_2 \left(\frac{R_r M}{L_r} x_1 - \frac{R_r}{L_r} x_3 \right).$$

With

$$L_f^2 h_1(x) = \frac{R_r}{L_r}\left(M f_1(x) - f_2(x) \right)$$
$$L_f^2 h_2(x) = \frac{M}{JL_r}\left(x_3 f_2(x) + x_2 f_3(x) \right). \tag{15}$$

The choice of these outputs leads to a complete linearization of order four ($r_1 + r_2 = N = 4$), where N is the system order [8, 9].

3.3 Diffeomorphism transformation

The change of nonlinear co-ordinates is given by the equations system according to [10–12]:

$$z_1 = h_1(x) = x_3$$
$$z_2 = L_f h_1(x) = f_3(x)$$
$$z_3 = h_2(x) = x_4$$
$$z_4 = L_f h_2(x) = f_4(x). \tag{16}$$

The application of the change of variables (16) to the system of equations (8) leads to the following writing:

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= L_f^2 h_1(x) + L_g L_f h_1(x) u_{ds} = v_1 \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= L_f^2 h_2(x) + L_g L_f h_2(x) u_{qs} = v_2 .
\end{aligned} \tag{17}$$

3.4 Nonlinear control law

To have a complete input/output linearization of order four in closed loop, it is necessary to apply the nonlinear state feedback, [13, 14].

$$\boldsymbol{U} = \boldsymbol{D}^{-1}(x)\left[\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \boldsymbol{A}(x)\right], \tag{18}$$

where

$$\boldsymbol{D}(x) = \begin{bmatrix} \dfrac{MR_r}{\sigma L_s L_r} & 0 \\ 0 & \dfrac{Mx_3}{J\sigma L_s L_r} \end{bmatrix}. \tag{19}$$

The decoupling matrix $\boldsymbol{D}(x)$ is not singular, $\det[\boldsymbol{D}(x)] \neq 0$, where:

$$\boldsymbol{D}^{-1}(x) = \frac{J\sigma L_s L_r}{MR_r x_3}\begin{bmatrix} \dfrac{x_3}{J} & 0 \\ 0 & R_r \end{bmatrix}, \quad \boldsymbol{A}(x) = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{bmatrix}. \tag{20}$$

The application of the law (18) to system of equation (17) leads to the linear model (21) Fig. 1

$$\dot{z}_1 = z_2, \quad \dot{z}_2 = v_1, \quad \dot{z}_3 = z_4, \quad \dot{z}_4 = v_2 . \tag{21}$$

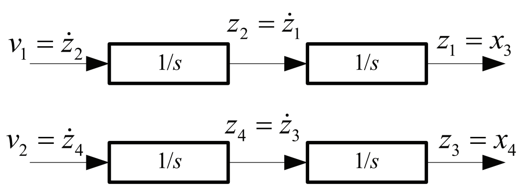

Fig. 1 – *Decoupled linear system.*

The linearization and decoupling for induction machine (IM) model is obtained by an appropriate selection of the control input $u = [u_{ds} \quad u_{qs}]^{\mathrm{T}}$ in the form:

$$\begin{bmatrix} u_{ds} \\ u_{qs} \end{bmatrix} = \boldsymbol{D}^{-1}(x) \begin{bmatrix} -L_f^2 h_1(x) + v_1 \\ -L_f^2 h_2(x) + v_2 \end{bmatrix}. \tag{22}$$

4 Trajectory Imposition Control

To follow reference trajectory of flux (z_{1ref}) and speed (z_{3ref}) with a certain dynamic, one imposes to the linearized system stable poles satisfying the desired performances (polynomial of Hurwitz). The inputs v_1, v_2 can be calculated by [15, 16]:

$$\begin{aligned} v_1 &= k_{11}\left(z_{1ref} - z_1\right) + k_{12}\left(\dot{z}_1 - \dot{z}_{ref}\right) + \ddot{z}_{1ref} \\ v_2 &= k_{21}\left(z_{3ref} - z_2\right) + k_{22}\left(\dot{z}_{3ref} - \dot{z}_3\right) + \ddot{z}_{3ref}. \end{aligned} \tag{23}$$

The error equations become:

$$\begin{aligned} \ddot{e}_1 + k_{12}\dot{e}_1 + k_{11}e_1 = 0 \\ \ddot{e}_2 + k_{22}\dot{e}_2 + k_{21}e_2 = 0. \end{aligned} \tag{24}$$

with $e_1 = z_{1ref} - z_1$ and $e_2 = z_{3ref} - z_3$.

The coefficients k_{ij} ($i = 1,2;\ j = 1,2$) are selected to satisfy the polynomial of Hurwitz:

$$\begin{aligned} k_{11} + k_{12}s + s^2 = 0 \\ k_{21} + k_{22}s + s^2 = 0. \end{aligned} \tag{25}$$

5 Simulation

The global nonlinear control with flux orientation for the induction machine is shown in Fig. 2.

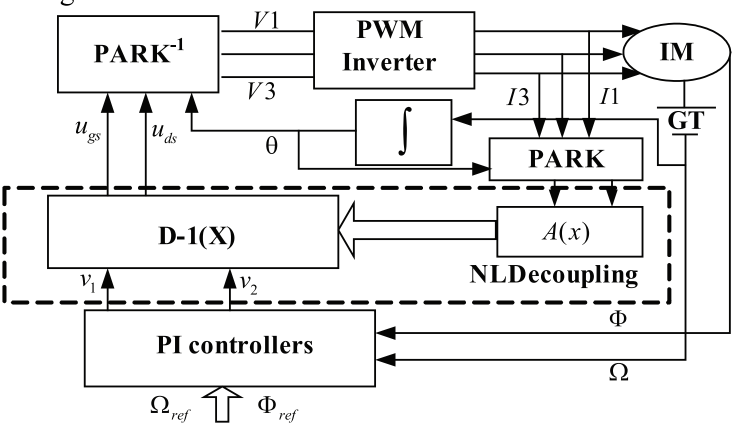

Fig. 2 – *General diagram of the Induction Machine NL control.*

5 Simulation Results

Fig. 3 illustrates the response speed of loadness machine, similar to a first-order system without being exceeded, with a response time of about 0.17s. There is, however, the rejection of disturbance which is applied to 2s later.

Perfect matching occurs when changing the reference speed. This confirms proper choice of the coefficients tuning controller nonlinear speed.

At the starting point (t=0), a peak torque electromagnetic at the machine load is 35 Nm, whereas after 2 seconds torque load drops to 10 Nm. The response to this load change with a dynamic torque is almost instantaneous, with a very low overrun and without oscillations.

The response of Rotor flux of induction machine in the 1st order system occurs along its reference path, without being exceeded, the response time being significantly smaller in the order of 0.09 seconds, whereby perfect decoupling between the flux and torque is observed.

When there is no load , the stator current absorbed by the machine shows an oscillation both at boot time and at speed change. Once loaded, the machine absorbs a current quasi-sinusoidal and r.m.s on the torque load.

At change of speed at 100 rad / s, the electromagnetic torque decreases reaching a negative value of -10N.m which corresponds to the system collapse.

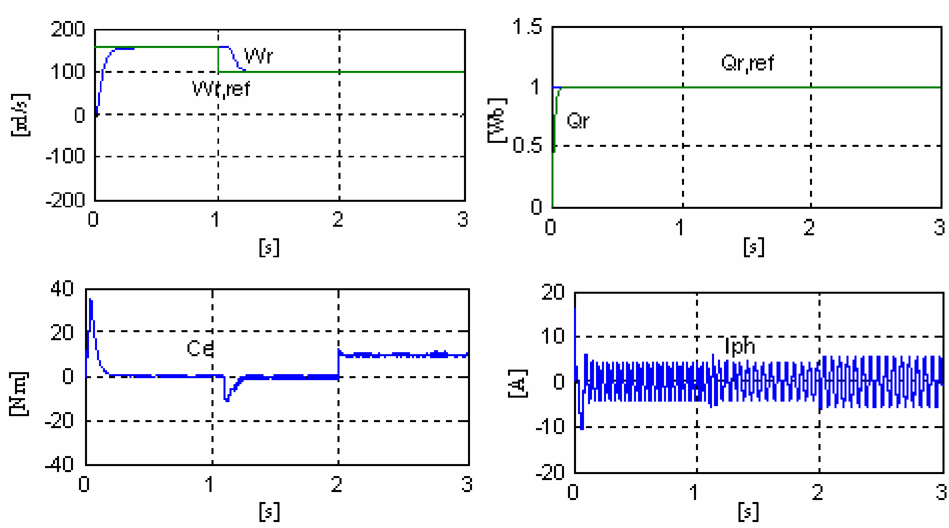

Fig. 3 – *Nonlinear control for an induction machine with a speed* +156rad/s *to* 100 rad/s *at* 1.00s *and load torque application at* 2.00s.

The reversal of rate of $156\,\text{rad/s}$ to $-156\,\text{rad/s}$ at $t = 1\,\text{s}$, according to Fig. 4, we note that the electromagnetic torque decreases instantly to a negative value of around –40Nm, which corresponds to an area of breaking and then switching to a rotation change.

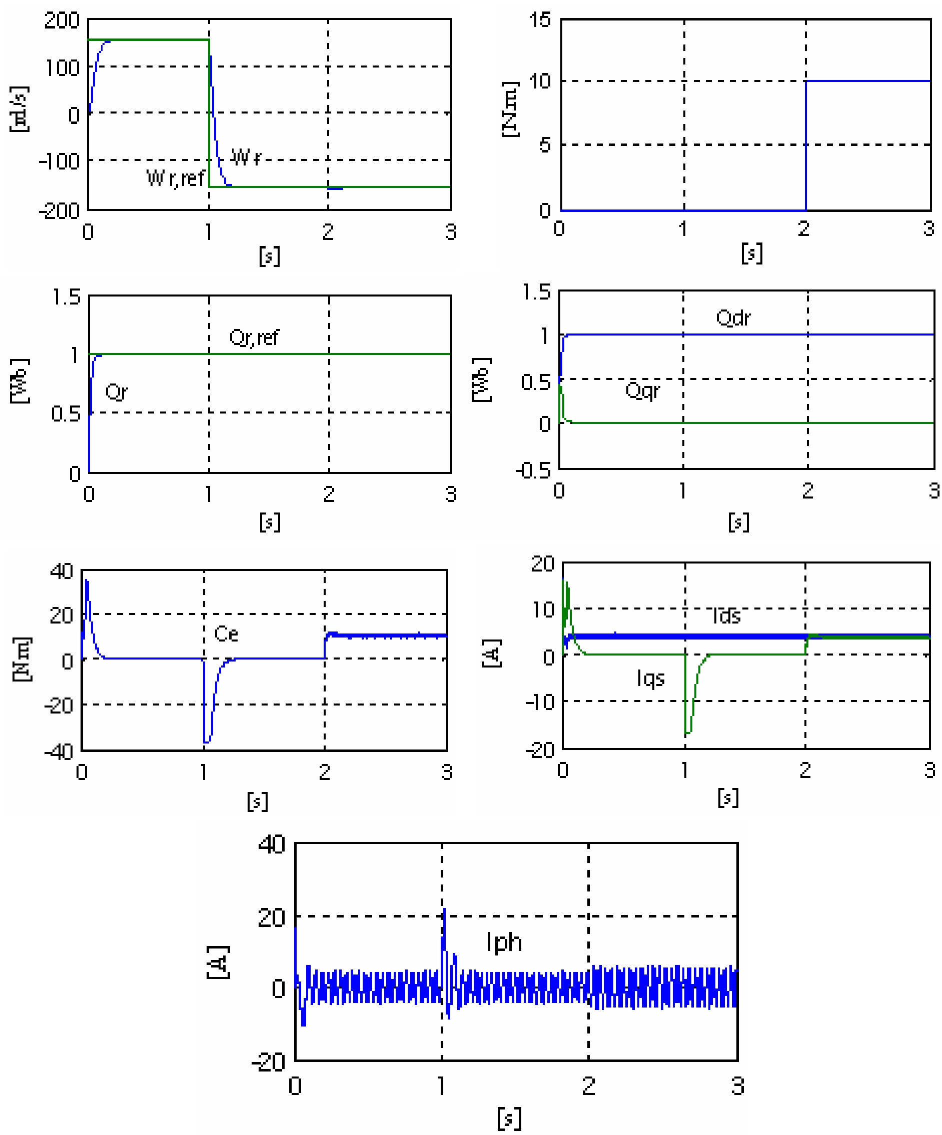

Fig. 4 – *Nonlinear control for an induction machine with a speed inversion* ($\pm\, 156\,\text{rad/s}$) *at* 1.00s *and load torque application at* 2.00s.

The flux does not have any influence due to the perfect decoupling. The simulation results show good performances for flux and the torque (speed), Fig. 4.

When the load torque is applied, we notice that there is no interaction between the two axes (d, q), which proves total dynamic decoupling between the two variables. The I_q current is proportional to the electromagnetic torque.

In addition, flux Φ_r is oriented in the d direction ($\Phi_{dr} = \Phi_r ; \Phi_{qr} = 0$).

The speed responses are without static error, without overshooting and with a very fast disturbance rejection.

7 Conclusion

In this paper, we have presented the nonlinear control applied to the induction machine having rotoric flux orientation. The change of nonlinear co-ordinates and a negative feedback NL permits returning the nonlinear behavior of the system to linear system. The disturbances rejection and decoupling of flux and torque are acceptable.

The field-oriented control technique supposes that the knowledge of the flux position is exact. The nonlinear control makes abstraction of flux position. The nonlinear regulator retains the same performance for a long time whereby we are not acquainted with uncertainty parameters.

It is well adapted to the problems of tracking trajectories and the problems of stabilization. The main limitations are the lack of robustness and the practical point of view, the requirement that all states are measurable.

The main disadvantage of the linearization order is that it is based on the knowledge of the exact model of system. Indeed, in most cases we can not know the exact model of the real system

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9 Induction Machine Parameters

$P = 1.5\,\mathrm{kW}$	$p = 2$
$U = 380/220\,\mathrm{V},\ 50\,\mathrm{Hz}$	$N = 1450\,\mathrm{tr/min}$
$I = 3/6\,\mathrm{A}$	$M = 0.258\,\mathrm{H}$
$R_s = 4.85\,\Omega$	$R_r = 3.81\,\Omega$
$L_s = 0.274\,\mathrm{H}$	$L_r = 0.274\,\mathrm{H}$
$J = 0.031\,\mathrm{Kgm}^2$	$f = 0.0114\,\mathrm{Nm/rad/s}$