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Design of Allpass-Based IIR Multi-Notch Filters with Identical Pole Radiuses

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Abstract: A design method for the allpass-based infinite impulse response multi-notch filters with identical pole radiuses is derived in the paper. According to the standard procedure when design of the allpass-based filters is considered, the multi-notch filters' magnitude response specifications are first formulated as the phase response specifications of the corresponding allpass filter. Then, for the specified identical pole radiuses, it is shown that unknown allpass filter coefficients can be obtained determined from the square system of linear equations. On the other hand, the minimum value of the pole radius, such that specifications of the the magnitude response are satisfied in all passbands, can be determined using the bisection method. Results of comparison with some of the existing design methods lead to the conclusion that proposed filters have higher area under the squared passbands magnitude response compared to filters with the same maximum pole radius. Furthermore, utilization of the proposed method can result in transfer functions that have the lowest possible maximum pole radius.

Keywords: Infinite impulse response multi-notch filter, Digital allpass filter, Magnitude response, Identical pole radiuses.

1 Introduction

Digital multi-notch filters are used in various applications [1 – 4] to remove sinusoidal interferences while passing the other input signal's spectral components relatively unchanged. Theoretically, an ideal multi-notch filter is characterized in Fourier domain by its magnitude response expressed as

$$\left|H_d(\mathrm{e}^{\mathrm{j}\omega})\right| = \begin{cases} 0, & \omega \in \left\{\omega_{n,k} \,|\, k = 1, 2, \ldots, K\right\}, \\ 1, & \text{otherwise}, \end{cases} \tag{1}$$

where $\omega_{n,k}$ for $k = 1, 2, \ldots, K$, are notch frequencies. However, zero notch-bandwidths of an ideal multi-notch filter are not practically realizable and would lead to infinite transient response duration to the sinusoidal interferences of frequencies equal to notch ones [3, 5, 6]. Therefore, non-zero notch-bandwidths,

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$\Delta\omega_k$, for $k = 1, 2, \ldots, K$, defined at attenuation level of $-20\log_{10}(\sqrt{2}/2) \approx 3$ dB, are also specified for the real multi-notch filter. Practically, multi-notch filter is acceptable if

$$\left|H\left(\mathrm{e}^{\mathrm{j}\omega}\right)\right| = 0, \quad \omega \in \left\{\omega_{n,k} | k = 1, 2, \ldots, K\right\}, \tag{2}$$

$$\left|H(\mathrm{e}^{\mathrm{j}\omega})\right| \geq \frac{\sqrt{2}}{2}, \quad \omega \in \mathcal{P} = \bigcup_{k=1}^{K} \mathcal{P}_k, \tag{3}$$

where $\mathcal{P}_k$ denotes set of frequencies in k-th passband

$$\mathcal{P}_k = \begin{cases} \left\{\omega | 0 \leq \omega \leq \omega_{l,1}\right\}, & k = 1, \\ \left\{\omega | \omega_{r,k-1} \leq \omega \leq \omega_{l,k}\right\}, & k = 2, \ldots, K, \\ \left\{\omega | \omega_{r,K} \leq \omega \leq \pi\right\}, & k = K+1, \end{cases} \tag{4}$$

where $\omega_{l,k}$ and $\omega_{r,k}$ are k-th left-hand and right-hand cutoff frequencies

$$\omega_{l,k} = \omega_{n,k} - \frac{\Delta\omega_k}{2}, \quad \omega_{r,k} = \omega_{n,k} + \frac{\Delta\omega_k}{2}, \tag{5}$$

respectively, for $k = 1, 2, \ldots, K$.

Digital multi-notch filters can be designed as finite or infinite impulse response (IIR) filters. If passbands phase response linearity is not of crucial importance, IIR multi-notch filters are preferred due to significantly lower computational complexity. There are several approaches to the IIR multi-notch filter design.

The conventional approach is to cascade several single-notch IIR filters [7, 8]. While the pole radius of notch filter can be easily determined from the specified notch-bandwidth, multi-notch filters obtained using cascading approach suffer from the uncontrollable gain between notch frequencies if they are not sufficiently separated and notch-bandwidths are not narrow [9, 10].

Optimum poles placement methods [4, 11 – 13] formulate the design problem as optimization problem in locations of unknown poles since locations of zeros are defined by the notch frequencies. Mentioned minimization problem is then solved either by the iterative Steiglitz-McBride scheme [14] or by using metaheuristic algorithms. The main disadvantage of design methods of this approach, beside the high computational complexity, is that obtained magnitude responses do not necessarily have the same extremal values in all passbands.

Finally, methods of the allpass-based approach [10, 15 – 19] are based on observation that transfer function of multi-notch filter can be expressed as $H(z) = 0.5(1 + A(z))$, where $A(z)$ is a stable allpass filter. This configuration allows the transformation of the multi-notch filter's magnitude response specifications into specifications of the allpass filter’s phase response. Then, a simple relation between the allpass filter coefficients and its phase response allows the transformed specifications to be expressed as linear constraints. Practical success of methods of this approach lies in their simplicity, as unknown coefficients are determined by solving a system of linear equations. Furthermore, allpass-based multi-notch filters can be realized by low-sensitive lattice structure [8, 10, 16].

The case when the allpass filter (characterizing the multi-notch filter) has identical pole radiuses is considered in this paper. It should be noted that non-adaptive design presented in [19] also assumes identical pole radiuses, however, values of pole radiuses are determined from the specified identical notch-bandwidths as $r = \sqrt{(1-\sin\Delta\omega)/\cos\Delta\omega}$, which do not guarantee the satisfaction of multi-notch filter's magnitude response specifications in passbands. On the other hand, the proposed design method does provide means for calculation of the minimum value of pole radiuses such that specifications in passbands are satisfied.

The rest of the paper is structured as follows. In Section 2, design problem of the allpass-based IIR multi-notch filter with identical pole radiuses is formulated, while the proposed method is presented in Section 3. Design examples and comparison with the existing methods are given in Section 4. Finally, remarks conclusions are drawn in Section 5.

2 Problem Formulation

Transfer function of considered allpass-based IIR multi-notch filter with K notch frequencies can be expressed as

$$H(z) = \frac{1 + A(z)}{2}, \tag{6}$$

where $A(z)$ is allpass filter transfer function of order $2K$,

$$A(z) = z^{-2K} \frac{1 + \sum_{k=1}^{2K} a_k z^k}{1 + \sum_{k=1}^{2K} a_k z^{-k}}, \tag{7}$$

with all poles assumed to have the same radius r, i.e., $a_{2K} = r^{2K}$. Obviously $r < 1$ to ensure stability.

Denoting the allpass filter’s phase response by $\theta(\omega)$,

$$\theta(\omega) = -2K\omega + 2\arctan\frac{\sum_{k=1}^{2K} a_k \sin(k\omega)}{1+\sum_{k=1}^{2K} a_k \cos(k\omega)}\,, \tag{8}$$

magnitude response of the corresponding multi-notch filter can be formulated as

$$\left|H(\mathrm{e}^{\mathrm{j}\omega})\right| = \left|\cos\frac{\theta(\omega)}{2}\right|. \tag{9}$$

Now, since $\theta(\omega)$ is monotonically decreasing function for $\omega \in (0,\ \pi)$, while $\theta(0) = 0$ and $\theta(\pi) = -2K\pi$ [8, 15], magnitude response specifications given by (2) and (3) are satisfied if

$$\theta(\omega_{n,k}) = -(2k-1)\pi\,, \tag{10}$$

$$-2(k-1)\pi - \pi/2 \le \theta(\omega_{l,k}) < -2(k-1)\pi\,, \tag{11}$$

$$-2k\pi < \theta(\omega_{r,k}) \le -2k\pi + \pi/2\,, \tag{12}$$

for $k = 1,\ 2,\ \ldots,\ K$.

Hence,

$$\cos\frac{\theta(\omega_{n,k})}{2} = 0\,, \tag{13}$$

$$(-1)^{k-1}\cos\frac{\theta(\omega_{l,k})}{2} \ge \frac{\sqrt{2}}{2}\,, \tag{14}$$

$$(-1)^{k}\cos\frac{\theta(\omega_{r,k})}{2} \ge \frac{\sqrt{2}}{2}\,, \tag{15}$$

and

$$(-1)^{k}\sin\frac{\theta(\omega_{l,k})}{2} > 0\,, \tag{16}$$

$$(-1)^{k}\sin\frac{\theta(\omega_{r,k})}{2} > 0\,, \tag{17}$$

for $k = 1,\ 2,\ \ldots,\ K$.

Therefore, IIR multi-notch filter design problem reduces to determination of allpass filter coefficients such that (13) – (17) are satisfied.

3 Design Method

Having in mind that cosine and sine of half of the phase response of the allpass filter with identical pole radiuses can be expressed as

$$\cos\frac{\theta(\omega)}{2} = \frac{R(\omega)}{\sqrt{R^2(\omega)+Q^2(\omega)}}\,, \tag{18}$$

$$\sin\frac{\theta(\omega)}{2} = \frac{Q(\omega)}{\sqrt{R^2(\omega)+Q^2(\omega)}}\,, \tag{19}$$

where

$$R(\omega) = \cos(K\omega) + \frac{a_K}{1+r^{2K}} + \sum_{i=1}^{K-1}\frac{a_{K+i}+a_{K-i}}{1+r^{2K}}\cos(i\omega)\,, \tag{20}$$

$$Q(\omega) = \frac{1-r^{-2K}}{1+r^{-2K}}\sin(K\omega) + \sum_{i=1}^{K-1}\frac{a_{K+i}-a_{K-i}}{1+r^{2K}}\sin(i\omega)\,, \tag{21}$$

utilization of (16) and (17), along with the following relation between coefficients a_{K+i} and a_{K-i} [19],

$$a_{K-i} = r^{-2i}a_{K+i}, \quad i = 1, 2, \ldots, K-1\,, \tag{22}$$

yields the following formulation of (13) – (17),

$$\boldsymbol{\Phi}_N \cdot \boldsymbol{u} = \boldsymbol{\gamma}_N\,, \tag{23}$$

$$\begin{bmatrix}\boldsymbol{\gamma}_L\\ \boldsymbol{\gamma}_R\end{bmatrix} \le \begin{bmatrix}\boldsymbol{\Phi}_L(r)\\ \boldsymbol{\Phi}_R(r)\end{bmatrix}\cdot\begin{bmatrix}\tilde{\boldsymbol{u}}\\ 1\end{bmatrix} < 0\,, \tag{24}$$

where: $\boldsymbol{u} = [u_i]$ is $K\times 1$ vector with elements,

$$u_i = \begin{cases} \dfrac{a_K}{1+r^{2K}}, & i = 1,\\[2ex] \dfrac{1+r^{-2(i-1)}}{1+r^{2K}}a_{K+i-1}, & i = 2, 3, \ldots, K, \end{cases} \tag{25}$$

$\tilde{\boldsymbol{u}} = [\tilde{u}_i = u_{i+1}]$ is $(K-1)\times 1$ vector, $\boldsymbol{\Phi}_N = [\phi_{ki}^{(N)}]$, $\boldsymbol{\Phi}_L(r) = [\phi_{ki}^{(L)}(r)]$ and $\boldsymbol{\Phi}_R(r) = [\phi_{ki}^{(R)}(r)]$ are $K\times K$ matrices with elements

$$\phi_{ki}^{(N)} = \cos\big((i-1)\omega_{n,k}\big)\,, \tag{26}$$

$$\phi_{ki}^{(L)}(r) = (-1)^{k-1}\frac{1-r^{-2i}}{1+r^{-2i}}\sin(i\omega_{l,k})\,, \tag{27}$$

$$\phi_{ki}^{(R)}(r) = (-1)^{k-1}\frac{1-r^{-2i}}{1+r^{-2i}}\sin(i\omega_{r,k})\,, \tag{28}$$

while $\boldsymbol{\gamma}_N = [\gamma_k^{(N)}]$, $\boldsymbol{\gamma}_L = [\gamma_k^{(L)}]$ and $\boldsymbol{\gamma}_R = [\gamma_k^{(R)}]$ are $K \times 1$ vectors with elements

$$\gamma_k^{(N)} = -\cos(K\omega_{n,k}) \,, \tag{29}$$

$$\gamma_k^{(L)} = (-1)^k \left[\cos(K\omega_{l,k}) + \sum_{i=1}^{K} u_i \cos\Big((i-1)\omega_{l,k}\Big) \right], \tag{30}$$

$$\gamma_k^{(R)} = (-1)^{k-1} \left[\cos(K\omega_{r,k}) + \sum_{i=1}^{K} u_i \cos\Big((i-1)\omega_{r,k}\Big) \right]. \tag{31}$$

Since inequality given by (24) is satisfied for r close to 1 and not satisfied for $r = 0$, it is reasonable to assume the existence of some r', $0 < r' < 1$, such that (24) is satisfied for every $r \geq r'$. Such r' can be obtained using the bisection method, and once determined, unknown coefficients of the allpass filter can be calculated using (25) and (22) for $r \geq r'$.

4 Design Examples

In this section comparison of filters obtained by the proposed method with filters obtained by methods from [10, 15 – 19] is performed in terms of maximum pole radius and the area under the squared passbands magnitude response

$$A_p = \int_{\omega \in \mathcal{P}} \left| H(\mathrm{e}^{\mathrm{j}\omega}) \right|^2 \mathrm{d}\,\omega \,. \tag{32}$$

Note that *Method I* from [10] is in fact the method discussed in [15, 16] with overcame limitations in regard to the tangent operation.

Utilization of mentioned design methods guarantees the exact satisfaction of notch frequencies positions. While magnitude response of filter designed using *Method I* from [10] satisfies $\left| H(\mathrm{e}^{\mathrm{j}\omega_{l,k}}) \right| = \sqrt{2}/2$ for $k = 1, 2, \ldots, K$, utilization of *Method II* from [10] results in transfer function satisfying $\left| H(\mathrm{e}^{\mathrm{j}\omega_{r,k}}) \right| = \sqrt{2}/2$ for $k = 1, 2, \ldots, K$. On the other hand, the method presented in [17] approximately satisfies both left-hand and right-hand cutoff frequencies positions, while utilization of *Method C-M* from [18] minimizes the maximum pole radius. It should be noted that utilization of methods from [10, 17, 19] does not guarantee the satisfaction of (3), i.e., magnitude response specifications may not be satisfied in all passbands.

IIR multi-notch filter with the following specifications: $\omega_{n,1} = 0.5\pi$, $\omega_{n,2} = 0.65\pi$, $\Delta\omega_1 = \Delta\omega_2 = 0.1\pi$, is considered in the first example. Poles locations and the areas under the squared passbands magnitude responses of filters obtained by the proposed and existing design methods from [10, 17 – 19] are given in second and third column of **Table 1**.

Table 1

Examples 1 and 2: Poles and areas under the passbands squared magnitude responses of obtained multi-notch filters.

	1st example		2nd example	
Method	Poles	A_p	Poles	A_p
Proposed 1, $r = r'$	$0.8802 \cdot e^{\pm j0.6403\pi}$ $0.8802 \cdot e^{\pm j0.5122\pi}$	2.3155	$0.9242 \cdot e^{\pm j0.7990\pi}$ $0.9242 \cdot e^{\pm j0.3947\pi}$ $0.9242 \cdot e^{\pm j0.1029\pi}$ $0.9242 \cdot e^{\pm j0.1934\pi}$	2.1822
Proposed 2	$0.9114 \cdot e^{\pm j0.6451\pi}$ $0.9114 \cdot e^{\pm j0.5062\pi}$	2.3938	$0.9724 \cdot e^{\pm j0.7999\pi}$ $0.9724 \cdot e^{\pm j0.3993\pi}$ $0.9724 \cdot e^{\pm j0.1003\pi}$ $0.9724 \cdot e^{\pm j0.1992\pi}$	2.3532
Method I [10]	$0.9114 \cdot e^{\pm j0.5159\pi}$ $0.7973 \cdot e^{\pm j0.6394\pi}$	2.2253	$0.8977 \cdot e^{\pm j0.7972\pi}$ $0.8807 \cdot e^{\pm j0.3846\pi}$ $0.9046 \cdot e^{\pm j0.1110\pi}$ $0.8511 \cdot e^{\pm j0.1811\pi}$	2.0097
Method II [10]	$0.8987 \cdot e^{\pm j0.6328\pi}$ $0.7857 \cdot e^{\pm j0.5205\pi}$	2.1923	$0.9039 \cdot e^{\pm j0.7988\pi}$ $0.9275 \cdot e^{\pm j0.3940\pi}$ $0.9372 \cdot e^{\pm j0.1902\pi}$ $0.8767 \cdot e^{\pm j0.1005\pi}$	2.1255
[19]	$0.8524 \cdot e^{\pm j0.6339\pi}$ $0.8525 \cdot e^{\pm j0.5201\pi}$	2.2414	$0.9095 \cdot e^{\pm j0.7985\pi}$ $0.9095 \cdot e^{\pm j0.3923\pi}$ $0.9095 \cdot e^{\pm j0.1903\pi}$ $0.9095 \cdot e^{\pm j0.1044\pi}$	2.1210
[17]	$0.8608 \cdot e^{\pm j0.6375\pi}$ $0.8714 \cdot e^{\pm j0.5158\pi}$	2.2781	$0.9094 \cdot e^{\pm j0.7996\pi}$ $0.9095 \cdot e^{\pm j0.3976\pi}$ $0.9462 \cdot e^{\pm j0.1999\pi}$ $0.9724 \cdot e^{\pm j0.1012\pi}$	2.1920
Method C-M [18]	$0.8802 \cdot e^{\pm j0.6403\pi}$ $0.8802 \cdot e^{\pm j0.5122\pi}$	2.3155	$0.9088 \cdot e^{\pm j0.7990\pi}$ $0.9235 \cdot e^{\pm j0.3947\pi}$ $0.9180 \cdot e^{\pm j0.1927\pi}$ $0.9235 \cdot e^{\pm j0.1034\pi}$	2.1570

From **Table 1** it can be observed that two filters are designed using the proposed method: first with the pole radiuses equal to r', and second with the pole radiuses equal to the maximum pole radius of filter obtained using *Method I* [10] (utilization of this method results in the highest pole radius in this example). The first proposed multi-notch filter exhibits higher A_p compared to all filters except the second proposed one. Additionally, the first proposed filter is the same as the one obtained by utilization of *Method C-M* [18]. From **Table 1** it can be also noted that pole radiuses of filter obtained by method [19] are lower compared to those of the first proposed filter, meaning that specifications of the passbands magnitude responses are not satisfied by method [19]. Passbands magnitude responses of the second proposed multi-notch filter, as well as of filters designed using *Method II* [10] and method from [19], are shown in Fig. 1.

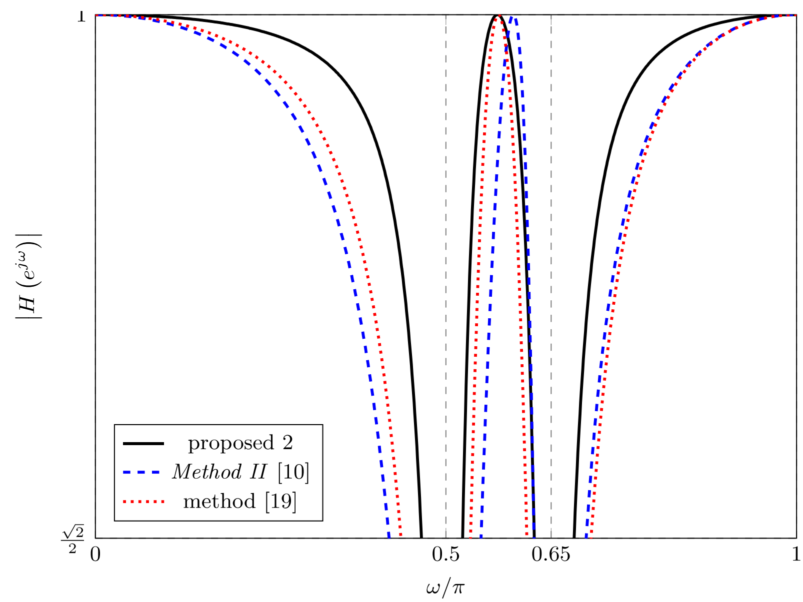

Fig. 1 – *Example 1. Magnitude responses of filters obtained using proposed (dashed line), Method II* [10] *(solid line) and method from* [19] *(dotted line).*

Specifications of the multi-notch filter, considered in the second example are: $\Delta\omega_1 = \Delta\omega_2 = \Delta\omega_3 = \Delta\omega_4 = 0.06\pi$, $\omega_{n,1} = 0.1\pi$, $\omega_{n,2} = 0.2\pi$, $\omega_{n,3} = 0.4\pi$ and $\omega_{n,4} = 0.8\pi$. Poles and the areas under the squared passbands magnitude responses of filters obtained using proposed and existing design methods from [10, 17 – 19] are given in last two columns of **Table 1**. As obvious from **Table 1**, two filters are designed by the proposed method: first with the pole radiuses equal to r', and second with the pole radiuses equal to the maximum pole radius of filter obtained using design method from [17] (utilization of this method results in the highest pole radius in this example). The first proposed multi-notch filter exhibits higher A_p compared to all filters except the second proposed one and the one designed using method [17]. In this example, however, utilization of *Method C-M* [18] results in slightly lower maximum pole radius compared to the proposed method (0.9235 compared to 0.9242). It can be observed that pole radiuses of filter obtained by method [19] are lower compared to those of the first proposed filter, meaning that specifications of the passbands magnitude responses are not satisfied by method [19]. Passbands magnitude responses of the first proposed filter, as well as of filters designed using *Method I* [10] and method from [17], are shown in Fig. 2.

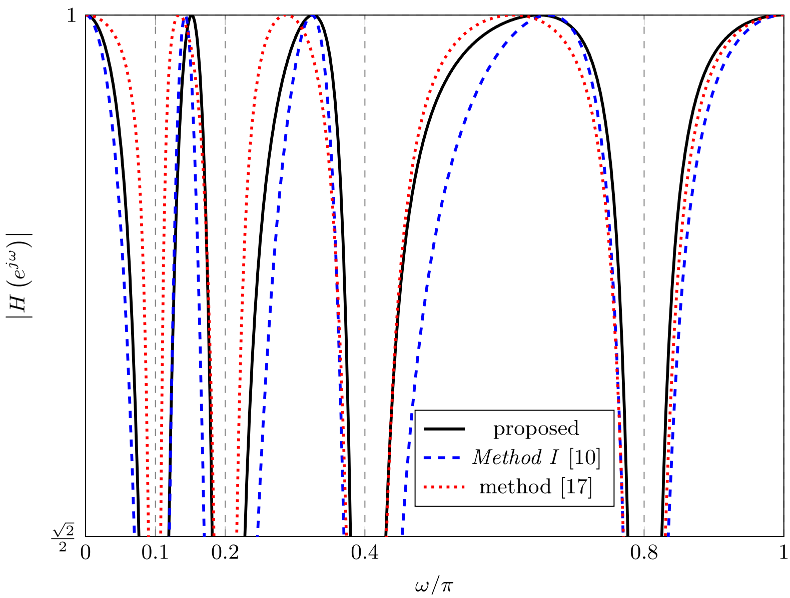

Fig. 2 – *Example 2. Passbands magnitude responses of filters obtained using proposed (dashed line), Method I* [10] *(solid line) and method from* [17] *(dotted line).*

5 Conclusion

A computationally efficient method for design of the allpass-based IIR multi-notch filter with identical pole radiuses is proposed in the paper. After the transformation of the multi-notch filter magnitude response specifications into those of the corresponding allpass filter phase response, it is shown that unknown filter coefficients can be obtained from a square system of linear equations, assuming the known value of the identical pole radiuses. On the other hand, minimum value of identical pole radiuses can be easily determined using the bisection method. Results of comparison with the existing methods reveal that filters with the highest stability margins (i.e., the lowest possible pole radiuses) can be obtained by the proposed design method. Furthermore, compared to the filters with the same maximum pole radius, proposed filters have higher area under the squared passbands magnitude responses.

7 References

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