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Numerical Computation of the Restoring Force in a Cylindrical Bearing Containing Magnetic Liquid

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Abstract: The present paper deals with the second order of magnetic levitation, applied to a cylindrical bearing holding a magnetized shaft and the magnetic liquid The magnetic restoring force acting on the shaft of the cylindrical bearing. was numerically evaluated, the liquid being considered a nonlinear medium.

Keywords: Second order levitation force, Magnetic liquid, Numerical simulation, 2D-FEM.

1 Introduction

The cylindrical bearing containing magnetic liquid represents one of the applications of the second order of magnetic levitation. The bearing with magnetic liquid and a permanent magnetized shaft has been proposed in [1], and analytically investigated in [2]. An approximately analytical expression for the levitation force acting on the magnetized shaft, assuming the liquid as a linear medium, has been proposed in [2, 3].

This paper evaluates numerically the magnetic field inside the magnetic liquid, the liquid being considered as a nonlinear medium. The magnetic characteristic of the liquid $B = f(H)$ was built in laboratory for a real magnetic liquid. The magnetic levitation force that acts on the shaft of the bearing was calculated numerically using the numerical values of the magnetic field inside the liquid. The results, presented in a graphical form, have been compared with the similar results using a magnetic liquid approximated as a linear medium.

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2 Theoretical Consideration

The sketch of a cross section of a cylindrical bearing with magnetic liquid is presented in Fig.1.

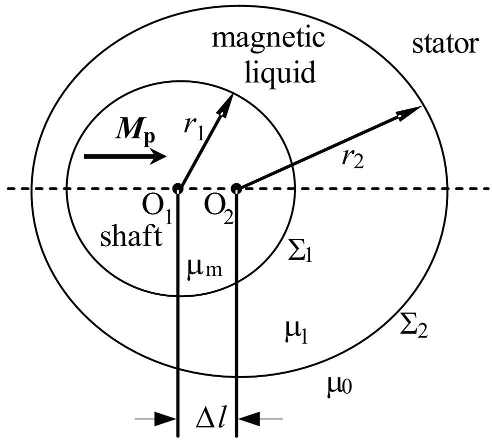


Fig. 1 – *Cross section of a cylindrical bearing.*

The shaft of r_1 radius consists of a permanent magnet with a constant permanent magnetization $\boldsymbol{M}_p$ and permeability μ_m. The stator of r_2 radius is considered nonmagnetic. The magnetic liquid, assumed as a nonlinear medium of permeability $\mu_l = \mu_l(H)$, occupies the space between the shaft (rotor) and the stator. The displacement between the rotor axis and stator axis, $O_1O_2 = \Delta l$, is in the same direction as $\boldsymbol{M}_p$. Due to the displacement Δl, the magnetic force directed in such manner as to provide shaft equilibrium (when the shaft is centered in bearing) acts on the magnetized shaft. The levitation force depends on the following: displacement Δl, magnetic properties of the magnetic liquid, permanent magnetization of the shaft, and geometrical design of the bearing (the values of the radius r_1 and r_2). The shaft is bounded by the Σ_1 surface and the stator is bounded in its inner part by the Σ_2 surface. Assuming the length of the shaft much greater than radius r_1 or r_2, the magnetic field can be considered to be plan-parallel.

The equation of the magnetic field can be obtained in all three domains by using the point form of Ampere's law:

$$\operatorname{rot} \boldsymbol{H} = 0 \text{ and } \operatorname{rot}_s \boldsymbol{H} = 0, \tag{1}$$

the relationship between $\boldsymbol{B}$, $\boldsymbol{M}$, and $\boldsymbol{H}$ expressed by:

$$\boldsymbol{B} = \mu \boldsymbol{H} + \mu_0 \boldsymbol{M}_p , \tag{2}$$

and the point form of the magnetic flux law:

$$\operatorname{div} \boldsymbol{B} = 0 . \tag{3}$$

As the consequence, the magnetic potential vector $\boldsymbol{A}$ satisfies a Laplace equation in all the three domains:

$$-\nabla^2 \boldsymbol{A} = 0 . \tag{4}$$

For the plan-parallel fields $\boldsymbol{A} = A(x, y)\boldsymbol{u}_z$,

$$\boldsymbol{B} = -\boldsymbol{u}_z \times \nabla \boldsymbol{A} \tag{5}$$

and (4) becomes the scalar form:

$$\nabla^2 A(x, y) = 0 . \tag{6}$$

As shown in [2,3], the force exerted on the unit length of the shaft is:

$$\boldsymbol{F}_m^* = -\oint_{\Sigma_1^*} \left(\int_0^H \boldsymbol{B} \, \mathrm{d} \boldsymbol{H} \right) \mathrm{d} \boldsymbol{S} + \oint_{\Sigma_1^*} \boldsymbol{H} (\boldsymbol{B} \boldsymbol{n}) \, \mathrm{d} \boldsymbol{S} , \tag{7}$$

with Σ_1^* representing the cylindrical surface of r_1 radius and unit length, attached to the magnetic liquid surrounding the shaft.

When we assumed the magnetic liquid as a linear medium of μ_l permeability:

$$\int_0^H \boldsymbol{B} \, \mathrm{d} \boldsymbol{H} = \mu_l \int_0^H \boldsymbol{H} \, \mathrm{d} \boldsymbol{H} = \frac{1}{2} \mu_l \boldsymbol{H}^2 \tag{8}$$

and (7) results in:

$$\boldsymbol{F}_m^* = -\frac{1}{2} \mu_l \oint_{\Sigma_1^*} \boldsymbol{H}^2 \, \mathrm{d} \boldsymbol{S} + \mu_l \oint_{\Sigma_1^*} \boldsymbol{H} (\boldsymbol{H} \boldsymbol{n}) d\boldsymbol{S} . \tag{9}$$

In order to establish the domain extension for the numerical simulation, we have adopted the model presented in Fig. 2. The magnetic field outward the permanent magnet shaft, $r > r_1$, is [2]:

$$H_{ext} = \frac{r_1^2 M_p}{(\mu_r + 1) r^2} , \tag{10}$$

with the maximum value for $r = r_1$:

$$H_{max} = \frac{M_p}{\mu_r + 1}. \tag{11}$$

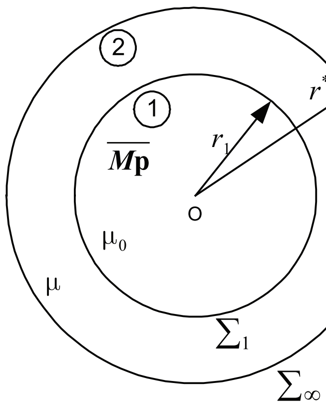


Fig. 2 – *The model used to establish the domain extension.*

Dividing (10) by (11), results in:

$$\frac{H_{ext}(r^*)}{H_{max}} = \frac{r_1^2}{r^{*2}}. \tag{12}$$

Considering that the boundary might be placed at a distance r^* where $H_{ext}\left(r^*\right) = \alpha H_{max}$ the following occurs (Fig.3):

$$r^* = \frac{1}{\sqrt{\alpha}} r_1. \tag{13}$$

The field beyond this distance has been neglected.

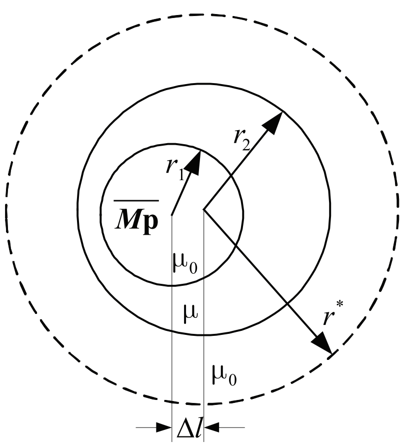


Fig. 3 – *The model used in numerical simulation.*

3 The Numerical Force Calculation

The plan parallel model used in numerical computation is shown in Fig. 4, with $r_1 = 10\,\text{mm}$, $r_2 = 11\,\text{mm}$ and $M_p = 136.4\ \text{kA/m}$. As the permeability in magnet μ_m is close to the μ_0, in the numerical computation we have assumed $\mu_m = \mu_0$.

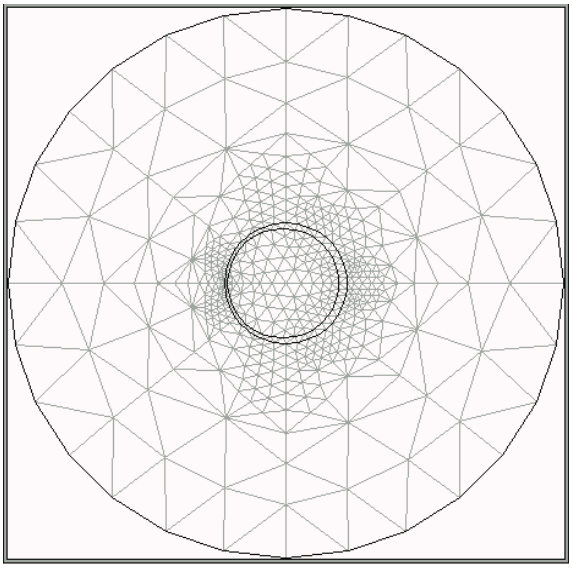
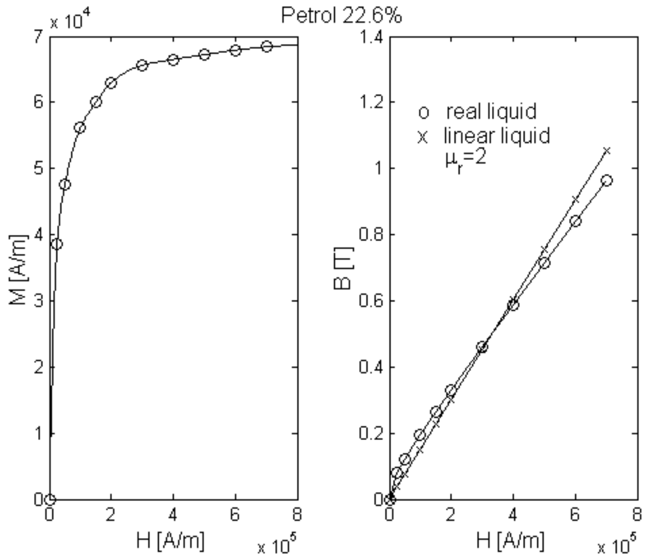


Fig. 4 – *The model used in numerical force calculation.*

The 2D-FEM of the MagNet 5.0 specialized program (*Infolytica*) was used to solve numerically the field problem. The solution is obtained iteratively using an effective technique known as the Conjugate Gradient Method. A Newton-Rhaphson algorithm has been used for the nonlinear magnetic liquid. The first order triangles and a number of about 10 000 nodes were used in simulation. On each triangle the vector magnetic potential **A** satisfies equation (6). Using a Cartesian system of coordinates, the magnetic flux density results in the form:

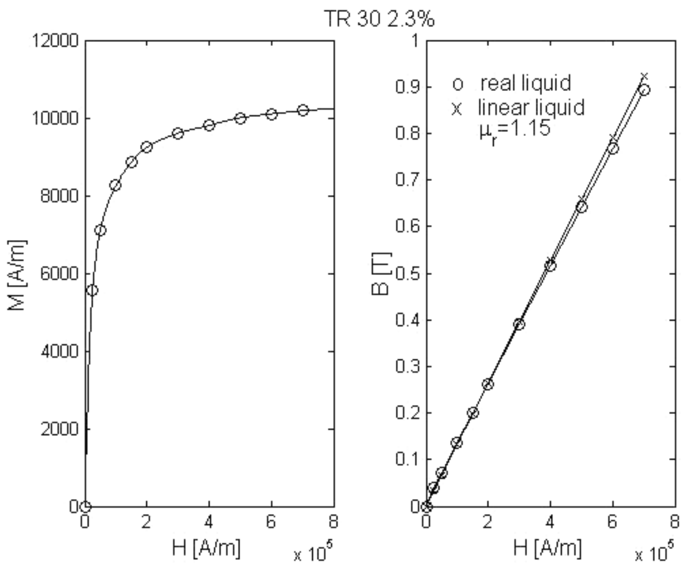
$$\boldsymbol{B} = B_x \boldsymbol{u}_x + B_y \boldsymbol{u}_y = \operatorname{rot} \boldsymbol{A} = \frac{\partial A}{\partial y} \boldsymbol{u}_x - \frac{\partial A}{\partial x} \boldsymbol{u}_y . \tag{14}$$

Figs. 5a and 6a show the dependence $M = M(H)$ for two magnetic liquids, one based on petrol, (22.6%), and the other one on transformer oil. The magnetic characteristic has been obtained in laboratory. Figs. 5b and 6b depict the magnetic curves of the liquid. "$\circ$" denotes the magnetic curve for the real magnetic liquid, "$\times$" denoting the linear curve connected with the magnetic liquid.



(a) (b)

Fig. 5 – *The magnetic characteristic for the petrol* 22.6%.



(a) (b)

Fig. 6 – *The magnetic characteristic for the transformers oil.*

The magnetic restoring force has been calculated numerically using (7) for the nonlinear magnetic liquid and (9) for the linear approximation of the magnetic liquid. The integration path represents the length of the circle of r_1 radius. In the numerical evaluation of the force, the path has been divided in 100 equal segments.

The Shimpson method and the MathCad medium have been used for numerical integration of the force. In all analyzed cases, as it was expected, the computed force had a component directed only towards the displacement.

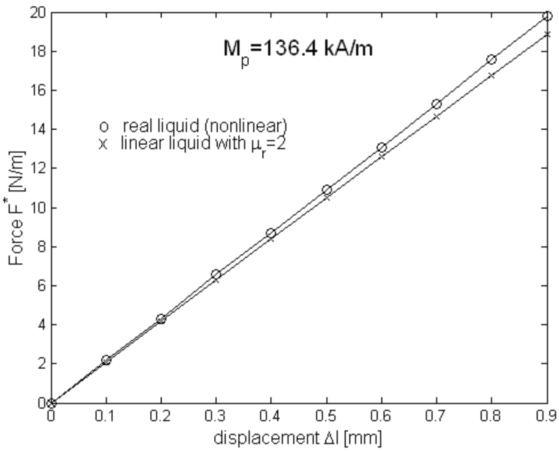


Fig. 7 – *The levitation force F versus the displacement* Δl *for the petrol based liquid.*

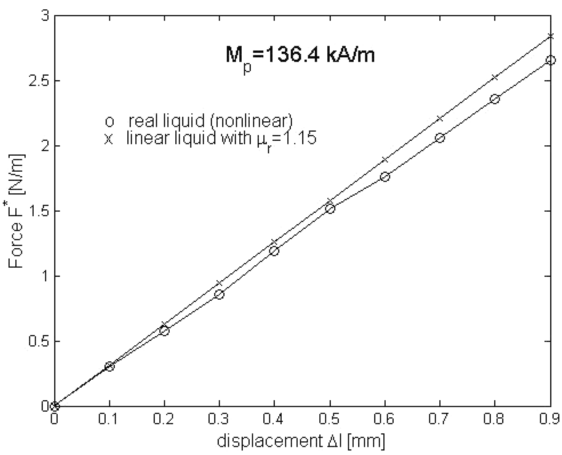


Fig. 8 – *The levitation force F versus the displacement* Δl *for the transformers oil.*

4 Conclusion

The results shown in a graphical form on Figs. 7 and 8 represent the restoring magnetic force acting on the unit length of the shaft versus displacement Δl for the two types of the applied magnetic liquid. "$\circ$" denotes force values obtained using real magnetic liquid, whereas "$\times$" denotes the values of the restoring force using a linear approximation for the magnetic liquid. The two pairs of curves are well correlated.

As a conclusion, the application of the linear approximation of liquid may provide good results in the numerical evaluation of the levitation force.

5 References

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