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Equivalent Circuit Model of Electromagnetic Behaviour of Wire Objects by the Matrix Pencil Method

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Abstract: The electromagnetic behaviour of thin-wire objects is an important engineering problem. A general analysis of such objects is done according to the moment method approach within an antenna-based electromagnetic model. This paper concerns possible extraction of parameters that would enable determination of an equivalent circuit model of such wire-structure objects. As an example, the authors focused on the electromagnetic behaviour of high frequency power converters and grounding conductors.

Keywords: Electromagnetic model, Numerical modelling, Matrix pencil method.

1 Introduction

Analysis of grounding systems at high frequencies is very important in EMC studies related to lightning. On the other hand, with the ever-increasing switching frequency power converters are also the subject of EMC/EMI analysis as sources of radiation emission in the surroundings. In both cases, in order to predict the transient behaviour, these objects are considered as thin-wire conductors, which are treated at high frequencies by electromagnetic models based on the antenna theory.

In this paper, the authors are interested in the possibility of extracting the parameters of equivalent circuit models of such wire-objects. More specifically, our analysis is related to transient analysis of:

1. Rectangular loop model of power converter;
2. Vertical grounding rod.

In both cases, the analysis is first performed in the frequency domain using developed electromagnetic antenna-based numerical models. In the second step, after transformation of the functions of interest to the time domain, the objective is to extract the poles and residues of functions using the 'matrix pencil' method.

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Using these parameters, it may be possible to later determine the equivalent circuit models. This analysis also enables observation of the influence of the wire-object's geometry on their transient behaviour that cannot be analyzed using circuit theory approaches.

2 Electromagnetic Analysis of Wire Objects at High Frequencies

2.1 Electromagnetic analysis of the converter circuit

The power converter circuit's construction is based on several elementary cells of commutation that may be represented by Fig. 1. The PCB trace printed over a dielectric layer circuit behaves like a transmitting antenna that radiates directly into the immediate surroundings. The geometry of such a converter circuit is considered to be a rectangular loop having length L and width W. PCB traces are modeled using the thin-wire approximation. Here, V_S is the source voltage and R_L represents the load. It is assumed that the switch is in position 1, so that the influence of PCB without supply is not taken into account.

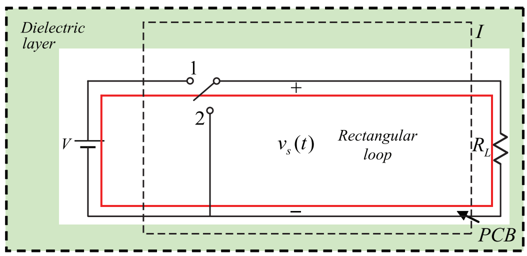


Fig. 1 – *Rectangular circuit loop model of the power converter.*

In our analysis, the switch and the DC source together are replaced by an equivalent voltage source of 1V step function with a rise time of the order of ns, which is placed on the position of the switch, $v_S(t)$ as shown on Fig. 2.

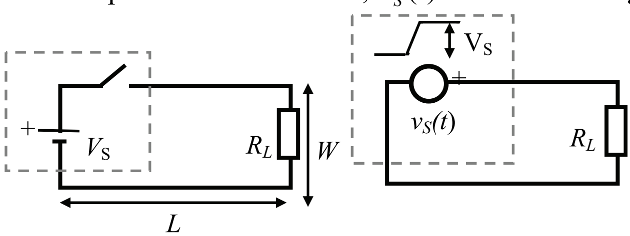


Fig. 2. – *Equivalent voltage source function* $v_S(t)$.

Our mathematical model for electromagnetic analysis of the power converter circuit given on Fig. 2 is based on rigorous formulations derived from the full set of Maxwell's equations, based on the theoretical background of microstrip antenna analysis. The model is developed in the frequency domain and takes into account all electromagnetic effects related to the geometry of the converter circuit and the influence of the dielectric board interfaces. More details about the mathematical model and the numerical results are given in [2].

In this paper the authors are more focused on the analysis of transient responses of the converter circuit when excited by voltage source excitation of the 1V step function with a high rise time of 1ns. Our objective was to investigate the influence of the circuit layout on the transient behaviour of the system. These computations were carried in the frequency domain in conjunction with the inverse Fourier transform method.

Firstly, using the developed electromagnetic model we obtained the transfer function of the converter's rectangular loop-circuit $H(f) = V_{RL}(f)/V_S$, with $V_S = 1\text{V}$ and $R_S = 0.1\,\Omega$ in the frequency domain of interest from 10MHz up to several GHz. In our analysis the load R_L was defined as 10kΩ to simulate an open circuit in order to obtain only the response of the rectangular circuit-loop itself. Our objective was to investigate the influence of the circuit layout on the transient behavior of the system.

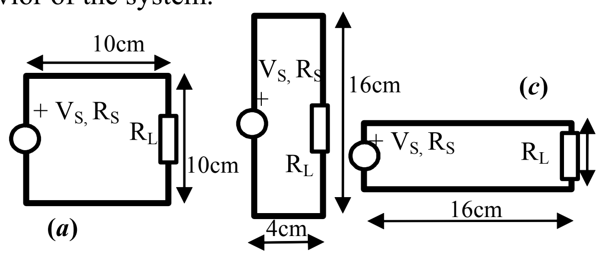


Fig. 3 – *Three configuration of the converter circuit.*

To cover the above mentioned high frequency range, it is assumed that the frequency of the voltage harmonic is in range from 10MHz to 2GHz. Three loop geometries are considered: (*a*) square loop (10×10cm), (*b*) rectangle loop (4×16cm) and (*c*) rectangle loop (16×4cm), as shown on Fig. 3. The length of the leads (perimeter of the circuit) is fixed to 40cm, while the equivalent thin-wire radius of the lead is 0.5mm. The dielectric board is defined by relative permittivity 4.7 and board thickness of 2.5mm. Two distinct positions of the equivalent source are assumed in the analysis: Case 1 - opposite to the load, Case 2 - in the middle of length *L*, as shown on Fig. 4.

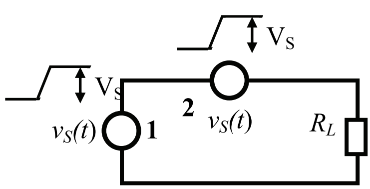


Fig. 4 – *Two positions of the source with respect to the load.*

2.1 Electromagnetic analysis of grounding rod

Consider a thin vertical grounding rod of radius a which is placed in homogeneous soil. It is assumed that the rod extends from depth $-d$ to $-(d+L)$ as shown on Fig. 5. The soil is characterized by permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$, permeability μ_0 and conductivity σ.

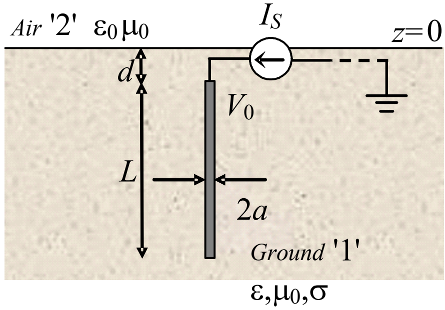


Fig. 5 – *Vertical grounding rod in homogeneous soil.*

The high frequency response of the grounding rod is analyzed by a complete electromagnetic model. It is based on integral equations derived from the Maxwell's equations and the influence of the earth is taken into account by an exact Sommerfeld formulation [1]. The current distribution is computed as a response to injected current IS. This leads to matrix equation

$$[Z][I] = [-Z_S I_S]. \tag{1}$$

where, the column matrix [I] represents the unknown currents to be determined, [Z] is the generalized impedance matrix, and [$-Z_S I_S$] represents the energization matrix.

The developed computer model is used to compute the impedance to ground $Z_g(f) = V_0(f)/I_S$, where $V_0(f)$ is the voltage between the feed point and remote ground, and $I_S = 1\text{A}$ is the injection current. The calculations are performed within frequency range from 1kHz to 100MHz. Our objective was to investigate the influence of the conductor length L and the soil conductivity σ on its transient behaviour. These computations were carried in the frequency domain in conjunction with the inverse Fourier transform method.

3 Introduction

After frequency-dependent functions $H(f)$ and $Z_g(f)$ were once computed, the results were converted to the time domain using the inverse fast Fourier transform algorithm. Thus we obtained $h(kT_S)$ and $z_g(kT_S)$, $k = 0, \ldots, N$; within N samples and T_S sampling period respectively.

Afterwards, using the 'matrix pencil' method [3] we determined their approximate representations:

$$\left.\begin{matrix} h(kT_S) \\ z_g(kT_S) \end{matrix}\right\} = y_{k+1} \approx \sum_{i=1}^{M} R_i z_i^k \quad \text{for } k = 0,\ldots,N-1 . \tag{2}$$

Here, R_i and z_i are residues and poles of the Matrix Pencil method basic function respectively. Each pole z_i may be represented by $z_i = e^{s_i T_S} = e^{(-\alpha_i + j\omega_i)T_S}$, where $i = 0,\ldots,M$. Here, R_i represents residues, α_i represents damping factors and ω_i are angular frequencies.

It is possible to define matrix $\boldsymbol{Y}$ of dimensions $(L+1)\times(N-L)$ by regrouping the terms y_k

$$\boldsymbol{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N-L} & y_{N-L+1} & \cdots & y_N \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 & \ldots & Y_L & Y_{L+1} \end{bmatrix} \tag{3}$$

and two sub-matrices

$$\boldsymbol{Y}_1 = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_L \end{bmatrix} \tag{4}$$

$$\boldsymbol{Y}_2 = \begin{bmatrix} Y_2 & Y_3 & \cdots & Y_{L+1} \end{bmatrix} . \tag{5}$$

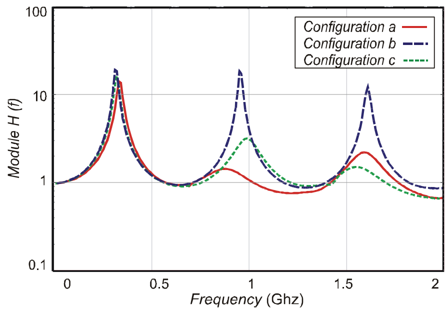
It is shown in [3] that $\boldsymbol{Y}_1 - \boldsymbol{Y}_2$ is diagnosable and permits precise determination of the z_i poles. Finally, in order to optimize the performances of the method, it is chosen that L is close to $N/2$, so that the choice of M is what determines the number of dominant poles in the identification process of the function $h(kT_S)$ and $z_g(kT_S)$. The above matrices are not generally square matrices, and the inverse matrix of $\boldsymbol{Y}_1$ is practically pseudo-inverse.

The corresponding time-domain responses $h(kT_S)$ and $z_g(kT_S)$ were subjects of the 'matrix pencil' method analysis. Thus we obtained their z_i poles which were compared with the frequency dependent functions $H(f)$ and $Z_g(f)$. The results are presented in the next section.

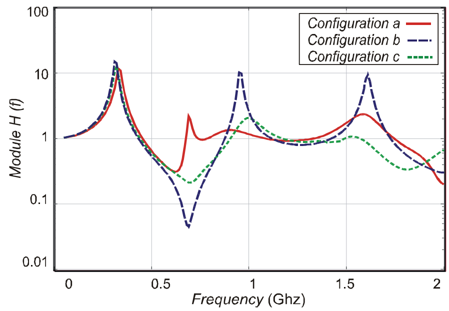
4 Numerical Results

4.1 Converter circuit

Figs. 6*a* and 6*b*, shows the transfer function $H(f)$ in the frequency range from 10MHz to 2GHz obtained for all three geometries with respect to the position of the source: 1 and 2 (opposite to the load and in the middle of length *L*, as given on Fig. 4). The curves in red, green and blue correspond to the three circuit-loop geometries considered: (*a*) rectangle loop (4×16cm); (*b*) square loop (10×10cm), and (*c*) rectangle loop (16×4cm).



(a) Case 1: *Source opposite to the load.*



(b) Case 2: *Source in the middle of length L.*

Fig. 6 – $H(f)$ *in frequency range from* 10MHz *to* 2GHz.

After transforming $H(f)$ in time we obtained samples of $h(kT_S)$, $k = 0,\ldots,N-1$, which were subject to the pencil method analysis. The results obtained which represent complex values of z_i are given in **Tables 1-a** and **1-b**.

Table 1-a

Values of α *and f in case of converter, source opposite to the load.*

Case 1	α in Np/ns	$f = \omega/(2\pi)$ in GHz
Cir. a	0.08 0.81 0.98	0.33 0.89 1.62
Cir. b	0.07 0.37 0.38	0.32 0.95 1.58
Cir. c	0.05 0.05 0.06	0.31 0.98 1.60

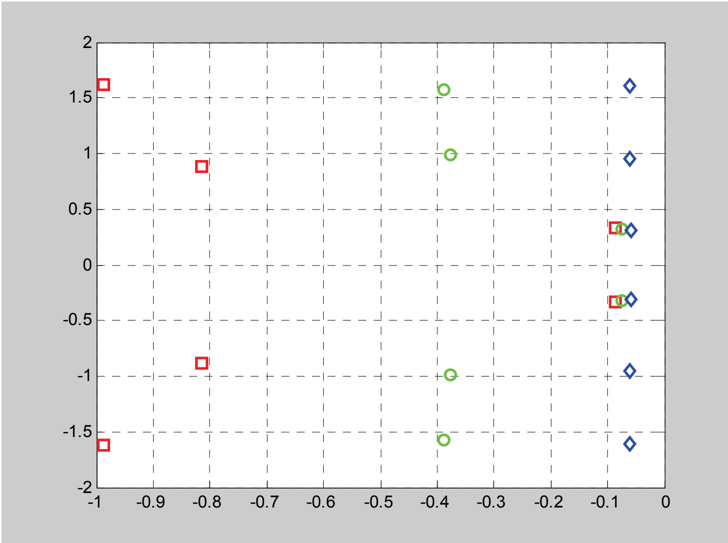
Table 1-b

Values of α *and f in case of converter, source in the middle of length L.*

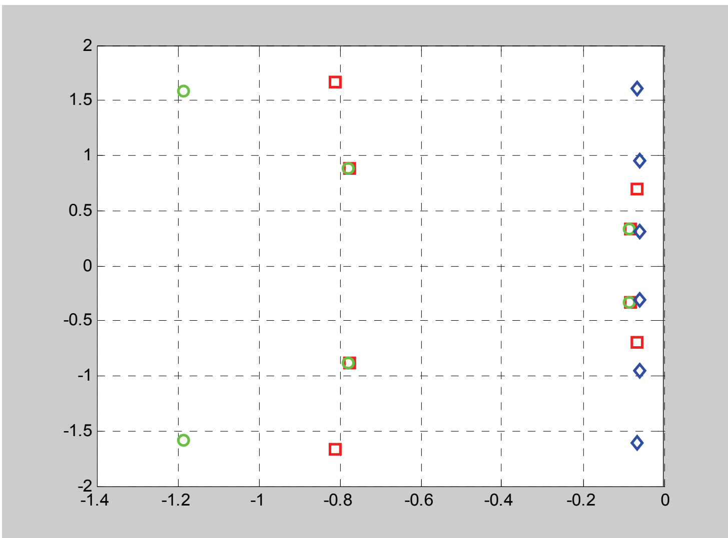
Case 1	α in Np/ns	$f = \omega/(2\pi)$ in GHz
Cir. a	0.08 0.07 0.77 0.81	0.33 0.69 0.88 1.66
Cir. b	0.08 0.78 1.18	0.33 0.88 1.58
Cir. c	0.06 0.06 0.07	0.31 0.95 1.6

Figs. 7*a* and 7*b*, gives a visual presentation of the results obtained using the pencil method.

Comparing Figs. 6 and 7 it may be observed that imaginary values of z_i in GHz (on the ordinate) correspond to pick frequencies of $H(f)$. Respectively, real values of z_i (on the abscissa) correspond to the damping factors.



(a) poles z_i of $h(t)$: Case 1- *Source opposite to the load.*



(b) poles z_i of $h(t)$: Case 2: *Source in the middle of length L.*

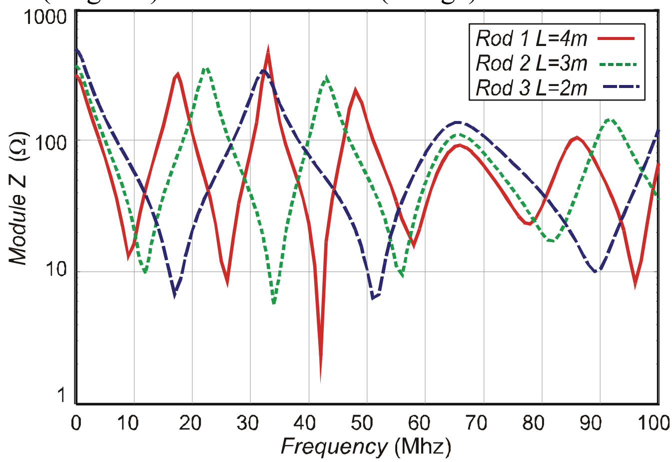
Fig. 7 – z_i *poles of* $h(t)$: *values on the ordinate – frequency in* GHz, *values on the abscissa – attenuation in* Np/ns.

It is shown that not only the geometry of the circuit but also the position of the source (switch with respect to the load) influence the transient behaviour of the converter circuit.

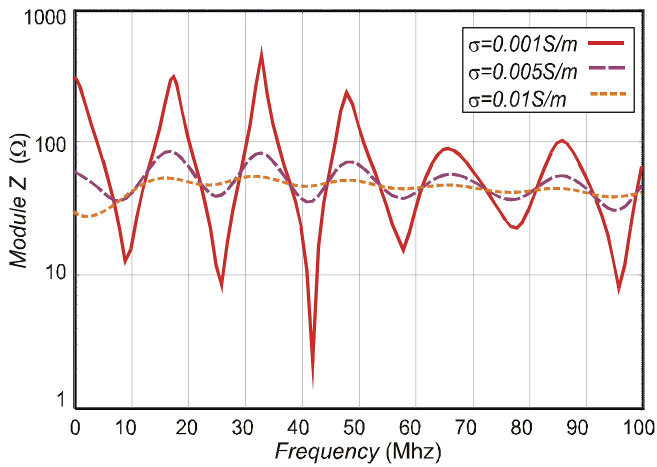
4.2 Grounding rod

Figs. 8*a* and 8*b*, represents the grounding impedance $Z_g(f)$ in the frequency range from 1kHz to 100MHz.

On Fig. 8a, the curves in red, green and blue correspond to the rod lengths considered: $L = 4\,\text{m}$, $L = 3\,\text{m}$ and $L = 2\,\text{m}$. The depth is $d = 0.5\,\text{m}$. Fig. 8-b shows the impedance $Z_g(f)$ in the case of a grounding rod of $L = 4\,\text{m}$ when placed in homogeneous soil with various conductivity: $\sigma = 0.001\,\text{S/m}$ (red), $\sigma = 0.005\,\text{S/m}$ (magenta) and $\sigma = 0.01\,\text{S/m}$ (orange).

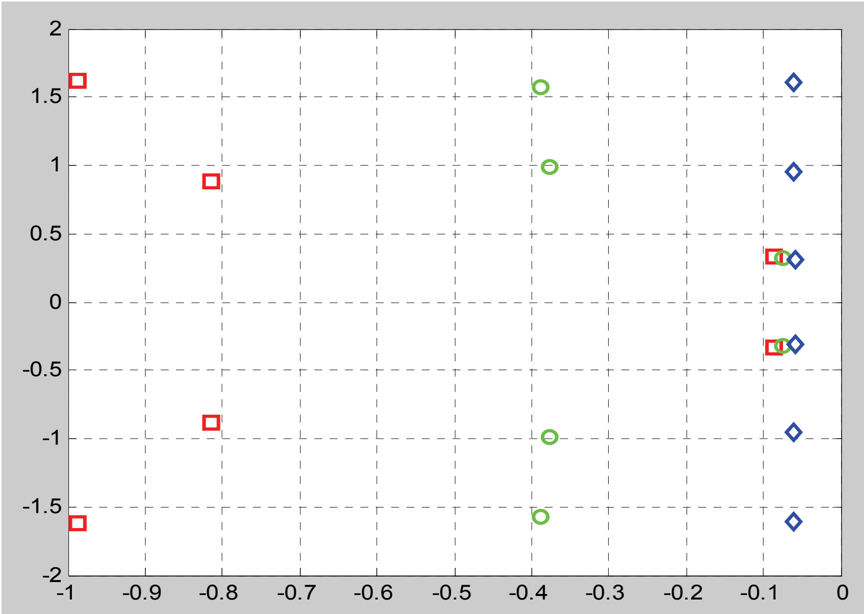


(a) Rod of $L = 4\,\text{m}$, $L = 3\,\text{m}$ and $L = 2\,\text{m}$ in soil with $\sigma = 0.001\,(S/m)$.

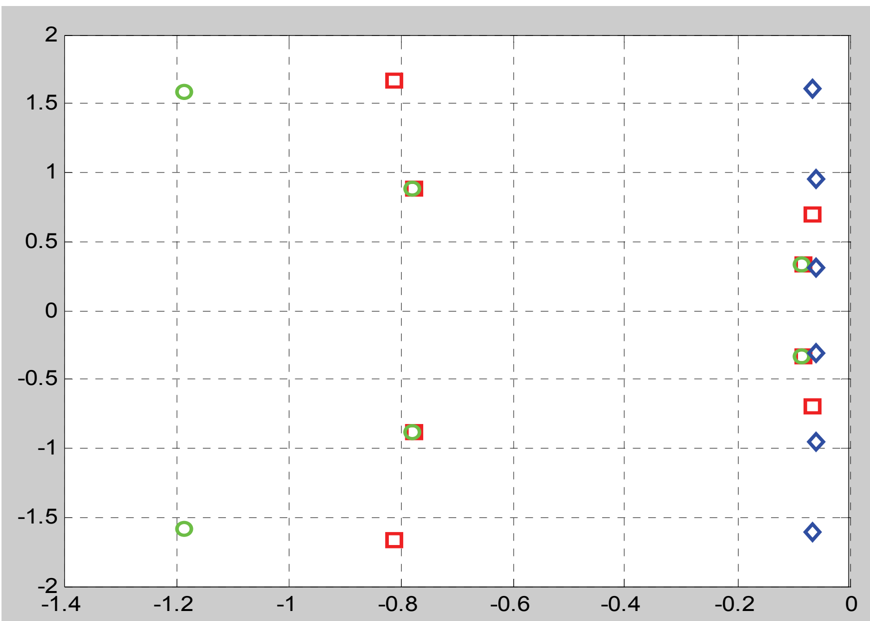


(b) Rod of $L = 4\,\text{m}$ in soil with $\sigma = 0.001, 0.005$ and $0.01\,(S/m)$.

Fig. 8 – $Z_g(f)$ *in the frequency range from* 1kHz *to* 100MHz.



(a) poles z_i of $z_g(t)$: *Various lengths of the grounding rod.*



(b) poles z_i of $z_g(t)$: *Various conductivity of the soil.*

Fig. 9 – z_i *poles of* $z_g(t)$: *values on the ordinate – frequency in* MHz, *values on the abscissa – attenuation in* Np/μs.

The corresponding values of z_i poles obtained using the pencil method are given in **Tables 2-a** and **2-b**. The results are visualized on Figs. 9a and 9b. The influence of the rod length as well as the influence of the soil conductivity may also be observed. Real values of z_i (on the abscissa) correspond to attenuation factors.

Table 2-a

Values of α *and f in the case of a converter, where the source is opposite to the load.*

Rod L(m)	α in Np/μs	$f = \omega/(2\pi)$ in MHz
4	7 5 8 15 13	17.5 32.5 48.2 65.6 87.9
3	10 8 18 22	22.2 42.8 63.6 89.6
2	11 22 82	32.1 62.8 104.3

Table 2-b

Values of α *and f in the case of a converter, the source is in the middle of length L.*

σ(S/m)	Rod $L = 4$m	Rod $L = 4$m
0.005	25 23 24 31 25	16.9 32.5 47.5 64.1 84.7
0.01	50 46 43 45 35	14.6 30.9 46.5 64.7 86.7

5 Conclusion

In this paper, we have applied the 'matrix pencil' method to determine the poles of specific frequency-domain functions that represent the transient behaviour of two distinct thin-wire objects. The results obtained in this way were compared with those obtained in the frequency domain using a complete electromagnetic approach. The main objective of this work was to present the possibility of parameter extraction that would enable determination of an equivalent circuit model of such wire-structure objects.

6 References

[1] L. Grcev, V. Arnautovski-Toseva: Grounding Systems Modeling for High Frequencies and Transients: Some Fundamental Considerations, in Proc. IEEE Bologna Power Tech., Bologna, Italy, June 23-26, 2003.

[2] V. Arnautovski-Toseva, Y. Rousset, K. El Khamlichi Drissi, L. Grcev: Radiated EMI from Power Converters, Serbian Journal of Electrical Engineering, Vol. 2, No. 2, Nov. 2005, pp. 117-124.

[3] T.K. Sarkar, O. Pereira: Using the Matrix Pencil Method to Estimate the Parameters of Sum of Complex Exponentials, IEEE Antennas and Propagation Magazine, Vol. 37, No. 1, Feb. 1995, pp. 48-55.